# Sidewalk Stones

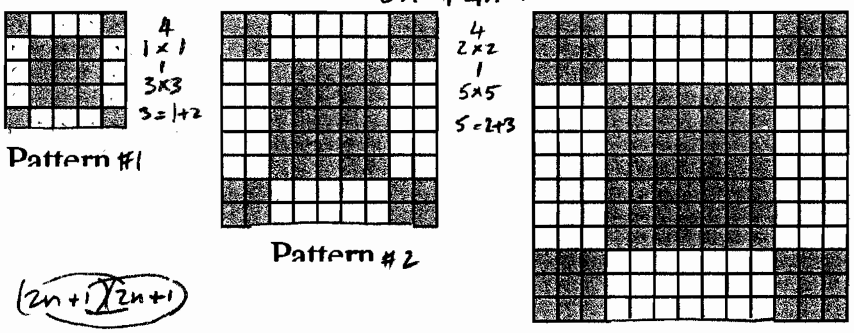
**T1**

In Prague some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns that are in various sizes.

GREY $4(n^2) + (2n+1)^2 = 4n^2 + 4n^2 + 4n + 1$
$= 8n^2 + 4n + 1$

WHITE
$4(n)(n+n+1)$
$4n(2n+1)$
$8n^2 + 4n$




Pattern #1

Pattern #2

Pattern #3

4
3×3
1
7×7
7=3+4

1 → 9
2 → 25
3 → 49

$(2n+1)(2n+1)$

How many blocks of each kind will pattern #*n* need? GREY $= 4n^2 + (2n+1)^2 = 8n^2 + 4n + 1$ ✓ 3
white $= 4n(2n+1) = 8n^2 + 4n$ ✓ 2

Which pattern has a total of 841 grey blocks? 10 ✓ 2

$n = 10$ white $= 8(10)^2 + 4(10)$
$= 800 + 40$
$= 840$

How many white blocks has that pattern? ✓ 1

$841 - 1 = 840$

840 white blocks

Explain your work and show your calculations.

Taking into account the # of white and grey blocks in the diagrams above, as well as the pattern numbers of the diagrams, I used inductive reasoning to create the formula grey $= 4(n^2) + (n+n+1)^2 = 8n + 4n + 1$ and white $= 4(n)(n+n+1) = 8n^2 + 4n$ where $n =$ pattern number. I then substituted 841 for grey and isolated n using factoring (shown) on next page. The pattern number is 10. Since the number of grey blocks is 1 more than the number of white blocks in a pattern. If pattern #10 has 841 grey blocks, pattern #10 also has 840 white blocks.

**Please continue your work on the page opposite**

T1

## Sidewalk Stones (continued)

WHITE

$4(n)(n+n+1)$

$4n(2n+1)$

GREY

$4(n^2)+(n+n+1)^2$

$4n^2+(2n+1)^2$

$4n^2+4n^2+4n+1$

$8n^2+4n+1$

GREY − WHITE = difference.

$(8n^2+4n+1)-(8n^2+4n)=1$

841 GREY

$841 = 8n^2+4n+1$
$-841 \quad\quad -841$

$0 = 8n^2+4n-840$

$0 = 8(n^2+\frac{1}{2}n-105)$

$0 = 8(n+10)(n+10\frac{1}{2})$

$-13.125 \cdot 8 = -105$
$-17.5 \cdot 6 = -105$
$-10 \cdot 10.5 = -105$

$-105$
$-10 \quad 10.5$
$\frac{1}{2}$

Pattern #10 WHITE

$841-1 = 840$
grey

✓

2

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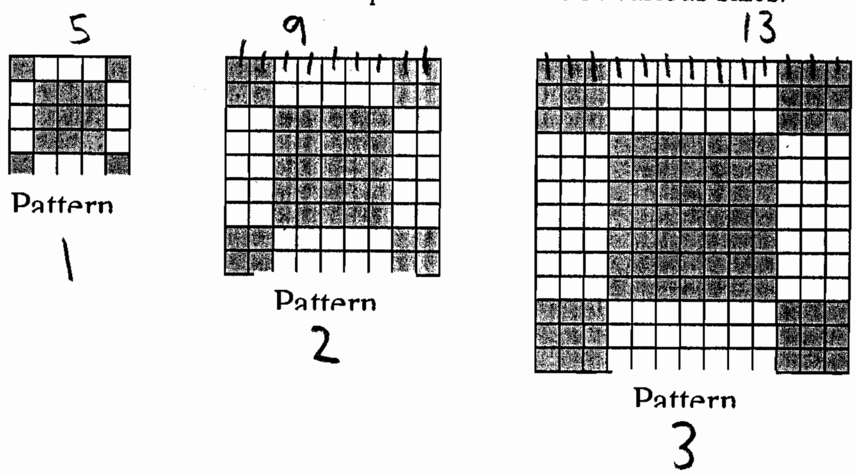



How many blocks of each kind will pattern #*n* need? 3

White : $8n^2+4n$ ✓ Grey: $8n^2+4n+1$ ✓ 2

Which pattern has a total of 841 grey blocks? Pattern 10 ✓ 2

How many white blocks has that pattern? 840 ✓ 1

Explain your work and show your calculations.

on page 9

**Please continue your work on the page opposite**

## Sidewalk Stones (continued)

$(4n+1)(4n+1)$

$16n^2+4n+4n+1$

$16n^2+8n+1$

$(2n+1)(2n+1)$

$4n^2+2n+2n+1$

$4n^2+4n+1$

$16n^2+8n\cancel{+1}-4n^2-4n^2-4n\cancel{-1}$

$8n^2+4n$

✓

$4n^2+4n^2+4n+1$

(1)

$841=8n^2+4n+1$

$n=10$







9

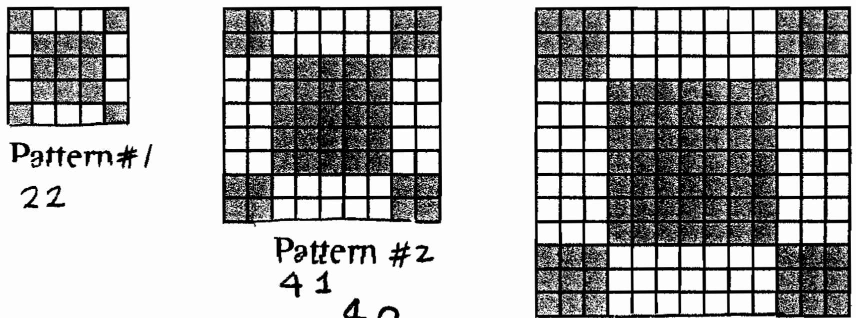
# Sidewalk Stones **T3**

In Prague some sidewalks are made of small square blocks of stone.

$4n^2+(1+2n)^2$

The blocks are in different shades to make patterns that are in various sizes.

3



Pattern #1
22

Pattern #2
41
40

Pattern #3 84
85

2

$n\cdot(2n+1)n^4\cdot 3n\cdot 4$

How many blocks of each kind will pattern #*n* need?

Which pattern has a total of 841 grey blocks? 10 ✓ 2

How many white blocks has that pattern? 840 ✓ 1

Explain your work and show your calculations.

$4n^2+(1+2n)^2$ = # of grey blocks

$n\cdot(2n+1)\cdot 4$ = # of white blocks ✓

$4(10)^2+(1+2(10))^2=841$

$10\cdot(2(10+1)\cdot 4=840$ ✓ 2

For the shaded $4n^2$ is the shaded corners The for corners

## Please continue your work on the page opposite

## Sidewalk Stones (continued)

are equal to the pattern # squared
The center shaded area is one greater than the pattern number multiplied by 2
The area of the white squares is in 4 areas.
The equation for one of the areas is $n \cdot (2n+1)$
$n$ = the height and $2n+1$ equals the length.
Multiply that by 4 because there are four of them.





-10

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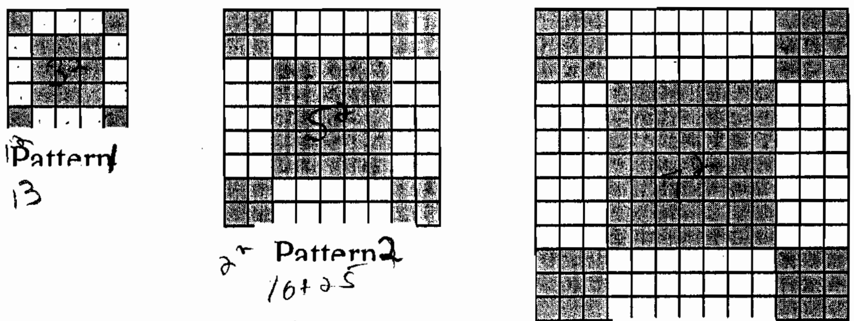

1n Pattern 1 13

$2^2$ Pattern 2 16+25 41

$3^2$ Pattern 3

How many blocks of each kind will pattern #*n* need? $4n^2+4n+2 \to (8n^2+4n+1)$ 3

black: $4n^2+(2n+1)^2 \to 4n^2+4n^2+4n+1$

white: $4(n \times (2n+1)) = 4(2n^2+n) = (8n^2+4n)$ × 2

Which pattern has a total of 841 grey blocks? 7thpattern 0

more work → on next page

$16n^2+8n+1 = 841$ × $16n^2+8n = 840$ $2n^2+n=105$ $2n^2+n-105=0$ $840 \div 8 = 105$

How many white blocks has that pattern? 420 × 0

$8(7)^2+4(7) = 8(49)+28 = 392+28 = 420$

Explain your work and show your calculations.

In the first Q, I realized that pattern # relates to the corner block. I saw that in pattern 1, the corner block was equal to pattern#, n, multiplied by itself. So, it was $1^2$ for 1, $2^2$ for 2, $3^2$ for 3, so forth. ~~take~~ There were four corners so I multiplied by four, $4n^2$. Then I saw that the center group of black blocks were ev... odd # squared except 1. So I quickly related the corner block to the center group. I saw that it was $(2n+1)^2$. (Continued →)

**Please continue your work on the page opposite**

T4

$$\begin{array}{r} \overset{4}{105} \\ \times\ 8 \\ \hline 840 \end{array}$$

2. $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\dfrac{-1 \pm \sqrt{1-4(2)(-105)}}{2(2)}$ $\dfrac{-1 \pm \sqrt{841}}{4}$

$\dfrac{-1 \pm 29}{4} = \dfrac{28}{4}$ or $\dfrac{-30}{4}$

$7$ or $\cancel{-7\frac{1}{2}}$

I added them together ~~was~~ which was $4n^2+(2n+1)^2$. For the White blocks, I saw that they formed 4 rectangles. I found that the width came from the pattern # and the length came from the sqrt of the # of black blocks in the center. Pattern # an is n, sqrt of # of black blocks in center is $2n+1$. There are 4 rectangles. So $4 \times ((2n+1) \times n)$ is # of white blocks. This simplifies to $8n^2+4n$.

2







7

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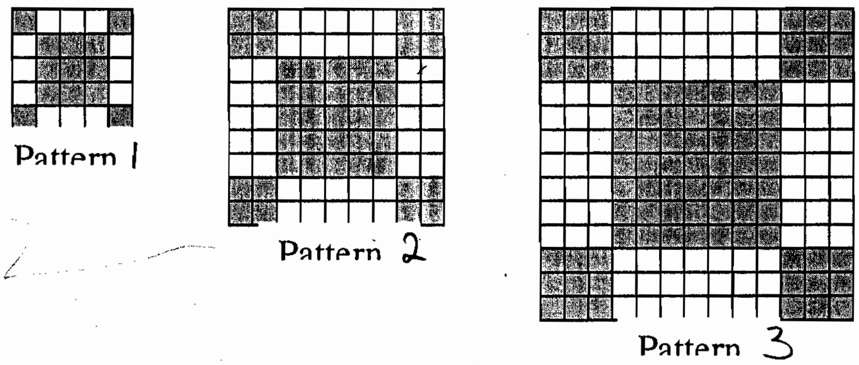


How many blocks of each kind will pattern #$n$ need? 3

grey = $4n^2 + (2n+1)^2$ ✓

white = $8n^2 + 4n$ ✓

Which pattern has a total of 841 grey blocks? ✓ # 10 2

How many white blocks has that pattern? ✓ 840 1

Explain your work and show your calculations.

(1) grey - $4n^2 + (2n+1)^2$

white - $8n^2 + 4n$

(2) $4n^2 + (2n+1)(2n+1) = 841$

$4n^2 + 4n^2 + 4n + 1 = 840$

$8n^2 + 4n - 840 = 0$ (continued on next page)

**Please continue your work on the page opposite**

## Sidewalk Stones (continued)

$2n^2 + n - 210 = 0$

$n = \frac{-1 \pm \sqrt{1+1680}}{4}$

$n = \frac{-1 \pm 41}{4}$

$n = 10$ or $\cancel{-\frac{42}{4}}$

$\boxed{n=10} \longrightarrow$ pattern $\boxed{\#10}$

(3) $8(10)^2 + 4(10)$

$800 + 40$

$\boxed{840}$

(1)