Sidewalk Stones

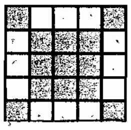
T1

In Prague some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns that are in various sizes.

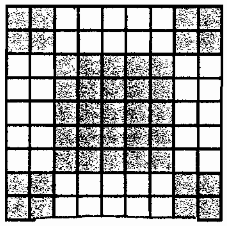
GREY $4(n^2) + (2n+1)^2 = 4n^2 + 4n^2 + 4n + 1$
$= 8n^2 + 4n + 1$

WHITE
$4(n)(n+n+1)$
$4n(2n+1)$
$8n^2 + 4n$



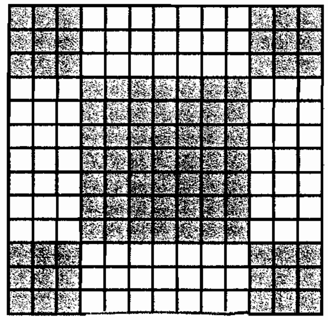
4
1×1
1
3×3
3 = 1+2

Pattern #1



4
2×2
1
5×5
5 = 2+3

Pattern #2



4
3×3
1
7×7
7 = 3+4

Pattern #3

1 → 9
2 → 25
3 → 49

$(2n+1)(2n+1)$

How many blocks of each kind will pattern #*n* need? Grey $= 4n^2 + (2n+1)^2 = 8n^2 + 4n + 1$
white $= 4n(2n+1) = 8n^2 + 4n$

Which pattern has a total of 841 grey blocks?

$n = 10$ white $= 8(10)^2 + 4(10)$
$= 800 + 40$
$= 840$

10

How many white blocks has that pattern?

$841 - 1 = 840$

840 white blocks

Explain your work and show your calculations.

Taking into account the # of white and grey blocks in the diagrams above, as well as the pattern numbers of the diagrams, I used inductive reasoning to create the formula grey $= 4(n^2) + (n+n+1)^2 = 8n + 4n + 1$ and white $= 4(n)(n+n+1) = 8n^2 + 4n$ where n = pattern number. I then substituted 841 for grey and isolated n using factoring (shown) on next page. The pattern number is 10. Since the number of grey blocks is 1 more than the number of white blocks in a pattern. If pattern #10 has 841 grey blocks, pattern #10 also has 840 white blocks.

Please continue your work on the page opposite

Sidewalk Stones (continued)

WHITE

$4(n)(n+n+1)$

$4n(2n+1)$

GREY

$4(n^2)+(n+n+1)^2$

$4n^2+(2n+1)^2$

$4n^2+4n^2+4n+1$

$8n^2+4n+1$

GREY − WHITE = difference.

$(8n^2+4n+1)-(8n^2+4n)=1$

841 GREY

$841 = 8n^2+4n+1$

$-841 \quad -841$

$0 = 8n^2+4n-840$

$0 = 8(n^2+\frac{1}{2}n-105)$

$0 = 8(n+10)(n+10\frac{1}{2})$

$-13.125 \cdot 8 = -105$

$-17.5 \cdot 6 = -105$

$-10 \cdot 10.5 = -105$

-105

$-10 \quad 10.5$

$\frac{1}{2}$

Pattern #10 WHITE

$841-1 = 840$

grey

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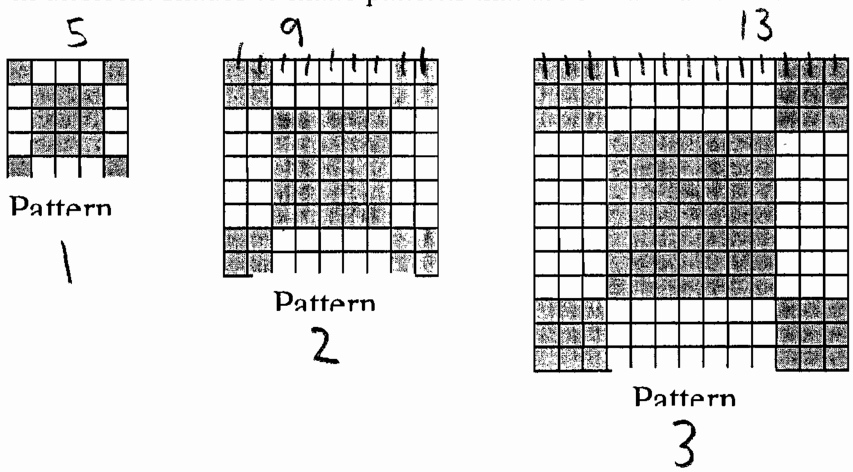


How many blocks of each kind will pattern #*n* need?

White : $8n^2+4n$ Grey: $8n^2+4n+1$

Which pattern has a total of 841 grey blocks? Pattern 10

How many white blocks has that pattern? 840

Explain your work and show your calculations.

on page 9

Please continue your work on the page opposite

T2

Sidewalk Stones (continued)

$(4n+1)(4n+1)$

$16n^2+4n+4n+1$

$16n^2+8n+1$

$(2n+1)(2n+1)$

$4n^2+2n+2n+1$

$4n^2+4n+1$

$16n^2+8n\cancel{+1}-4n^2-4n^2-4n\cancel{-1}$

$8n^2+4n$

$4n^2+4n^2+4n+1$

$841=8n^2+4n+1$

$n=10$

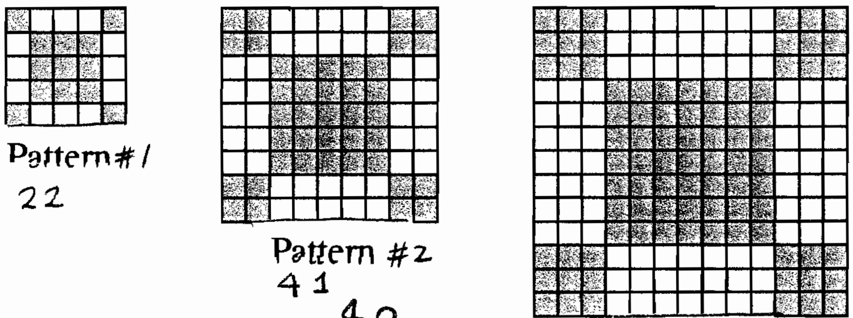
Sidewalk Stones

T3

In Prague some sidewalks are made of small square blocks of stone.

$4n^2 + (1+2n)^2$

The blocks are in different shades to make patterns that are in various sizes.



Pattern #1
22

Pattern #2
41
40

Pattern #3
85
84

$n \cdot (2n+1) n^4 \cdot 3n \cdot 4$

How many blocks of each kind will pattern #*n* need?

Which pattern has a total of 841 grey blocks? 10

How many white blocks has that pattern? 840

Explain your work and show your calculations.

$4n^2 + (1+2n)^2$ = # of grey blocks

$n \cdot (2n+1) \cdot 4$ = # of white blocks

$4(10)^2 + (1+2(10))^2 = 841$

$10 \cdot (2(10+1) \cdot 4 = 840$

For the shaded $4n^2$ is the shaded corners. The for corners

Please continue your work on the page opposite

Sidewalk Stones (continued)

are equal to the pattern # squared
The center shaded area is one greater than the pattern
number multiplied by 2
The area of the white squares is in 4 areas.
The equation for one of the areas is $n \cdot (2n+1)$
n = the height and $2n+1$ equals the length.
Multiply that by 4 because there are four of them.

Sidewalk Stones

T4

In Prague some sidewalks are made of small square blocks of stone.

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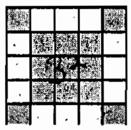
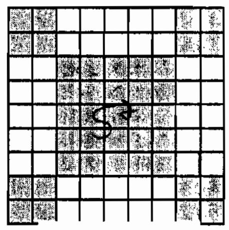
1 n



Pattern 1

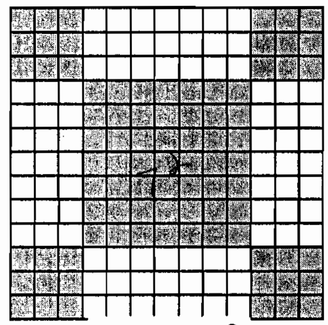
13



2² Pattern 2

16+25

41



3² Pattern 3

How many blocks of each kind will pattern #*n* need? $4n+2 \rightarrow$ $(8n^2+4n+1)$

black: $4n^2+(2n+1)^2 \rightarrow 4n^2+4n^2+4n+1$

white: $4(n \times (2n+1)) = 4(2n^2+n) = (8n^2+4n)$

Which pattern has a total of 841 grey blocks? 7th pattern

$16n^2+8n+1=841 \quad 16n^2+8n=840 \quad 2n^2+n=105 \quad 2n^2+n-105=0 \quad 840 \div 8 = 105$

more work -> on next page

How many white blocks has that pattern? 420

$8(7)^2+4(7)=8(49)+28=392+28=420$

Explain your work and show your calculations.

In the first Q, I realized that pattern # relates to the corner block. I saw that In pattern 1, the corner block was equal to pattern #, n, multiplied by itself. So, it was 1² for 1, 2² for 2, 3² for 3 so forth ~~take~~ There were four corners so I multiplied by four, 4n². Then I saw that the center group of black blocks were all odd # squared except 1. So I quickly related the corner block to the center group. I saw that it was (2n+1)². (Continued ->)

Please continue your work on the page opposite

T4

Sidewalk Stones (continued)

$$\begin{array}{r} \overset{4}{105} \\ \times\ \ 8 \\ \hline 840 \end{array}$$

2. $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\frac{-1 \pm \sqrt{1-4(2)(-105)}}{2(2)}$ $\frac{-1 \pm \sqrt{841}}{4}$

$\frac{-1 \pm 29}{4} = \frac{28}{4}$ or $\frac{-30}{4}$

7 or $-\cancel{7\frac{1}{2}}$

I added them together ~~was~~ which was $4n^2+(2n+1)^2$. For the White blocks, I saw that they formed 4 rectangles. I found that the width came from the pattern # and the length came from the sqrt of the # of black blocks in the center. Pattern # ~~an~~ is n, sqrt of # of black blocks in center is $2n+1$. There are 4 rectangles. So $4 \times (2n+1) \times n)$ is # of white blocks. This simplifies to $8n^2+4n$.

Sidewalk Stones **T5**

In Prague some sidewalks are made of small square blocks of stone.

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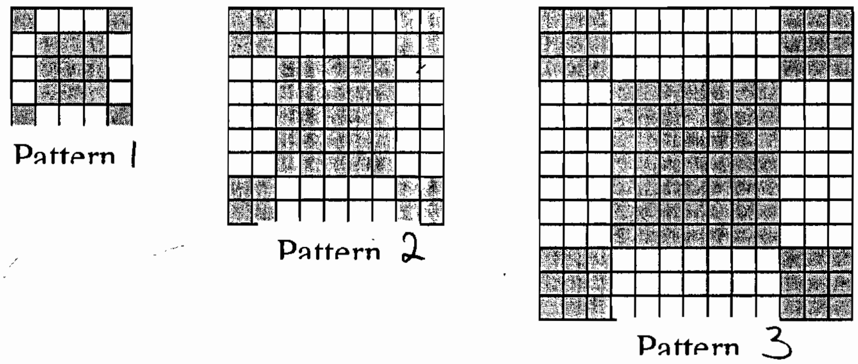

How many blocks of each kind will pattern #*n* need?

grey $= 4n^2 + (2n+1)^2$

white $= 8n^2 + 4n$

Which pattern has a total of 841 grey blocks? # 10

How many white blocks has that pattern? 840

Explain your work and show your calculations.

(1) grey $- 4n^2 + (2n+1)^2$

white $- 8n^2 + 4n$

(2) $4n^2 + (2n+1)(2n+1) = 841$

$4n^2 + 4n^2 + 4n + 1 = 840$

$8n^2 + 4n - 840 = 0$ (continued on next page)

Please continue your work on the page opposite

T5

Sidewalk Stones (continued)

$2n^2 + n - 210 = 0$

$n = \dfrac{-1 \pm \sqrt{1+1680}}{4}$

$n = \dfrac{-1 \pm 41}{4}$

$n = 10$ or $\cancel{-4.2}$

$\boxed{n=10} \longrightarrow$ pattern $\boxed{\#10}$

(3) $8(10)^2 + 4(10)$

$800 + 40$

$\boxed{840}$