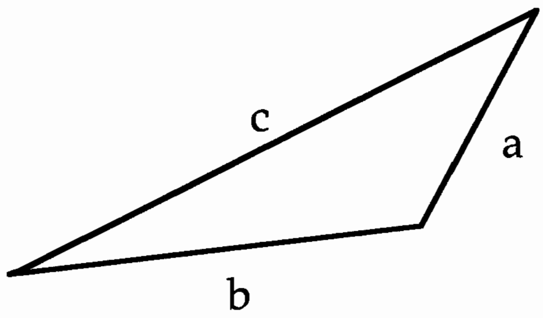
# Triangular Frameworks

**T1**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meters*, 3 *meters* etc and he always keeps one rod of each length in stock.




This diagram shows one of Joe's triangular frameworks.

*a, b, c* are all integers and $c > b > a$.

That is, *c* is the longest side, *a* is the shortest side and *a, b, c* are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 different frameworks ✓ 1

2

7, 6, 5
7, 6, 4
7, 6, 3
7, 6, 2 } 4

7, 5, 4
7, 5, 3 } 2

✓

2 sides > other side
so only these

T1

## Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

c=6 6,5,4 6,5,3 6,5,2 (3) 6,4,3 (1) $\boxed{4}$

c=5 5,4,3 5,4,2 (2) ~~5,3,~~ $\boxed{2}$

c=4 4,3,2 (1) $\boxed{1}$ c can not < 2

c=8 8,7,6 8,7,5 8,7,4 8,7,3 8,7,2 (5) 8,6,5 8,6,4 8,6,3 (3) 8,5,4 (1) $\boxed{9}$

c=9 9,8,7 9,8,6 9,8,5 9,8,4 9,8,3 9,8,2 (6) 9,7,6 9,7,5 9,7,4 9,7,3 (4) 9,6,5 9,6,4 (2) ~~9,6,4~~ ~~9,5~~ $\boxed{12}$

can not use a 1meter rod.

| c | n | |
|---|---|---|
| 4 | 1 | +1 |
| 5 | 2 | +2 |
| 6 | 4 | +3 +4 |
| 7 | 6 | +3 |
| 8 | 9 | +3 +6 |
| 9 | 12 | |

1, 2 +3, 3+1, 4+2 +5, 5+3+1, 6+4+2

3. Write down any generalizations you can make.

$\frac{(c-3)(c-1)}{4}$ if c is odd $\frac{(c-2)^2}{4}$ if c is even

I can't find one rule.

2



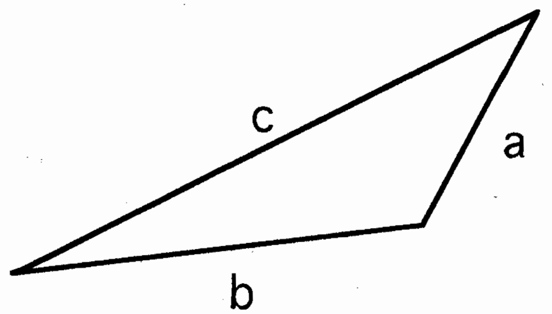


10

# Triangular Frameworks T2

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meters*, 3 *meters* etc and he always keeps one rod of each length in stock.



This diagram shows one of Joe's triangular frameworks.

$a, b, c$ are all integers and $c > b > a$.

That is, $c$ is the longest side, $a$ is the shortest side and $a, b, c$ are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 ways ✓

1

| c | b | a | c | b | a | c | b | a |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 5 ✓ | 7 | 5 | 4 ✓ | ~~7~~ | ~~4~~ | ~~3~~ |
| 7 | 6 | 4 ✓ | 7 | 5 | 3 ✓ | ~~7~~ | ~~4~~ | ~~2~~ |
| 7 | 6 | 3 ✓ | ~~7~~ | ~~5~~ | ~~2~~ | ~~7~~ | ~~4~~ | ~~1~~ |
| 7 | 6 | 2 ✓ | ~~7~~ | ~~5~~ | ~~1~~ | | | |
| ~~7~~ | ~~6~~ | ~~1~~ | | | | | | |

✓ 2

# T2

## Triangular Frameworks (continued)

2. Investigate this situation for other values of $c$.

$c=8$ : 9 ways

$8-2-1=5$ $5+3+1=9$

$8-3-2=3$

$8-4-3=1$ bigger c is more combinations

$c=4$ : 1 ways

$4-2-1=1$

$c=3$

smallest way = 1 for $c=4$

| c | b | a |
|---|---|---|
| 8 | 7 | 6 |
| 8 | 7 | 5 |
| 8 | 7 | 4 |
| 8 | 7 | 3 |
| 8 | 7 | 2 |
| ~~8~~ | ~~7~~ | ~~1~~ |

| c | b | a |
|---|---|---|
| 8 | 6 | 5 |
| 8 | 6 | 4 |
| 8 | 6 | 3 |
| ~~8~~ | ~~6~~ | ~~2~~ |
| ~~8~~ | ~~6~~ | ~~1~~ |

| c | b | a |
|---|---|---|
| 8 | 5 | 4 |
| ~~8~~ | ~~5~~ | ~~3~~ |
| ~~8~~ | ~~5~~ | ~~2~~ |
| ~~8~~ | ~~5~~ | ~~1~~ |

| c | b | a |
|---|---|---|
| 4 | 3 | 2 |

| c | b | a |
|---|---|---|
| 3 | 2 | 1 |

impossible

3. Write down any generalizations you can make.

total ways $= n \cdot c - n(n+2)$

$c > n+1+n$

① $c-2-1$
② $c-3-2$
③ $c-4-3$
ⓝ $c-(n+1)-n$

Total Ways $n \cdot c - \frac{n(n+1+2)}{2} - \frac{(n+1)n}{2}$

$= n \cdot c - n(n+2)$

$c > n+1+n$ $\quad c > 2n+1$

ex: $c=7$ $\quad 7 > 2n+1$

$n<3$ $\quad n=2$

Total Ways $= 2 \cdot 7 - 2(4) = 14 - 8 = 6$





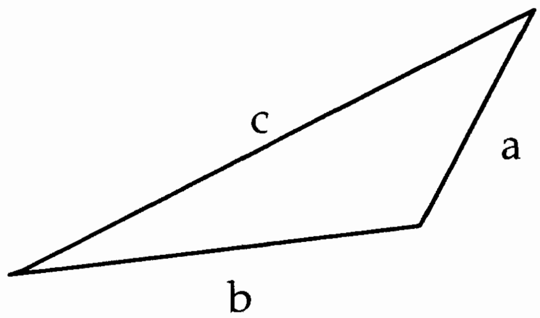
1 1 1 1 (1) 9

# Triangular Frameworks

# T3

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meters*, 3 *meters* etc and he always keeps one rod of each length in stock.




This diagram shows one of Joe's triangular frameworks.

*a, b, c* are all integers and $c > b > a$.

That is, *c* is the longest side, *a* is the shortest side and *a, b, c* are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

c>b>a   b+a MUST be bigger than c ✓   1

7 7 6 7 5

674

673

672 ✓

5 4   2

5 3

6 ways ✓

## Triangular Frameworks (continued)

2. Investigate this situation for other values of $c$.

c > 4 > b > a
3 2

c > 5 4 3
4 2

c > 6 5 4
5 3
5 2
4 3

c > 7

c > 8 7 6
7 5
7 4
7 3
7 2
6 5
6 4
6 3
5 9

c > 9 8 7
8 6
8 5
8 4
8 3
8 2
7 6
7 5
7 4
7 3
6 5
6 4

c > 10 9 8
9 7
9 6
9 5
9 4
9 3
9 2
8 7
8 6
8 5
8 4
8 3
7 6
7 5
7 4
6 5

| c | ways | |
|---|---|---|
| 4 | 1 | |
| | | 1 |
| 5 | 2 | |
| | | 2 |
| 6 | 4 | |
| | | 2 |
| 7 | 6 | |
| | | 3 |
| 8 | 9 | |
| | | 3 |
| 9 | 12 | |
| | | 4 |
| 10 | 16 | |

The larger c is, the more triangles can be made ✓

Smallest c is 4 with one triangle ✓

c = 3 is impossible as c > 2 > 1 but 2+1 = 3 so no triangle

✓

3. Write down any generalizations you can make.

even numbers go up in square numbers

| c | ways | |
|---|---|---|
| 4 | 1 | $1^2$ |
| 6 | 4 | $2^2$ |
| 8 | 9 | $3^2$ |
| 10 | 16 | $4^2$ |

$\frac{(c-2)^2}{4}$ = ways ✓

$10-2 = 8$

$8^2 = \frac{64}{4} = 16$ ✓

odds

| c | ways |
|---|---|
| 5 | 2 |
| 7 | 6 |
| 9 | 12 |

$7-2 = 5^2 = \frac{25}{4} = 6+$

$7-1 = 6^2 = \frac{36}{4} = 9$

$7-3 = 4^2 = \frac{16}{4} = 4$

$\frac{(7-1)(7-3)}{4} = \frac{6 \cdot 4}{4} = 6$ ✓

$\frac{(c-1)(c-3)}{4}$ = ways ✓





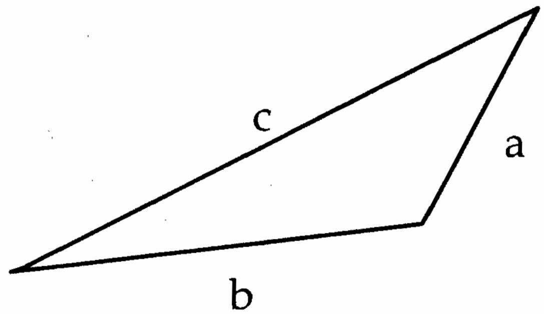


1
1
1
1
1
2
10

# Triangular Frameworks **T4**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meters*, 3 *meters* etc and he always keeps one rod of each length in stock.




This diagram shows one of Joe's triangular frameworks.

$a, b, c$ are all integers and $c > b > a$.

That is, $c$ is the longest side, $a$ is the shortest side and $a, b, c$ are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

$a+b > c$  $a+b>7$  $a < b$  $c$  1

$5 < 6$

$4 < 6$

$3 < 6$

$2 < 6$  6 ways  2

$4 < 5$

$3 < 5$

~~2 4~~

## Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

c = 8

| | a | b | c | a | b | c | | c |
|---|---|---|---|---|---|---|---|---|
| | 2 | 7 | 8+ | 2 | 8 | 9+ | 2 3 | 4+ |
| | 3 | 7 | | 3 | 8 | | | |
| | 4 | 7 | | 4 | 8 | | 1 way | |
| 5 | 5 | 7 | | 5 | 8 | | | |
| | 6 | 7 | | 6 6 | 8 | | | |
| | 3 | 6 | | 7 | 8 | | 3 4 | 5+ |
| 3 | 4 | 6 | | 6 | 7 | | 2 4 | |
| | 5 | 6 | | 5 | 7 | | | |
| | 4 | 5 | | 4 4 | 7 | | | |
| 1 | | | | 3 | 7 | | | |
| | | | | 4 | 6 | | | |
| | | | | 2 5 | 6 | | | |
| | 9 ways | | | 12 ways | | | | |

As c increases so does the # triangles

c = 4 is smallest # of triangles made

when c = x b = x−1, a = x−2 are the largest lengths each can be

3. Write down any generalizations you can make.

when c is even # of ways is odd.

when c is odd # of ways is even

c=7 6 ways 3 lots of 6 + 2 lots of 5

c=8 9 ways 5 lots of 7 + 3 lots of 6 + 1 lot of 5

c=9 12 ways 6 lots of 8 + 4 lots of 7 + 2 lots of 6

c = 4 1 way 1 lot of 3

$\frac{(c-2)^2}{4}$ = even ways.

8−2 = 6 $6^2$ = 36 ÷ 4 = 9

4−2 = 2 $2^2$ = 4 ÷ 4 = 1

(1)

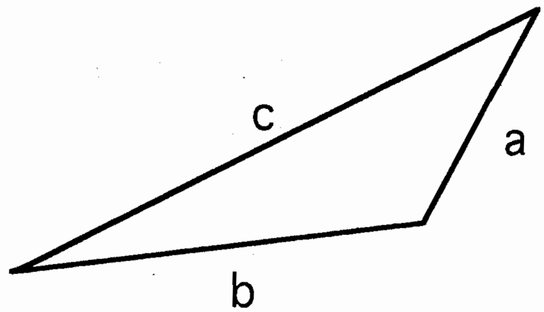
9

# Triangular Frameworks

**T5**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meter*s, 3 *meter*s etc and he always keeps one rod of each length in stock.




This diagram shows one of Joe's triangular frameworks.

*a, b, c* are all integers and $c > b > a$.

That is, *c* is the longest side, *a* is the shortest side and *a, b, c* are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 different variations can be made ✓ 1

$7 > 6 > 5$ $7 > 5 > 4$

$7 > 6 > 4$ $7 > 5 > 3$ ✓ 6 ways 2

$7 > 6 > 3$ 2

$7 > 6 > 2$

4

## Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

| | | | |
|---|---|---|---|
| 8>7>6 | 8>6>5 | 6>5>4 | 5>4>3 |
| 8>7>5 | 8>6>4 | 6>5>3 | 5>4>2 |
| 8>7>4 | ~~8>6>3~~ | 6>5>2 | 2 ways |
| 8>7>3 | 8>5>4 | 6>4>3 | |
| 8>7>2 | 4 | 4 ways | 2 ways |
| 5 ✓ | | | 4>2>1 |
| 9 ways | | | |

1
1
1

| | | | |
|---|---|---|---|
| −3 | 4 − 1 | ✓ | 1 |
| −3 | 5 − 2 | (+1) | |
| −2 | 6 − 4 | (+2) | |
| −1 | 7 − 6 | (+2) | |
| −1 | 8 − 9 | (+3) | |
| +2 3 | 9 − 12 | | |
| +3 6 | 10 − 16 | | |
| +4 10 | 11 − 21 | | |
| | 12 − 27 | | |

9>8>7 9>7>6
9>8>6 9>7>5
9>8>5 9>7>4
9>8>4 9>7>3
9>8>3 9>6>5
9>8>2 9>6>4

12

3. Write down any generalizations you can make.

$c > b > a$ $c - 1$ could be $c$ cannot $= 1$

$(a+b) > c$ $(c-2) + (c-b) > c$ $\frac{(c-3)(c-1)}{4}$ odds ✓

$\frac{(4-2)^2}{4} = \frac{4}{4} = 1$ ✓ $\frac{(c-2)^2}{4}$ = w even only ✓ ✓

$5-2 = \frac{9}{4}$

$6-2 = \frac{16}{4} = 1$ ✓ $(5-3)(5-1) = \frac{8}{4} = 2$ ✓
2 . 4

$8-2 = \frac{36}{4} = 9$ ✓ $(9-1)(9-3) = \frac{48}{4} = 12$ ✓
8 . 6

1

2







10