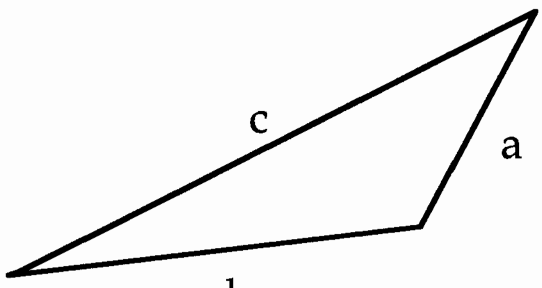
Triangular Frameworks **T1**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meter*s, 3 *meter*s etc and he always keeps one rod of each length in stock.



This diagram shows one of Joe's triangular frameworks.

a, b, c are all integers and $c > b > a$.

That is, c is the longest side, a is the shortest side and a, b, c are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 different frameworks

7, 6, 5
7, 6, 4
7, 6, 3
7, 6, 2
4

7, 5, 4
7, 5, 3
2

2 sides > other side
so only these

T1

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

c=6 6,5,4] 6,5,3] 3 6,5,2] 6,4,3] 1 $\boxed{4}$

c=5 5,4,3] 2 5,4,2] ~~5,3,~~ $\boxed{2}$

c=4 4,3,2 1 $\boxed{1}$ c can not < 2

c=8 8,7,6 8,7,5 8,7,4 8,7,3 8,7,2] 5 8,6,5 8,6,4 8,6,3] 3 8,5,4] 1 $\boxed{9}$

c=9 9,8,7 9,8,6 9,8,5 9,8,4 9,8,3 9,8,2] 6 9,7,6 9,7,5 9,7,4 9,7,3] 4 9,6,5 9,6,4] 2 $\boxed{12}$ ~~9,6,4~~ ~~9,5~~

can not use a 1meter rod.

c	n	
4	1	+1
5	2	+2
6	4	+3
7	6	+3
8	9	+3
9	12	

1, 2 +3, 3+1 +4, 4+2 +5, 5+3+1, 6+4+2

3. Write down any generalizations you can make.

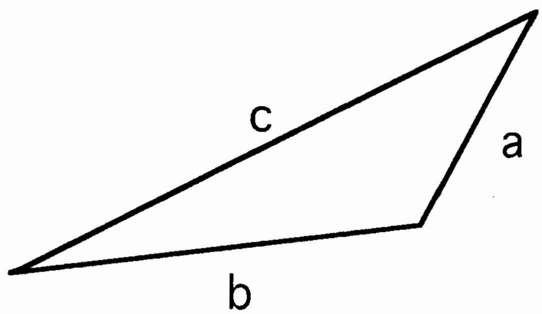
$\frac{(c-3)(c-1)}{4}$ if c is odd $\frac{(c-2)^2}{4}$ if c is even

I can't find one rule.

Triangular Frameworks **T2**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

He buys metal rods which have lengths 1 *meter*, 2 *meters*, 3 *meters* etc and he always keeps one rod of each length in stock.



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a, b, c are all integers and $c > b > a$.

That is, *c* is the longest side, *a* is the shortest side and *a, b, c* are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 ways

c b a
7 6 5 ✓
7 6 4 ✓
7 6 3 ✓
7 6 2 ✓
~~7 6 1~~

c b a
7 5 4 ✓
7 5 3 ✓
~~7 5 2~~
~~7 5 1~~

c b a
~~7 4 3~~
~~7 4 2~~
~~7 4 1~~

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

$c=8$: 9 ways $\quad\quad$ $c=4$: 1 ways

$8-2-1=5 \quad 5+3+1=9 \quad\quad 4-2-1=1$

$8-3-2=3 \quad\quad c=3$

$8-4-3=1$ bigger c is more combinations.

smallest way = 1 for c = 4

c	b	a	c	b	a	c	b	a
8	7	6	8	6	5	8	5	4
8	7	5	8	6	4	~~8~~	~~5~~	~~3~~
8	7	4	8	6	3	~~8~~	~~5~~	~~2~~
8	7	3	~~8~~	~~6~~	~~2~~	~~8~~	~~5~~	~~1~~
8	7	2	~~8~~	~~6~~	~~1~~			
~~8~~	~~7~~	~~1~~						

c b a
4 3 2

c b a
3 2 1 impossible

3. Write down any generalizations you can make.

total ways $= n \cdot c - n(n+2)$

$c > n+1+n$

① $c-2-1$
② $c-3-2$
③ $c-4-3$
ⓝ $c-(n+1)-n$

Total Ways $n \cdot c - \frac{n(n+1+2)}{2} - \frac{(n+1)n}{2}$

$= n \cdot c - n(n+2)$

$c > n+1+n \quad c > 2n+1$

ex: $c=7 \quad 7 > 2n+1$

$n<3 \quad n=2$

Total Ways $= 2 \cdot 7 - 2(4) = 14 - 8 = 6$

Triangular Frameworks **T3**

Joe uses metal rods to make triangular frameworks in which each side has a different length.

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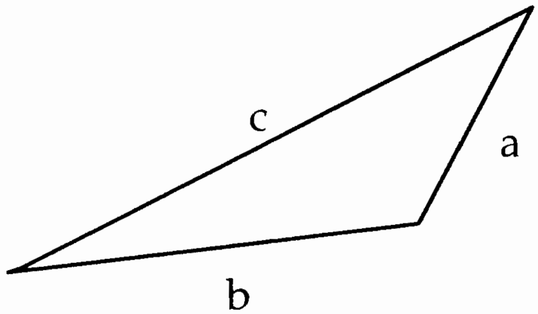

This diagram shows one of Joe's triangular frameworks.

a, b, c are all integers and $c > b > a$.

That is, c is the longest side, a is the shortest side and a, b, c are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

c>b>a   b+a MUST be bigger than c

7 7675

674

673

672

5 4

5 3

6 ways

T3

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

$c > 4 > b > a$
3 2

$c > 5$ 4 3
4 2

$c > 6$ 5 4
5 3
5 2
4 3

$c > 7$

$c > 8$ 7 6
7 5
7 4
7 3
7 2
6 5
6 4
6 3
5 4

$c > 9$ 8 7
8 6
8 5
8 4
8 3
8 2
7 6
7 5
7 4
7 3
6 5
6 4

$c > 10$ 9 8
9 7
9 6
9 5
9 4
9 3
9 2
8 7
8 6
8 5
8 4
8 3
7 6
7 5
7 4
6 5

c	ways	
4	1	> 1
5	2	> 2
6	4	> 2
7	6	> 3
8	9	> 3
9	12	> 4
10	16	

The larger c is, the more triangles can be made

Smallest c is 4 with one triangle

c = 3 is impossible as $c > 2 > 1$ but $2 + 1 = 3$ so no triangle

3. Write down any generalizations you can make.

even numbers go up in square numbers

c	ways	
4	1	1^2
6	4	2^2
8	9	3^2
10	16	4^2

$\frac{(c-2)^2}{4}$ = ways

$10 - 2 = 8$
$8^2 = \frac{64}{4} = 16$ ✓

odds

c	ways
5	2
7	6
9	12

$7 - 2 = 5^2 = \frac{25}{4} = 6+$

$7 - 1 = 6^2 = \frac{36}{4} = 9$

$7 - 3 = 4^2 = \frac{16}{4} = 4$

$\frac{(7-1)(7-3)}{4} = \frac{6 \cdot 4}{4} = 6$ ✓

$\frac{(c-1)(c-3)}{4}$ = ways

Triangular Frameworks

T4

Joe uses metal rods to make triangular frameworks in which each side has a different length.

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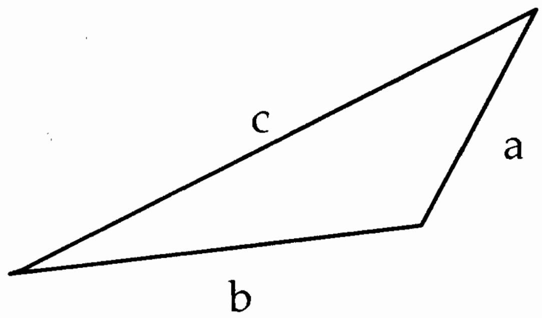

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That is, *c* is the longest side, *a* is the shortest side and *a, b, c* are whole numbers.

1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

$a+b>c$ $a+b>7$ $a < b$ c

$5 < 6$

$4 < 6$

$3 < 6$

$2 < 6$

$4 < 5$

$3 < 5$

~~2 4~~

6 ways

T4

Triangular Frameworks (continued)

2. Investigate this situation for other values of c.

$c = 8$

a	b	c		a	b	c		c
2	7	8+		2	8	9+		2 3 4+
3	7			3	8			1 way
4	7			4	8			
5	7			5	8			
6	7		6	6	8			
3	6			7	8			3 4 5+
4	6		3	6	7			2 4
5	6			5	7			
4	5		1	4	7		4	
				3	7			
				4	6			
				5	6		2	

5

9 ways

12 ways

As c increases so does the # triangles

c = 4 is smallest # of triangles made

when $c = x$ $b = x-1$, $a = x-2$ are the largest lengths each can be

3. Write down any generalizations you can make.

when c is even # of ways is odd.

when c is odd # of ways is even

c = 7 6 ways 3 lots of 6 + 2 lots of 5

c = 8 9 ways 5 lots of 7 + 3 lots of 6 + 1 lot of 5

c = 9 12 ways 6 lots of 8 + 4 lots of 7 + 2 lots of 6

c = 4 1 way 1 lot of 3

$\frac{(c-2)^2}{4}$ = even ways.

$8-2 = 6$ $6^2 = 36 \div 4 = 9$

$4-2 = 2$ $2^2 = 4 \div 4 = 1$

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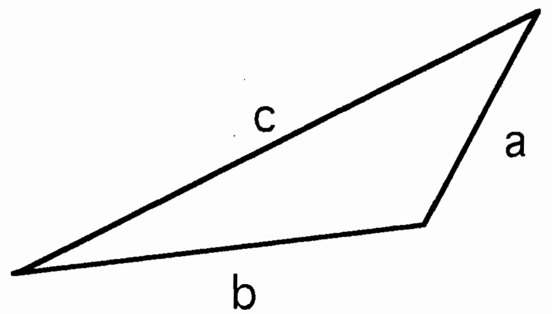

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1. How many different triangular frameworks can Joe make which have a longest side 7 meters long, using the rods he has in stock? Show your work.

6 different variations can be made

$7 > 6 > 5$
$7 > 6 > 4$
$7 > 6 > 3$
$7 > 6 > 2$
4

$7 > 5 > 4$
$7 > 5 > 3$
2

6 ways

Triangular Frameworks (continued)

2. Investigate this situation for other values of *c*.

8>7>6	8>6>5	6>5>4	5>4>3
8>7>5	8>6>4	6>5>3	5>4>2
8>7>4	~~8>6>3~~	6>5>2	2 ways
8>7>3	8>5>4	6>4>3	
8>7>2	4	4 ways	2 ways
5			4>2>1
9 ways			

9>8>7 9>7>6
9>8>6 9>7>5
9>8>5 9>7>4
9>8>4 9>7>3
9>8>3 9>6>5
9>8>2 9>6>4

12

−3	4 − 1		
−3	5 − 2	(+1)	
−2	6 − 4	(+2)	
−1	7 − 6	(+2)	
− 1	8 − 9	(+3)	
+2 3	9 − 12		
+3 6	10 − 16		
+4 10	11 − 21		
	12 − 27		

3. Write down any generalizations you can make.

$c > b > a$ $c-1$ could be c cannot $= 1$

$(a+b) > c$ $(c-2)+(c-b) > c$ $\frac{(c-3)(c-1)}{4}$ odds

$\frac{(4-2)^2}{4} = \frac{4}{4} = 1$ ✓ $\frac{(c-2)^2}{4}$ = w even only ✓

$5-2 = \frac{9}{4}$

$6-2 = \frac{16}{4} = 1$ ✓

$8-2 = \frac{36}{4} = 9$ ✓

$(5-3)(5-1) = \frac{8}{4} = 2$ ✓
2 . 4

$(9-1)(9-3) = \frac{48}{4} = 12$ ✓
8 . 6