

Proofs Of The Pythagorean Theorem?

T1

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

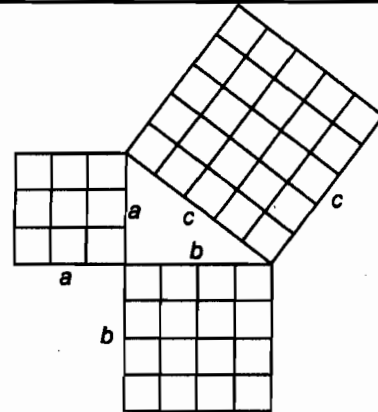
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

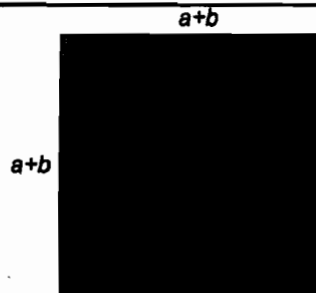
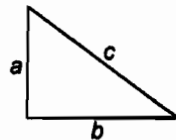
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



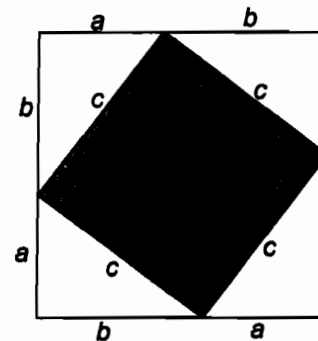
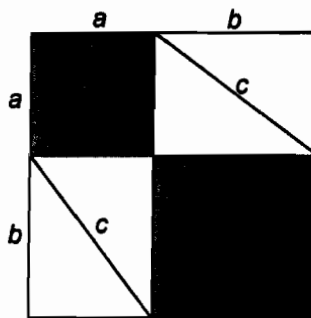
You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

In the second way you leave one large square hole. This has an area of c^2 .

Since these areas are equal

$a^2 + b^2 = c^2$

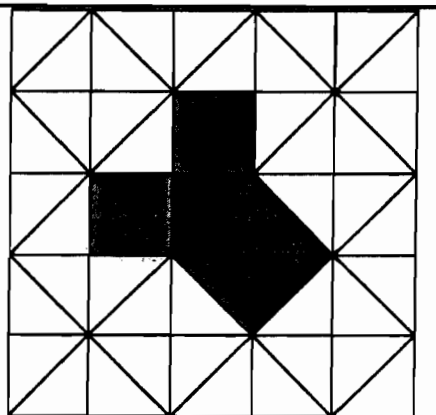


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number 2 ✓

3

This is because

It proves that the Pythagorean theorem is
always true, no matter what a, b, and c are

By arranging the triangles in different 2
way it accurately demonstrates that
 $a^2 + b^2 = c^2$ is always true if

My criticisms of the others are.

Attempt 1 only proves that $a^2 + b^2 = c^2$ if $a=3,$ 2

$b=4$ and $c=5$. ✓ Attempt 3 only proves that

$a^2 + b^2 = c^2$ if a and b are congruent and there

is a right angle. 3

Proofs Of The Pythagorean Theorem?

T2

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

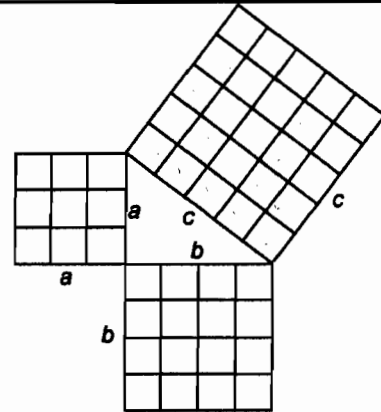
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

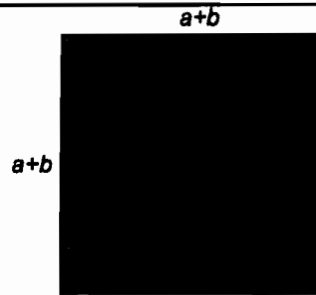
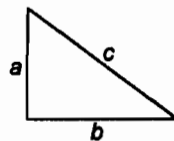
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

$$\text{So: } a^2 + b^2 = c^2$$



Attempt 2

Suppose that you start with **four** right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



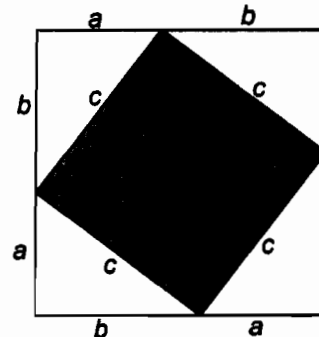
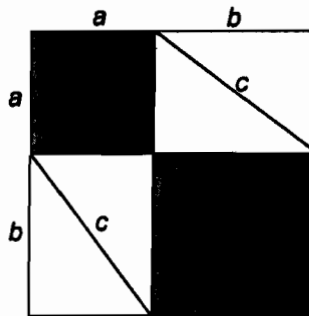
You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

In the second way you leave one large square hole. This has an area of c^2 .

Since these areas are equal

$$a^2 + b^2 = c^2$$

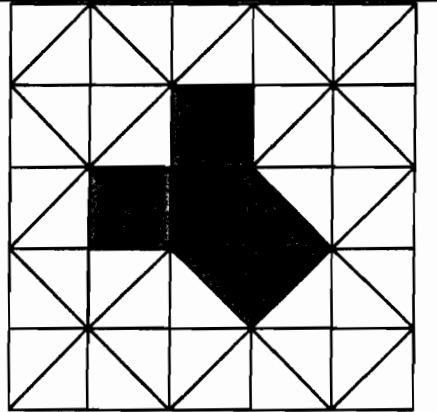


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number

2 ✓

3

This is because

I can easily understand and it works for every trio of numbers, not just 3,4,5. We can see the difference by the shading and labeling. We can also 'see' the squares.

2

My criticisms of the others are.

1. $a^2 + b^2 = c^2$ But it doesn't always equal
 $3^2 + 4^2 = 5^2$ It can be $5^2 + 12^2 = 13^2$ or other combinations.

2

3. It just doesn't make sense.

A

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Proofs Of The Pythagorean Theorem?

T3

Here are three attempts to prove the Pythagorean theorem.

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Explain your reasoning as fully as possible.

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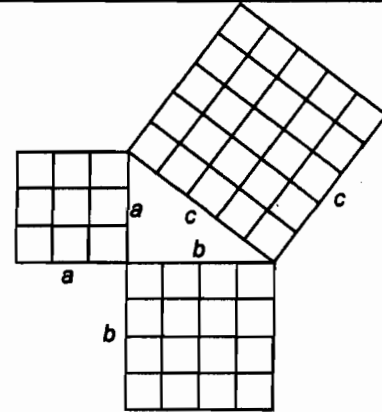
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

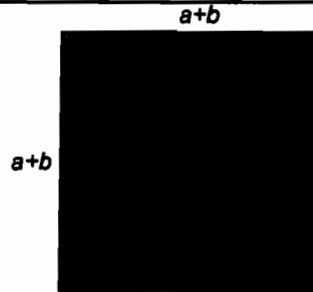
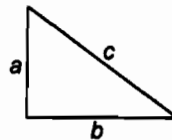
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with four right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



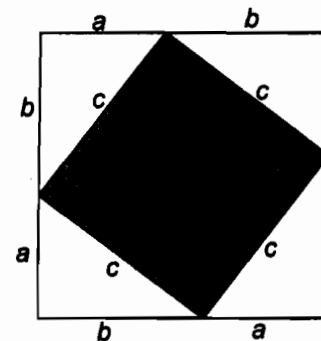
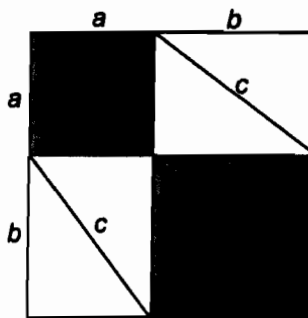
You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

In the second way you leave one large square hole. This has an area of c^2 .

Since these areas are equal

$a^2 + b^2 = c^2$

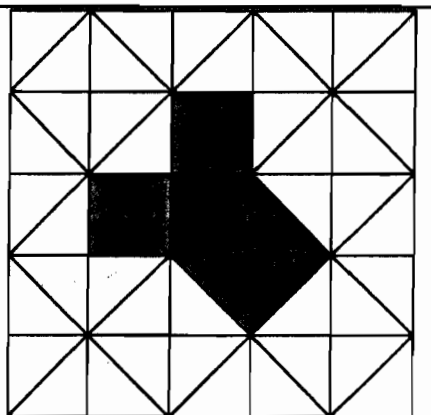


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number

2 ✓

3

This is because

It doesn't make any specific assumptions about the triangle
other than the fact that it is a right triangle. Sides a and b are
not necessarily congruent, and they don't state that they're in
a specific ratio.

2

My criticisms of the others are.

Attempt 3 doesn't work with a totally arbitrary right triangle

2

Instead, it works with an isosceles right triangle, and this has only
proved the theorem for isosceles right triangle. Attempt one makes

the assumption that the sides are in a 3:4:5 ratio, which isn't
always true for right angles

3

Proofs Of The Pythagorean Theorem?

T4

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

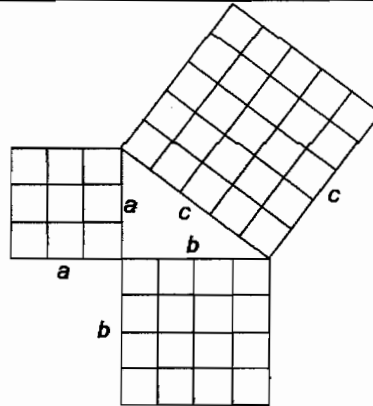
Attempt 1:

Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown. Divide these squares into smaller squares.

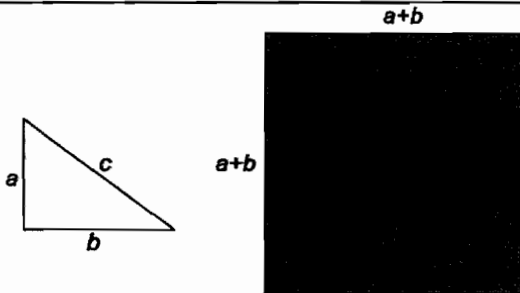
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with four right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



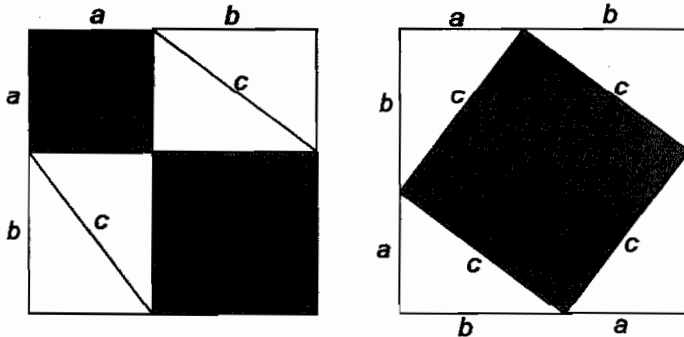
You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

In the second way you leave one large square hole. This has an area of c^2 .

Since these areas are equal

$a^2 + b^2 = c^2$

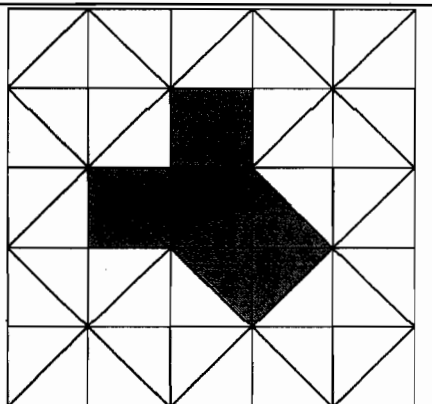


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number

2 ✓

3

This is because

It doesn't make any specific assumptions about the triangle other than the fact that it has a right triangle. sides a and b are not necessarily congruent, and they don't state that they're in a specific ratio, only $a^2 + b^2 = c^2$ ✓

2.

My criticisms of the others are.

Attempt 3 only works for an isosceles right triangle not for any right triangle. ✓

3

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8

Proofs Of The Pythagorean Theorem?

T5

Here are three attempts to prove the Pythagorean theorem.

Look carefully at each attempt. Which is the best 'proof' ?

Explain your reasoning as fully as possible.

Attempt 1:

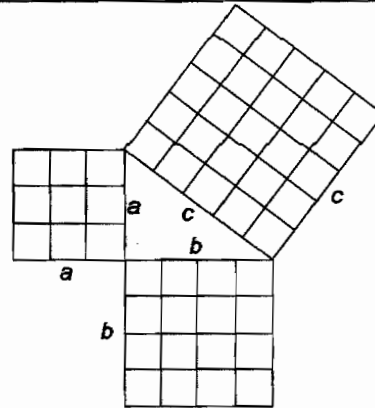
Suppose a right triangle has sides of length a , b and c

Draw squares on the three sides as shown.

Divide these squares into smaller squares.

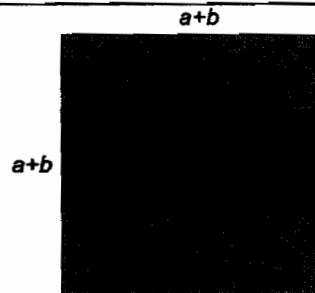
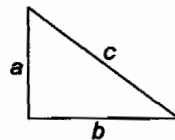
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.

So: $a^2 + b^2 = c^2$



Attempt 2

Suppose that you start with four right triangles with sides of length a , b and c and a square tray with sides of length $a+b$.



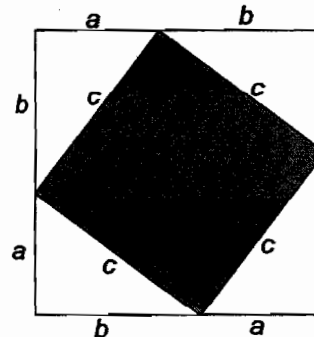
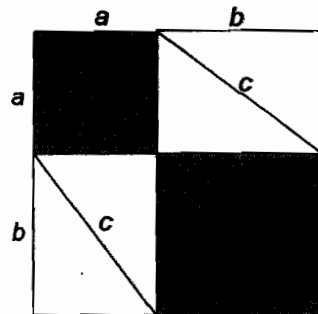
You can arrange the triangles into the tray in two different ways as shown here.

In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.

In the second way you leave one large square hole. This has an area of c^2 .

Since these areas are equal

$a^2 + b^2 = c^2$

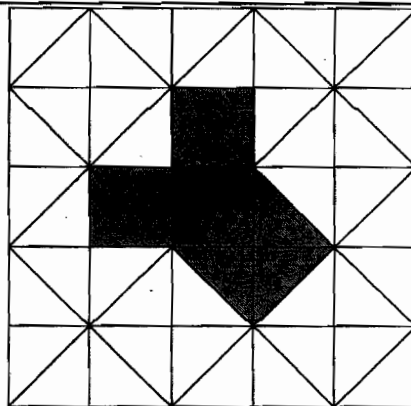


Attempt 3:

The proof of the Pythagorean theorem is clear from this diagram.

The squares on the two shorter sides of the black triangle are each made from two congruent triangles.

These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number

2 ✓ ✓

3

This is because

The area of the 4Δ s + the area of the black square
= the entire square $4(\frac{1}{2}ab) + c^2 = (a+b)^2$

$$2ab + c^2 = a^2 + 2ab + b^2$$

$$c^2 = a^2 + b^2 \quad \checkmark \quad 2$$

You don't have to assume any side ratios and a and b can be any number as long as there is a right angle.

My criticisms of the others are.

For attempt 1, a , b and c have to be whole numbers 2

You have to know the ratio of the sides in order for the squares to be able to be split into smaller whole

squares. For attempt 3, the right triangle must be

45-45-90. It is isosceles and does not apply to any other types of triangles. ✓ 3

(10)