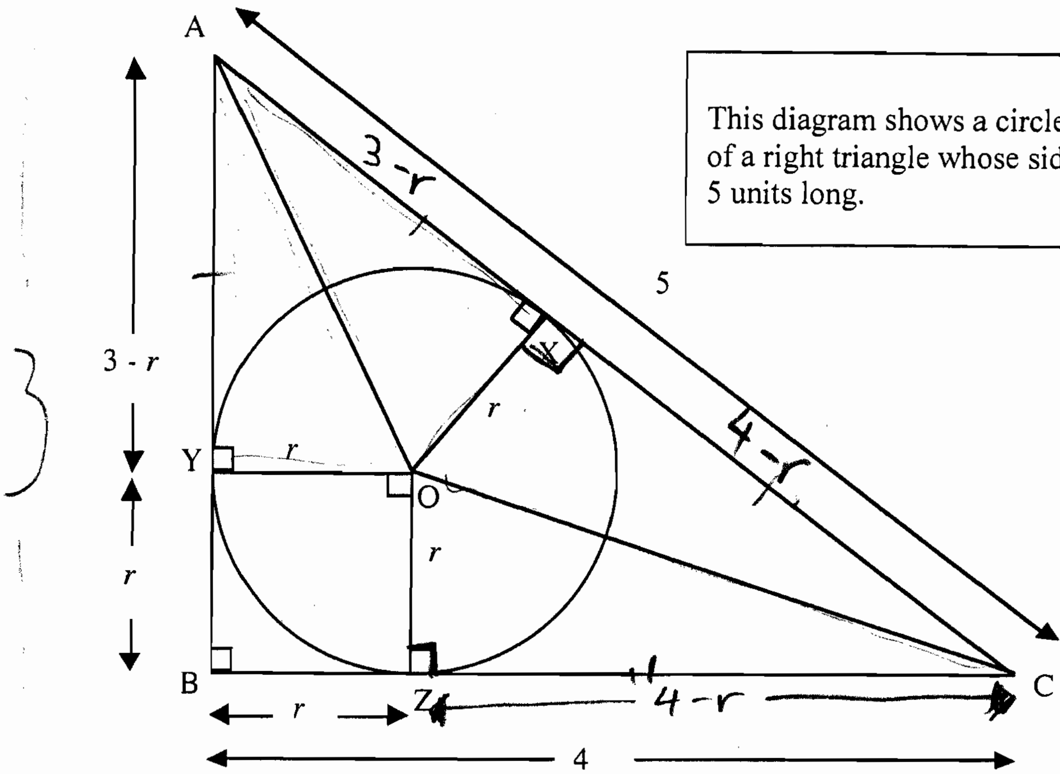


This diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

1. Prove that triangles AOX and AOY are congruent.

① r = r, because radii of a ⊙ are ≅. ② ∠AYO & ∠AXO are rt. ∠s
③ AO = AO, Reflexive Prop. ④ Hypotenuse Leg, △AOX ≅ △AOY

2. What can you say about the measures of the line segments CX and CZ?

They are congruent, Using Hypotenuse Leg. (same procedure as above)

T1

3. Find r, the radius of the circle. Explain your work clearly and show all your calculations.

$(3-r)+(4-r)=5$

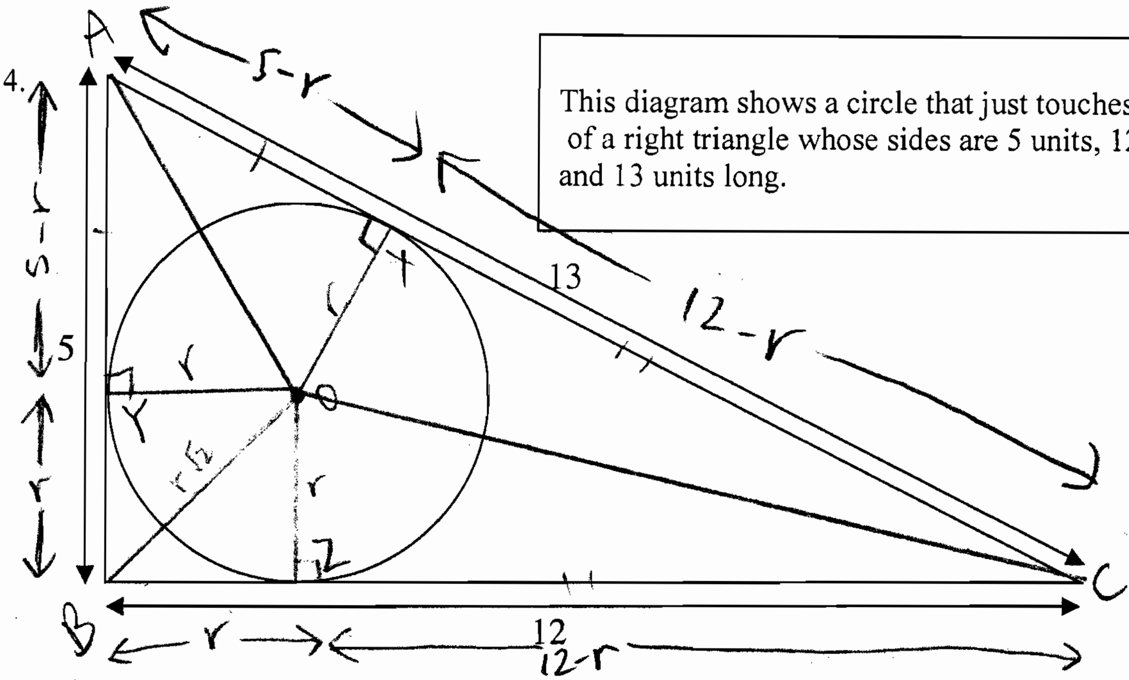
$3+4-r-r=5$

$7-2r=5-7$

$-2r=-2$

$r=1$

4.



This diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long.

Draw construction lines as in the previous task, and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.

$(5-r)+(12-r)=13$

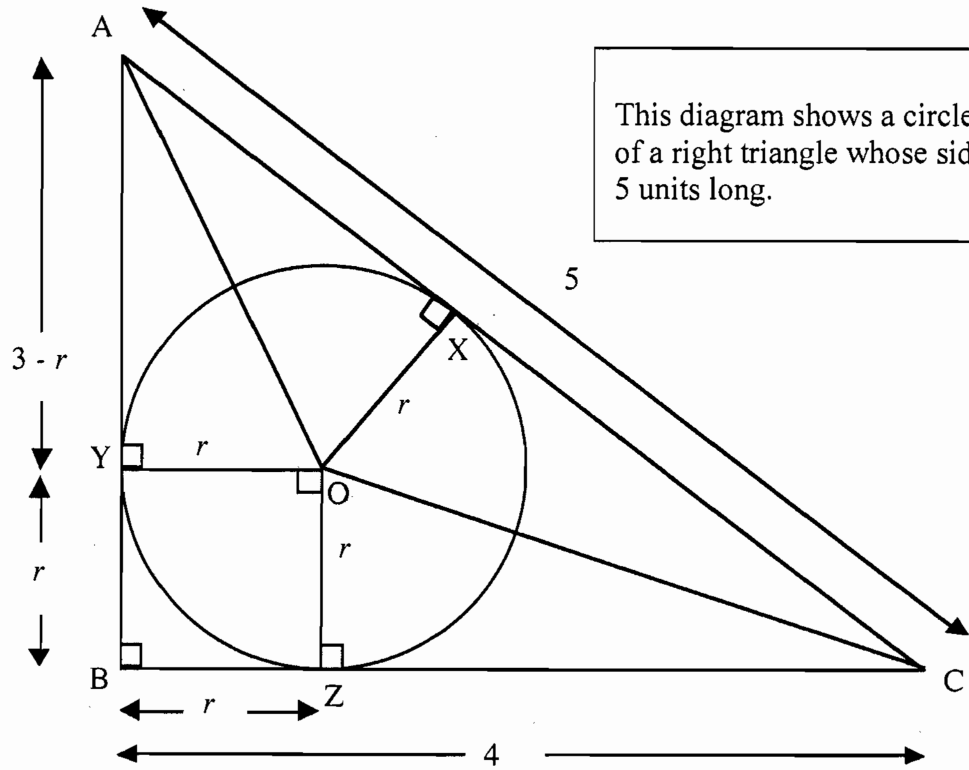
$5+12-r-r=13$

$\cancel{17}-2r=13-17$

$-2r=-4$

$r=2$

Circles in Triangles



This diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

1. Prove that triangles AOX and AOY are congruent.

$\overline{AO} \cong \overline{AO}$ by Reflexive $\overline{YO} \cong \overline{XO}$ because they equal r. $\triangle AOY \cong \triangle AOX$ by HL postulate.

2. What can you say about the measures of the line segments CX and CZ?

They are congruent

3. Find r, the radius of the circle. Explain your work clearly and show all your calculations.

$\overline{AX} = 3 - r$

$\overline{CX} = 5 - (3 - r) = 2 + r$

$\overline{CZ} = 2 + r$

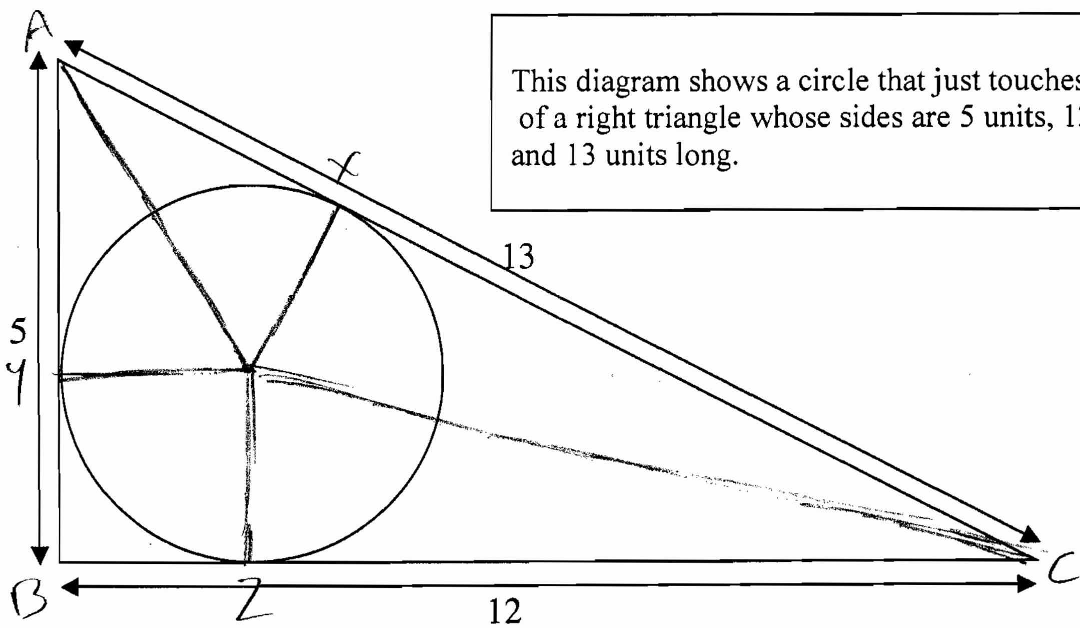
$2 + r + r = 4$

$2r + 2 = 4$

$r + 1 = 2$

$r = 1$

4.



This diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long.

Draw construction lines as in the previous task, and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.

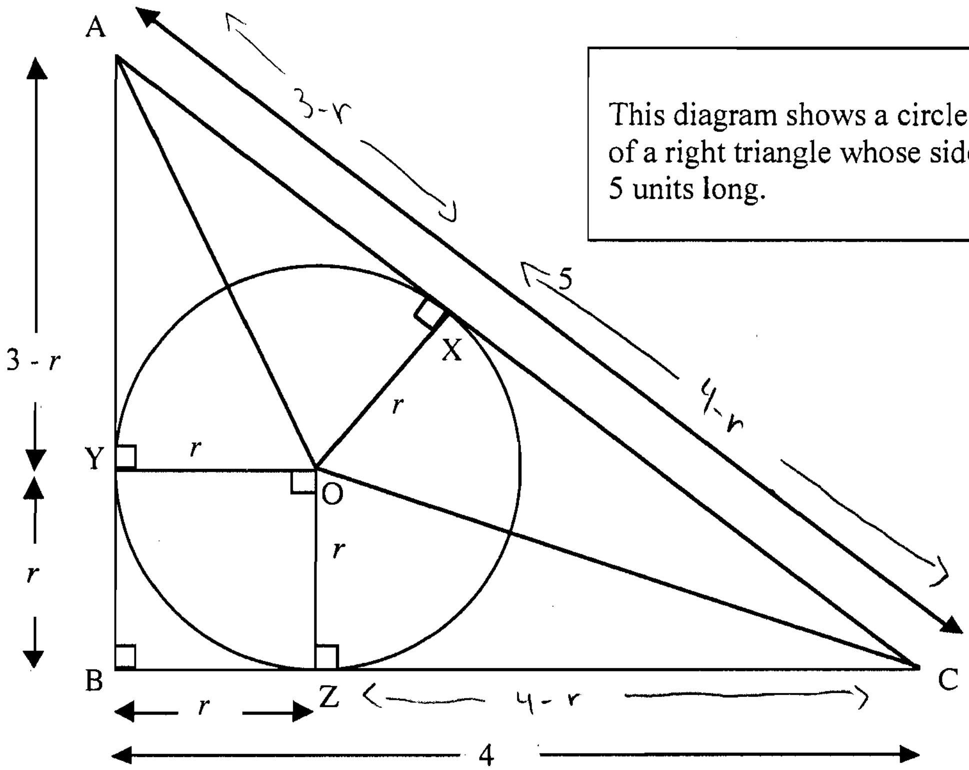
$AX = \overline{AY} = 5 - r$ | $8 + r + r = 12$

$\overline{CX} = 13 - (5 - r) = 8 + r$ | $2r = 4$

$CZ = 8 + r$ | $r = 2$

Circles in Triangles **T3**

Diagram:



This diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

1. Prove that triangles AOX and AOY are congruent.

Statements	Reasons
1) Diagram ~~Shown~~	1) Given
2) $\angle AYO$ = rt. $\angle$, ~~$\angle AXO$ = rt. $\angle$~~, ⊙O with radii $\overline{OY}$ & $\overline{OX}$	2) Shown from diagram
3) $\overline{AO} \cong \overline{AO}$	3) Reflexive ~~Property~~
4) $\overline{OY} \cong \overline{OX}$	4) All radii of a circle are $\cong$
5) $\triangle AOX \cong \triangle AOY$	5) HL Postulate.

Using a two-column proof, I proved that triangles AOX & AOY are congruent.

2. What can you say about the measures of the line segments CX and CZ?

Statements	Reasons
1) Diagram Shown	~~1) Given~~
2) $\angle AXO$ = rt. $\angle$, $\angle OZC$ = rt. $\angle$, ⊙O with radii $\overline{OX}$ & $\overline{OZ}$	2) Shown from diagram
3) $\overline{OC} \cong \overline{OC}$	3) Reflexive Property
4) $\overline{OX} \cong \overline{OZ}$	4) ~~All radii~~ of a circle are $\cong$
5) $\angle AXO$ Supp. to $\angle CXO$; $\angle AXC$ = St. $\angle$	5) Assume from diagram
6) $\angle AXO = 90°$	6) def. rt. $\angle$
7) $\angle AXC = 180°$	7) def. st. $\angle$
8) $\angle CXO = 90°$	8) Subtract. Prop.
9) $\angle CXO$ = rt. $\angle$	9) def. rt. $\angle$
10) $\triangle CXO \cong \triangle CZO$	10) HL
11) $\overline{CX} \cong \overline{CZ}$	11) CPCTC

I CAN SAY ~~THAT LINE~~ SEGMENTS $\overline{CX}$ AND $\overline{CZ}$ ARE CONGRUENT. I HAVE A TWO-COLUMN PROOF TO SUPPORT THAT STATEMENT.

T3

3. Find r, the radius of the circle. Explain your work clearly and show all your calculations.

R, the radius of the circle, is 1 unit long. I know that: $\overline{AO} \cong \overline{AO}$ (Reflex Pr.), $\angle AYO$ & $\angle AXO$ are Rt. $\angle$s, and $\overline{OY} \cong \overline{OX}$ (Radii $\cong$). So, because of HL, $\triangle AYO \cong \triangle AXO$. I also know that: $\overline{OC} \cong \overline{OC}$ (Reflex Pr.), $\angle OXC$ & $\angle OZC$ are Rt. $\angle$s and $\overline{OZ} \cong \overline{OX}$ (Radii $\cong$). So, because of HL, $\triangle ZOC \cong \triangle XOC$. Using CPCTC, I know that $\overline{AY} \cong \overline{AX}$ & $\overline{ZC} \cong \overline{XC}$. As labeled in the diagram, $\overline{AY} = (3-r)$. So, $\overline{AX} = (3-r)$ (def. $\cong$ segm.). As labeled in the diagram, $\overline{BC} = (4)$ and $\overline{BZ} = (r)$. Using subtraction, I know that $ZC = (4-r)$. So, because of def. $\cong$ segm., $\overline{XC} = (4-r)$. As labeled in the diagram, $\overline{AC} = 5$, and $\overline{AX} + \overline{XC} = \overline{AC}$. Using substitution, $(3-r) + (4-r) = 5$. Simplifying, you get $r = 1$.

Calculations:

$(3-r) + (4-r) = 5$

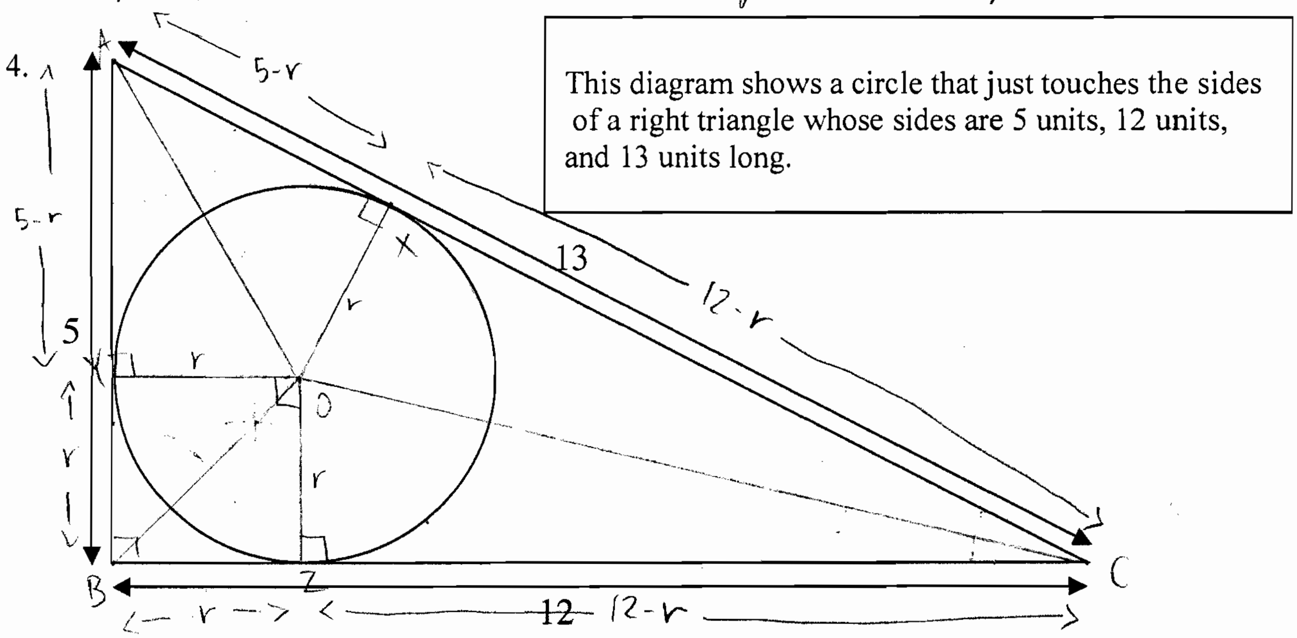
$3 - r + 4 - r = 5$

$7 - 2r = 5$

$2 = 2r$

$r = 1$

4.



This diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long.

Draw construction lines as in the previous task, and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.

R, the radius of the circle, is 2 units long. I know that $\overline{AO} \cong \overline{AO}$, $\overline{CO} \cong \overline{CO}$ (Reflex. Pr.), $\overline{YO} \cong \overline{OX}$, $\overline{OZ} \cong \overline{OX}$ (Radii $\cong$), and $\angle AYO$, $\angle AXO$, $\angle OZC$, and $\angle OXC$ rt. $\angle$. So, because of HL, $\triangle AYO \cong \triangle AXO$ & $\triangle OZC \cong \triangle OXC$. Using CPCTC, $\overline{AY} \cong \overline{AX}$ & $\overline{ZC} \cong \overline{XC}$. Using construction lines from previous task, I know that YB & $BZ = r$. Using subtraction, I know that $\overline{AY} = (5-r)$ & $\overline{ZC} = (12-r)$. Using substitution ($\cong$ segm.), I know that $\overline{AX} = (5-r)$ & $\overline{XC} = (12-r)$. Also, in the diag., $\overline{AX} + \overline{XC} = \overline{AC}$. Using substitution, $(5-r) + (12-r) = 13$. Simplifying, you get $r = 2$.

Calculations:

$(5-r) + (12-r) = 13$

$5 - r + 12 - r = 13$

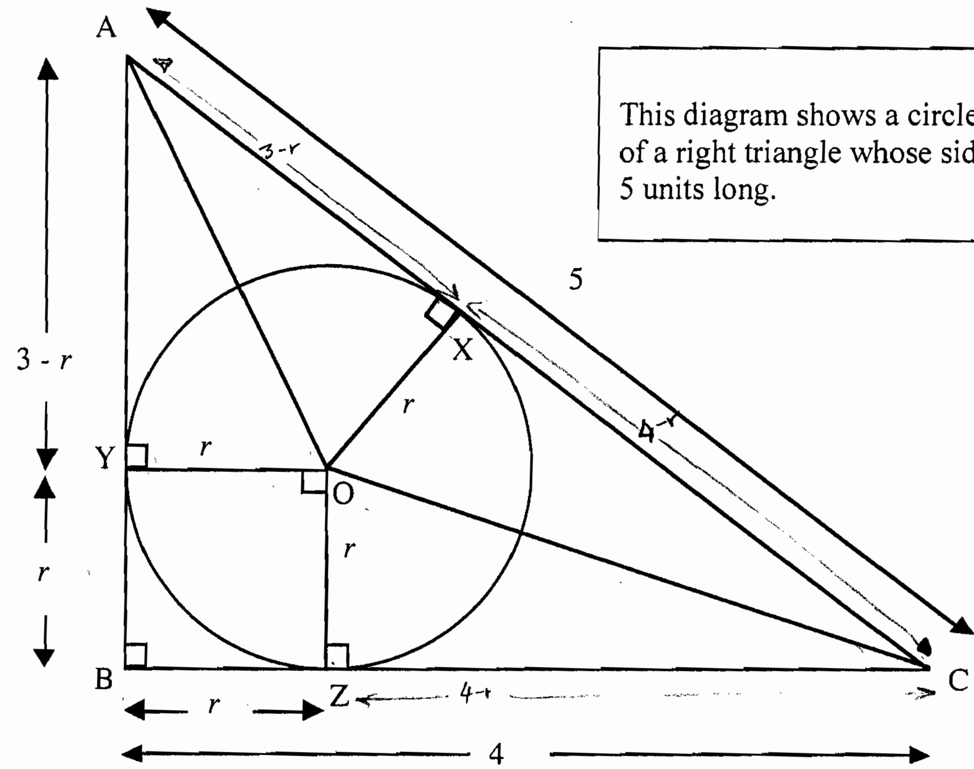
$17 - 2r = 13$

$4 = 2r$

$r = 2$

Circles in Triangles

T4



This diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

1. Prove that triangles AOX and AOY are congruent.

AO ≅ AO   YO ≅ XO because they are radius to same circle.
by Reflexive   HL postulate

2. What can you say about the measures of the line segments CX and CZ?

In △s OZC and OXC   OX = OZ = r radii of same circle ; OC = OC as HL postulate

∠OZC and ∠OXC = 90° A given fact so △OZC ≅ △OXC. This means that ZC = XC = r-1

T4

3. Find r, the radius of the circle. Explain your work clearly and show all your calculations.

$k = \frac{1}{2}(a+b+c)$ $\quad$ radius $= \frac{\sqrt{k(k-a)(k-b)(k-c)}}{k}$

$k = \frac{1}{2}(3+4+5)$ $\quad = \frac{\sqrt{6(6-3)(6-4)(6-5)}}{6}$

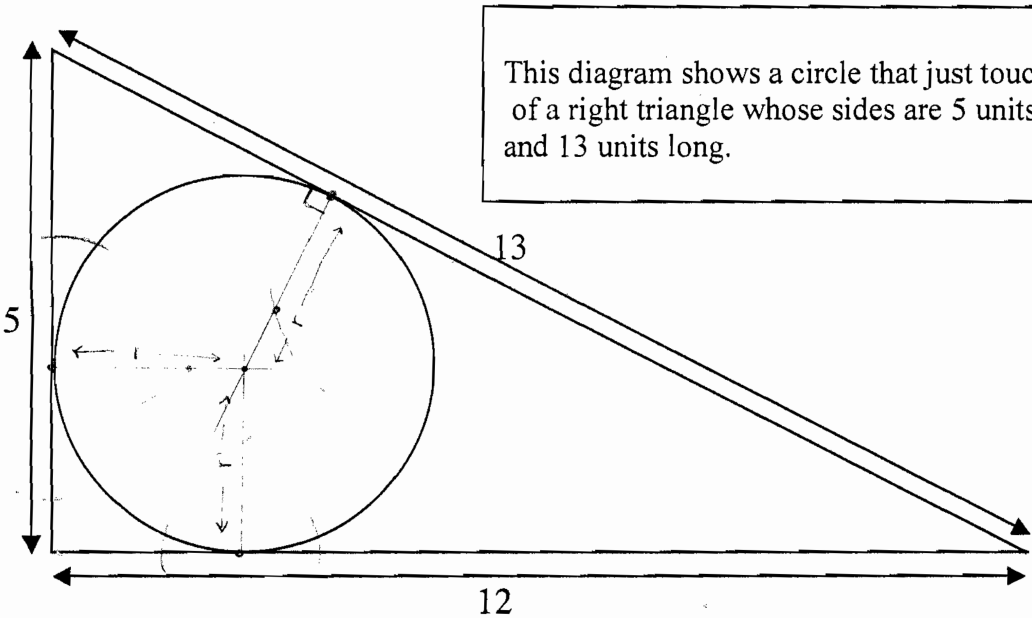
$k = \frac{1}{2}(12)$ $\quad = \frac{\sqrt{6 \cdot 3 \cdot 2 \cdot 1}}{6}$

$k = 6$ $\quad = \frac{\sqrt{36}}{6}$

$= \frac{6}{6}$

radius = 1

4.



This diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long.

Draw construction lines as in the previous task, and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.

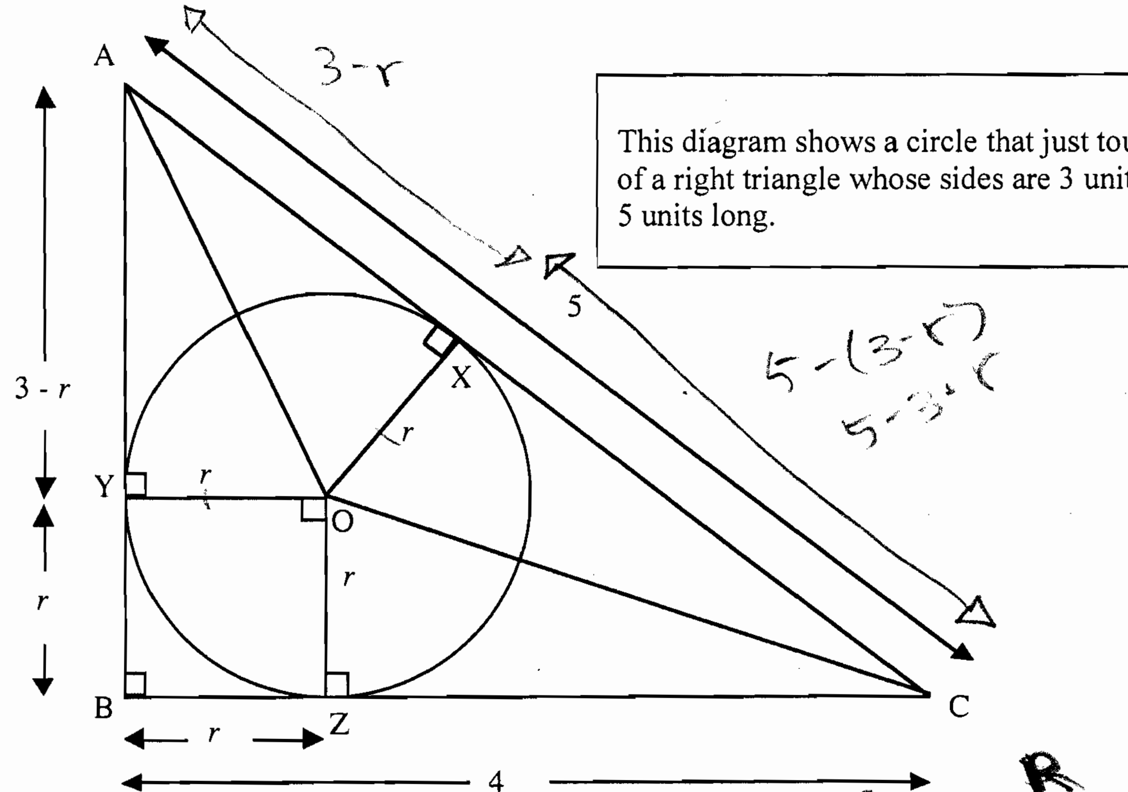
I drew the lines ⊥ to the point where the circle meets the triangle side any line drawn from the point of intersection to the end of the circle. The radius is 2 units long.

$k = \frac{1}{2}(5+12+13)$ $\quad$ radius $= \frac{\sqrt{15(15-5)(15-12)(15-13)}}{15}$

$k = \frac{1}{2} \cdot 30$ $\quad = \frac{\sqrt{15 \cdot 10 \cdot 3 \cdot 2}}{15}$

$k = 15$ $\quad = \frac{\sqrt{900}}{15} = 2$

Circles in Triangles **T5**



This diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

1. Prove that triangles AOX and AOY are congruent.

S	R
1. OO; ∠AYO and ∠AXO are rt. ∠s	1. Given
2. $\overline{YO} \cong \overline{XO}$	2. [illegible]
3. $\overline{AO} \cong \overline{AO}$	3. Reflexive
4. ∠AYO ≅ ∠AXO	4. [illegible]
5. △AYO ≅ △AXO	5. HL postulate

2. What can you say about the measures of the line segments CX and CZ?

They are congruent because △OCZ and △OCX are ≅
so $4 - r = 5 - 3 + r$; $4 - 5 + 3 = 2r$; $2 = 2r$; $r = 1$
so $\overline{CZ} = 3$ and $\overline{CX} = 3$

3. Find *r*, the radius of the circle. Explain your work clearly and show all your calculations.

Because $\triangle OCZ \cong \triangle OCX$ from HL postulate, $\overline{XC}$ and $\overline{ZC}$ are $\cong$. $\overline{XC} = 5-3+r$; $\overline{XZ} = 4-r$

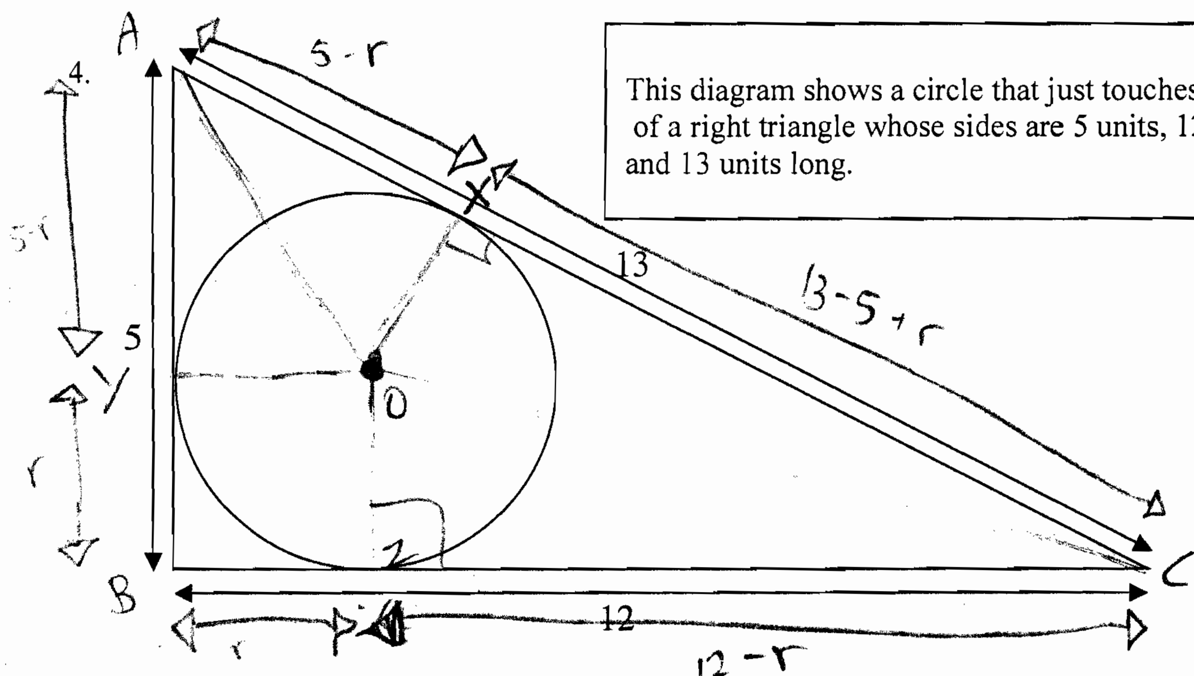
Since $\overline{XC} \cong \overline{XZ}$ by CPCTC $\Rightarrow$ then $5-3+r=4-r$

$= 2+r=4-r$

$= 2r=2$

$= r=1$

4.



This diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long.

Draw construction lines as in the previous task, and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.

$\triangle OZC \cong \triangle OXC$ from HL postulate; so $\overline{XC} \cong \overline{ZC}$

$12-r = 13-5+r$

$2r=4$

$r=2$