CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Representing Data with Box Plots

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Representing Data with Box Plots

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to interpret data using frequency graphs and box plots. In particular this unit aims to identify and help students who have difficulty figuring out the data points and spread of data from frequency graphs and box plots. It is advisable to use the lesson: *Representing Data with Frequency Graphs* before this one.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

S-ID: Summarize, represent, and interpret data on a single count or measurement variable.

This lesson also relates to **all** the *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

The unit is structured in the following way:

- Before the lesson, students individually complete an assessment task that is designed to reveal their current understanding and difficulties.
- A whole-class introduction gives guidance to students on how to work during the collaborative activity. Students then work in pairs or threes on a discussion task, matching frequency graphs to box plots. The lesson ends with a whole-class discussion.
- In a follow-up lesson students work alone on a similar task to the introductory assessment task.

MATERIALS REQUIRED

- Each student will need a copy of the tasks: *Cell Phones 2* and *Cell Phones 2 (revisited),* the handout *Matching a Box Plot to a Frequency Graph*, a rule, mini-whiteboard, pen, and eraser.
- Each small group of students will need *Card Set: Frequency Graphs* and *Card Set: Box Plots* (both sets cut-up before the lesson), a large sheet of paper for making posters, and a glue stick.
- You may want to copy the two *Card Sets* onto transparencies to be used on an overhead projector to support whole-class discussions. There is also a projector resource to help with whole-class discussions.
- If you are teaching the lesson as a sequel to the lesson *Representing Data with Frequency Graphs* you will not need *Card Set: Frequency Graphs* or a sheet of paper for making a poster, as students' existing posters include these cards.

TIME NEEDED

Approximately 15 minutes before the lesson, a 1-hour lesson, and 20 minutes in a follow-up lesson. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Cell Phones 2* (15 minutes)

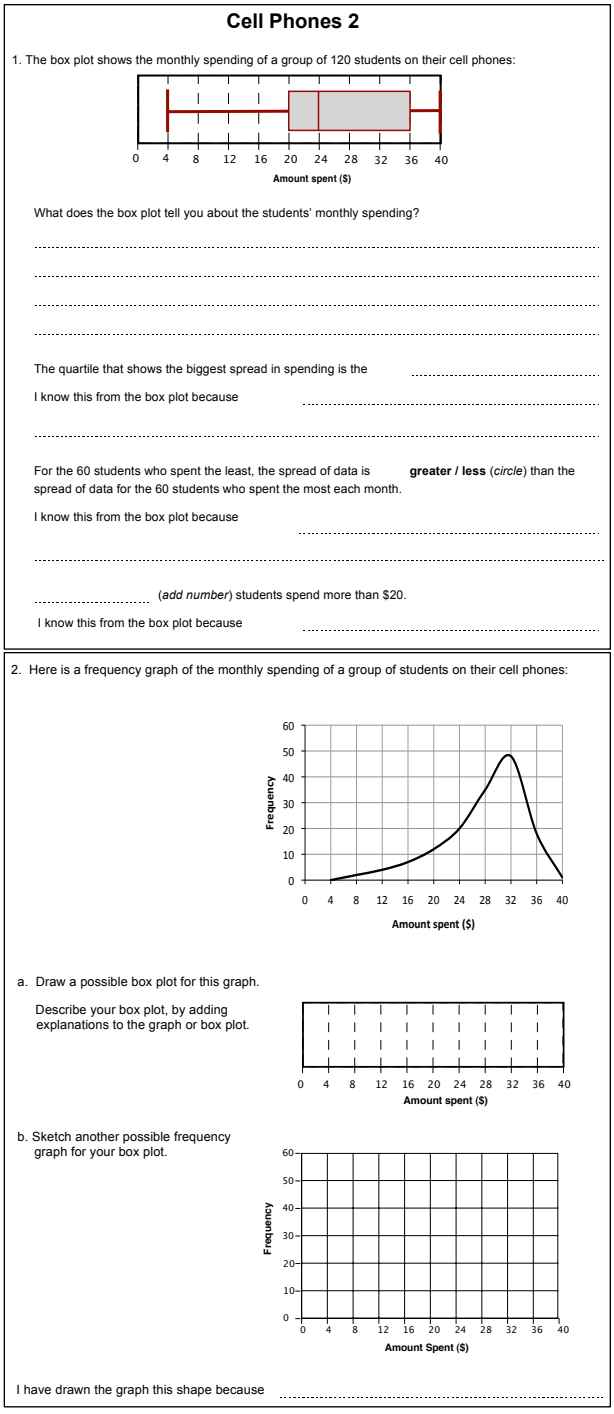
Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the next lesson.

Give each student a copy of *Cell Phones 2* and a rule.

> *Read through the questions and try to answer them as carefully as you can.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.



Cell Phones 2

1. The box plot shows the monthly spending of a group of 120 students on their cell phones:

What does the box plot tell you about the students' monthly spending?

.....

.....

.....

.....

The quartile that shows the biggest spread in spending is the .....

I know this from the box plot because .....

.....

For the 60 students who spent the least, the spread of data is **greater / less** (*circle*) than the spread of data for the 60 students who spent the most each month.

I know this from the box plot because .....

.....

..... (*add number*) students spend more than $20.

I know this from the box plot because .....

2. Here is a frequency graph of the monthly spending of a group of students on their cell phones:

a. Draw a possible box plot for this graph.

Describe your box plot, by adding explanations to the graph or box plot.

b. Sketch another possible frequency graph for your box plot.

I have drawn the graph this shape because .....

Assessing students' responses

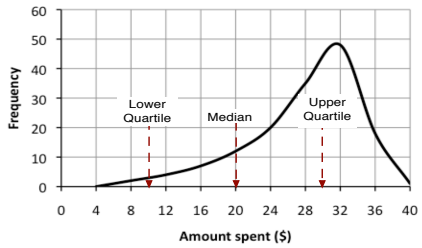
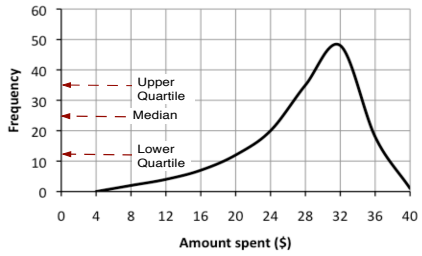
Collect students' responses to the task and note what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on pages T-3 and T-4. We suggest that you make a list of your own questions, based on your students' work, using the ideas on the following page. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight those questions relevant to each student's work.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students in the follow-up lesson.

Common issues	Suggested questions and prompts
Struggles to get started or the description does not include data values (Q1)	• What data values are represented on a box plot? • What do the vertical/horizontal lines represent?
Unable to extract information about the spread of data from the box plot (Q1)	• The two horizontal lines on the box plot are of different lengths. What does this mean? • How many students spend between $20 and $24 a month? How do you know?
Does not understand how to figure out the lower and upper quartiles or the median from the graph (Q2a) For example: The student divides $40 into four equal parts:  Or: The student divides 48 (the maximum frequency) into four equal parts:  	• What does the median represent? • How can you show the median value on the graph? [Draw a vertical line to divide in half the area under the graph. The median amount is the value at the point this line intersects the *x*-axis.] • What does the lower quartile represent? • What does the upper quartile represent? • How can you show the lower/ upper quartile on the graph? [Draw vertical lines to divide in quarters the area under the graph.]
Draws a box plot but provides limited or incorrect explanation of its shape (Q2a)	• Suppose you were to explain your box plot to someone unfamiliar with this type of work. How could you make the math clear, to help the student to understand? • How did you figure out the median amount spent from your graph? [At the point where the area under the curve is split in half.] • How did you figure out the lower and upper quartile from your graph? [At the point where the area under the curve is split in quarter, three quarters.]

Does not sketch a graph (Q2b) or the student does not address all the features of the box plot (Q2b)	• What must remain the same in the second graph? What can change? • What are the maximum and minimum values spent each month? How can you show these on your graph? • What proportion of students spends less than $25 a month? How can you show this on your graph?
Has written little explanation (Q2b)	• How do you know the quartiles/median values?

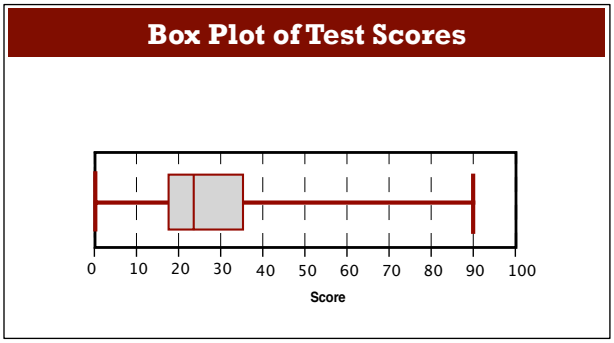
SUGGESTED LESSON OUTLINE

Whole-class introduction (15 minutes)

Give each student a mini-whiteboard, pen, and eraser.

Throughout the introduction encourage students to first tackle a problem individually and only then discuss it with a neighbor. In that way students will have something to talk about. Maximize participation in the whole-class discussion by asking all students to show you solutions on their mini-whiteboards. Select a few students with interesting or contrasting answers to justify them to the class. Encourage the rest of the class to challenge these explanations.

Show Slide P-1 of the projector resource:



Explain to students that the box plot represents the scores of students in a test for which the maximum score was 100.

Students are to write on their mini-whiteboards all the information they can derive from the box plot. After a couple of minutes ask students to show you their answers. Ask one or two students to justify their answers. Even if their explanations are incorrect or only partially correct, write them next to the box plot. Encourage students to challenge these interpretations and then replace them with new ones.

You may want to ask students about the five summary values (minimum, maximum, lower and upper quartiles, and median) and a selection of the following questions:

What does the box plot tell you about this test?
Did the students find it difficult or easy? How can you tell?

Did anyone achieve a score of 100? [No.]

What is the range of scores? [90.]
What is the range of scores for the middle 50% of students? [18.]

The horizontal line to the right of the box is longer than the one to its left. What does this tell you about the spread of scores? [The spread of the top 25% of the scores is bigger than the spread of the bottom 25% of scores. However, we cannot tell from the box plot the density of the data.]

The median is not in the middle of the box. What does this tell you about the distribution of scores? [The range of scores of the second quartile is smaller than the range of scores of the third.]

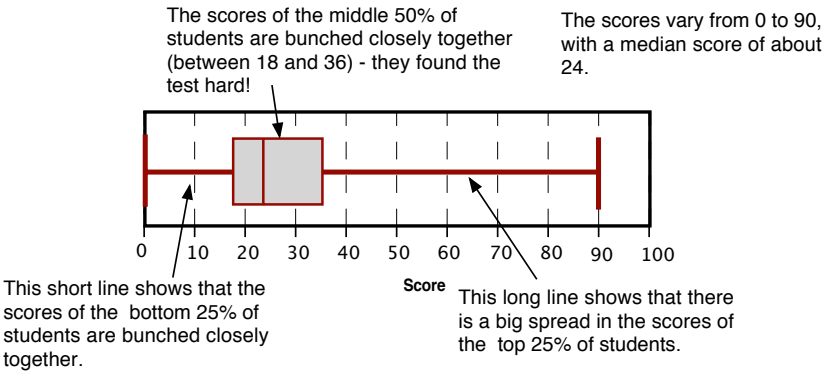
Approximately what proportion of students achieved a score under 36? [75%.]

Can you figure out the mean or mode score from a box plot? [No.]

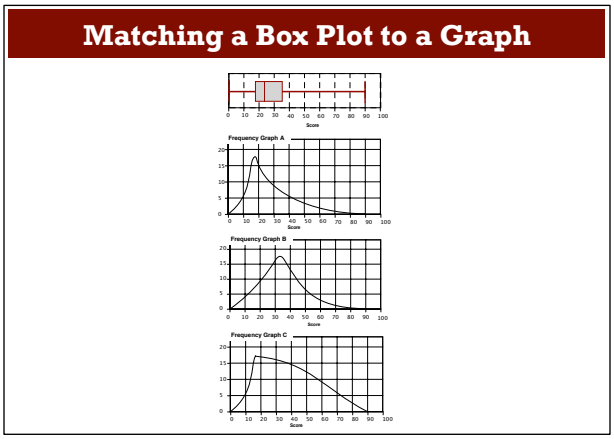
Can you figure out the number of students who took the test from a box plot? [No.]

The intention is that students start to think about more than just the summary values, they begin to explore how the box plots represent the varying density of the data.

The box plot may end up looking like this:



Give each student a copy of *Matching a Box Plot to a Frequency Graph* and show Slide P-2 of the projector resource:



Ask the students to select the graph that best fits the box plot. *[A]*

After a few minutes ask one or two students to justify their answers. Encourage others to challenge these explanations.

> *How can you figure out the median score from the graph?*
> *Does this correspond to the median of the boxplot?*
>
> *How can you figure out the lower or upper quartile score from the graph?*
> *Does this correspond to the quartiles of the boxplot?*

To answer these questions, the area under the graph needs to be divided in half/quarters by vertical lines extending from the *x*-axis.

This introduction models the depth of reasoning students should work to during the collaborative task.

Collaborative activity: Matching Card Sets to Box Plots (20 minutes)

Organize the class into groups of two or three students.

Give each group the *Card Set: Box Plots*. If this lesson is a sequel to the lesson *Representing Data with Frequency Graphs* give back the posters the students produced and ask them to match the new

cards with the ones already on their poster. Otherwise give students *Card Set: Frequency Graphs* and ask them to match these cards with the *Box Plot* cards.

Take turns at matching pairs of cards that you think belong together.

Each time you do this explain your thinking clearly and carefully.

Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.

You both need to be able to agree on and explain the placement of every card.

Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.

Write your reasons for each match on the poster.

These instructions are summarized on Slide P-3 of the projector resource, *Matching Cards*.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student reasoning.

Note different student approaches to the task

In particular, notice any difficulties students encounter with what they are doing and the ways they justify and explain to each other. Do students check to see if their match is correct? Do they just focus on the summary values? When figuring out a measure from a graph, do students confuse the values on the *x*-axis and *y*-axis? Are students using the correct mathematical language? Do they talk about the spread of the data? What do they do if they get stuck?

You can then use this information to focus your questioning in the whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards a particular categorization. Instead, ask questions to help students to reason together. You may want to use some of the questions and prompts from the *Common issues* table.

If a student struggles to get started, encourage them to ask a specific question about the task. Articulating the problem in this way can sometimes offer a direction to pursue that was previously overlooked. However, if the student needs their question answered, ask another member of the group for a response.

You may want to ask students about the five summary values and a selection of the following questions:

What does the width of this box tell you about the scores?

What does the width of this horizontal line tell you about the scores?

Show me a box plot in which a lot of students/a few students found the test easy? How do you know?

Which box plot shows a lot of students of a similar ability? How do you know?

If you find one student has matched two cards, challenge another student in the group to provide an explanation.

Daisy matched these cards. Tyrrell, why does Daisy think these two cards go together?

If you find the student is unable to answer this question, ask them to discuss the work further. Explain that you will return in a few minutes to ask a similar question.

If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold a brief whole-class discussion.

Sharing posters (10 minutes)

As students finish matching the cards, ask one student from each group to visit another group's poster.

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, copy your matches onto a piece of paper.

Go to another group's desk and check to see which matches are different from your own.

If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider as a group whether to make any changes to your poster.

You may want to use Slide P-4 of the projector resource, *Sharing Posters*, to display these instructions.

When students are satisfied, give them a glue stick. They are to glue the matched cards onto the poster.

Whole-class discussion (15 minutes)

You may want to use transparencies of the cards or Slide P-5 of the projector resource to support the discussion.

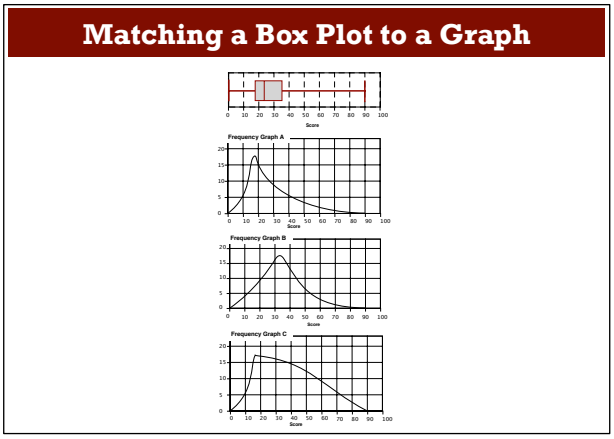
The intention is that this discussion focuses on the justification of a few examples, rather than checking students all have the correct solutions. You could first select a pair of cards that most groups matched correctly, as this should encourage good explanations. Then select one or two matches that most groups found difficult. In trials students have had difficulty matching graphs E, F, and G.

How did you decide to match this card?

Can someone else put that into their own words?

Could this card be matched with another one?

After discussing a few matches, if you have time show students Slide P-2 of the projector resource again:



Ask the following question:

Could there be other graphs that also matched the box plot? [Yes.]

Ask students to sketch a graph either on their mini-whiteboard or on top of one of the graphs on the *Matching a Box Plot to a Frequency Graph* handout.

Ask several students with different sketches to show and explain their graphs to the class. Encourage the rest of the class to ask questions and challenge these explanations. In this discussion you may want to ask the following questions in turn:

What does a box plot tell you for definite about the data?

What does a box plot not tell you about the data?

Do you know if the scores in the top quartile are evenly distributed? What about outliers?

Suppose the median is 34 instead of 24. How would this change the graph? Sketch it.

Again ask several students with different sketches to show and explain their graphs to the class.

Could there be other box plots that matched the Graph A? [No.]

If students think there is more than one box plot that will match *Graph A* then ask them to sketch one and justify it.

Follow-up lesson: reviewing the assessment task (20 minutes)

Return the original assessment *Cell Phones 2* to the students, together with a copy of *Cell Phones 2 (revisited)*. If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Look at your original responses and the questions (on the board/written on your script.)
Answer these questions and revise your response.

When you revise your work, write as if you are explaining the solutions to someone unfamiliar with this type of math.

Now look at the new task sheet, Cell Phones 2 (revisited).
Use what you have learned to answer these questions.

Some teachers give this as a homework task.

SOLUTIONS

Assessment task: *Cell Phones 2*

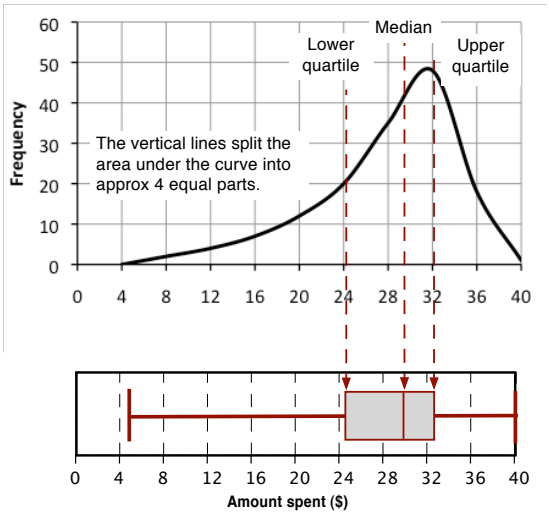
1. The minimum amount spent each month is $4 and the maximum amount spent is $40, giving a range of $36. The lower quartile is $20, the upper quartile is $36 and the median amount spent is $24. This means 30 students spent between $4 and $20, another 30 spent between $20 and $24, a third 30 spent between $24 and $36, and the rest spent between $36 and £40. The box plot shows there is a much greater spread of the amount spent in the lower quartile compared to the upper quartile. Half the students spent under $24.

   The quartile that shows the biggest spread in spending is the lower quartile. I know because this quartile has the biggest range of spending.

   For the 60 students who spent the least, the spread of data is **greater** than the spread of data for the 60 students who spent the most each month. I know this because this half of the data has the biggest range of spending.

   90 students spend more than $20. I know this because the lower quartile is $20. This means 30 students spend less than £20.

2a. Here is the box plot for the frequency graph.

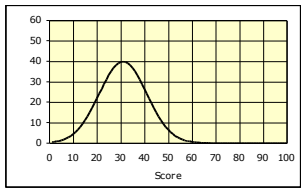


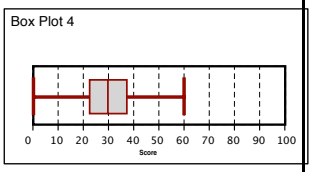
   The median should be less than the mode of $31. The median should be slightly closer to the upper quartile than the lower quartile. By way of explanation, students should split the area under the frequency graph into four equal areas, marking on the lower and upper quartiles and the median.

2b. There are many possible graphs students could sketch.

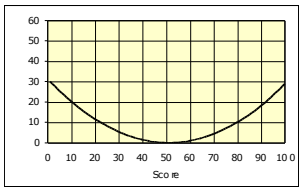
Lesson task:

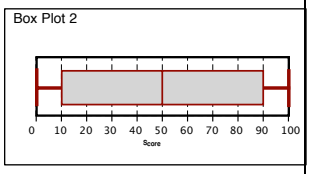
Graph A



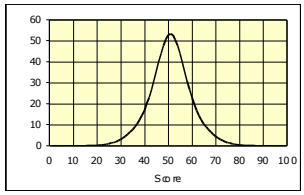


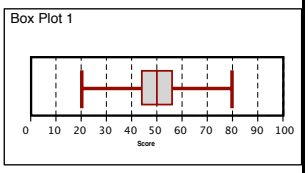
Graph B



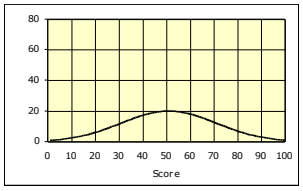


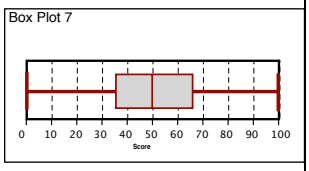
Graph C



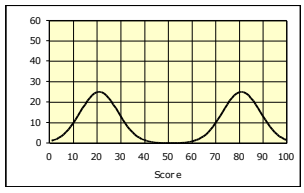


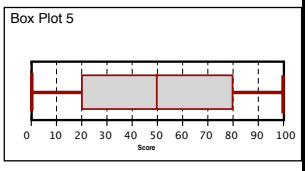
Graph D



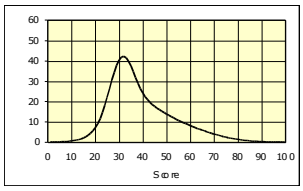


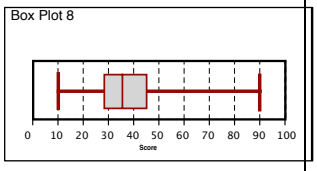
Graph E



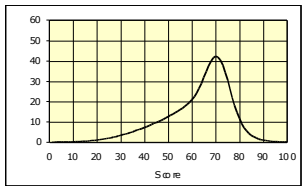


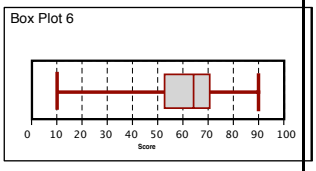
Graph F



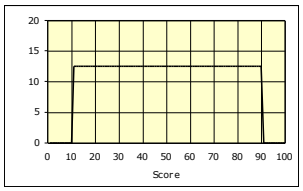


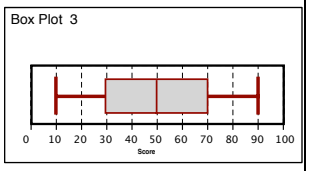
Graph G





Graph H





Assessment task: *Cell Phones 2 (revisited)*

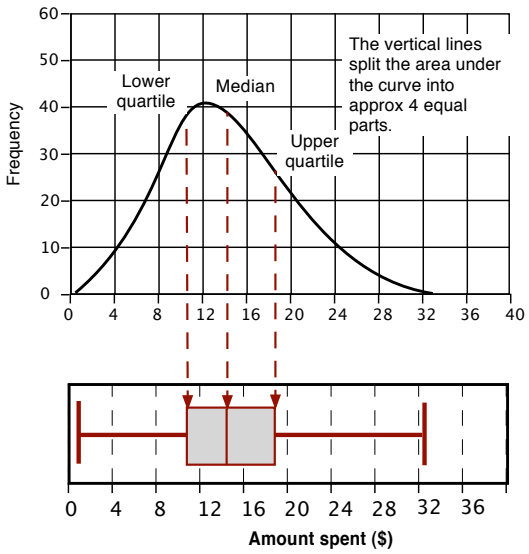
1. The minimum amount spent each month is $0 and the maximum amount spent is $36, giving a range of $36. The lower quartile is $12, the upper quartile is $32 and the median amount spent is $28. This means 40 students spent between $0 and $12, another 40 spent between $12 and $28, a third 40 spent between $28 and $32, and the rest spent between $32 and £36. The box plot shows there is a much greater spread of the amount spent in the lower quartiles compared to the upper quartiles. Half the students spent under $28. The biggest spread of spending is in the second quartile.

   The quartile that shows the biggest spread in spending is the second quartile. I know because this quartile has the biggest range of spending.

   For the 80 students who spent the least, the spread of data is **greater** than the spread of data for the 80 students who spent the most each month. I know this because this half of the data has the biggest range of spending.

   120 students spend less than $32. I know this because the upper quartile is $32. This means 40 students spend more than £32.

2a. Here is the box plot for the frequency graph.



The median should be greater than the mode of $12. By way of explanation, students should split the area under the frequency graph into four equal areas, marking on the lower and upper quartiles and the median.

2b. There are many possible graphs students could sketch.

Cell Phones 2

1. The box plot shows the monthly spending of a group of 120 students on their cell phones:

What does the box plot tell you about the students' monthly spending?

The quartile that shows the biggest spread in spending is the

I know this from the box plot because

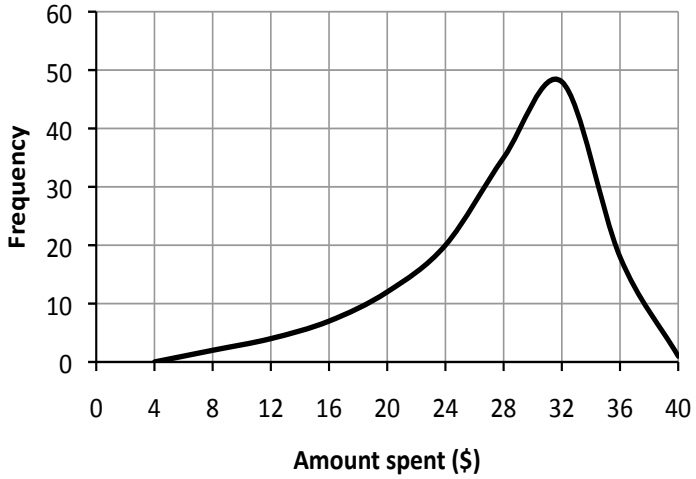
For the 60 students who spent the least, the spread of data is **greater / less** (*circle*) than the spread of data for the 60 students who spent the most each month.

I know this from the box plot because

(*add number*) students spend more than $20.

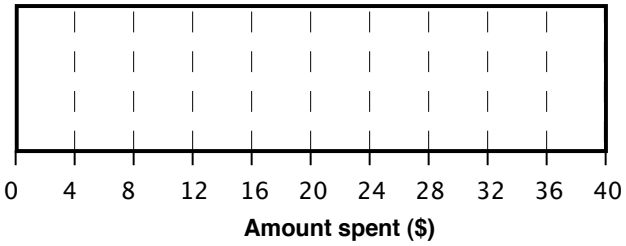
I know this from the box plot because

2. Here is a frequency graph of the monthly spending of a group of students on their cell phones:

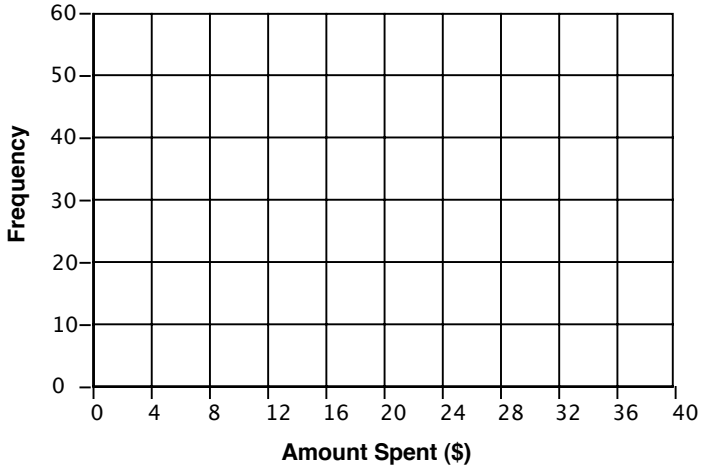


a. Draw a possible box plot for this graph.

Describe your box plot, by adding explanations to the graph or box plot.

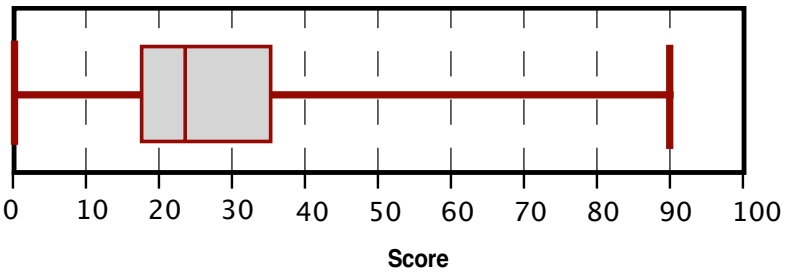


b. Sketch another possible frequency graph for your box plot.

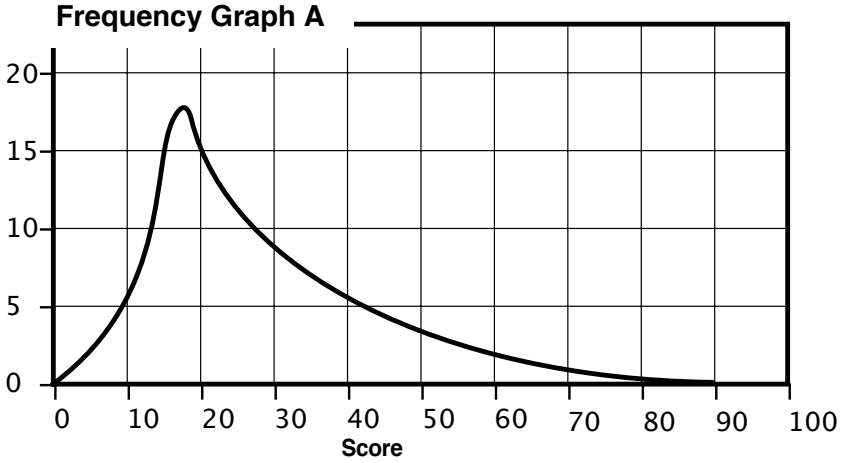


I have drawn the graph this shape because

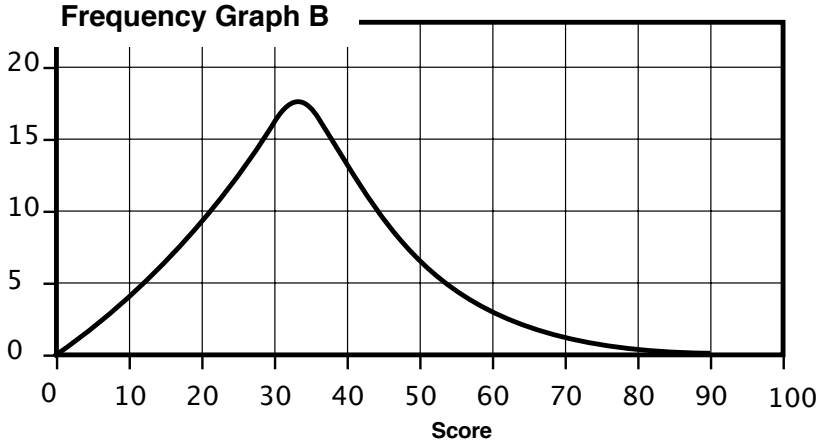
Matching a Box Plot to a Frequency Graph



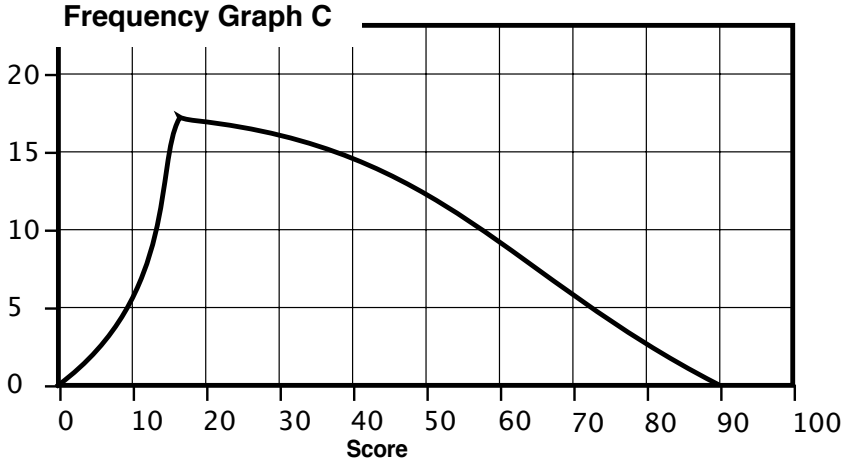
Frequency Graph A



Frequency Graph B

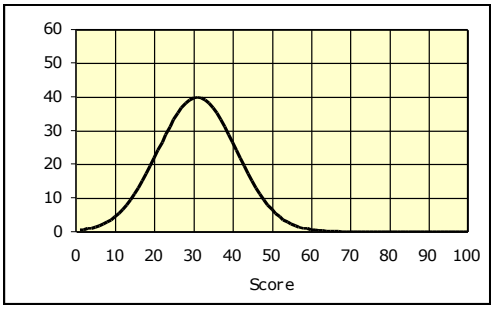


Frequency Graph C

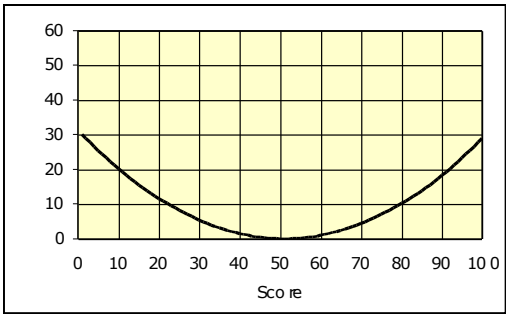


Card Set: Frequency Graphs

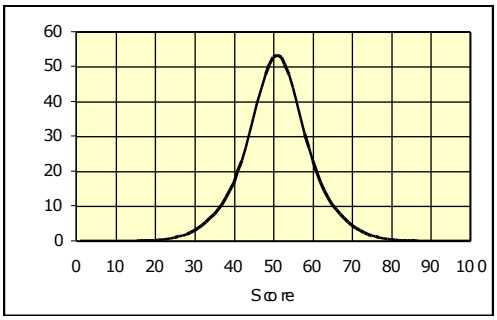
Frequency Graph A



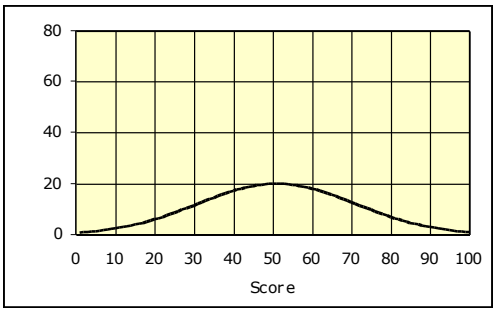
Frequency Graph B



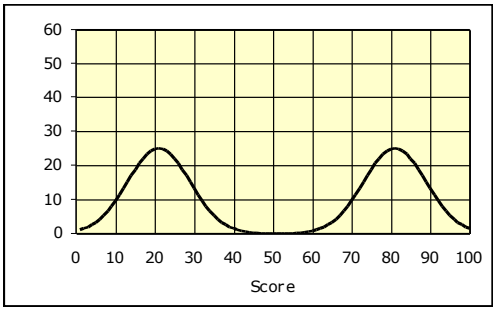
Frequency Graph C



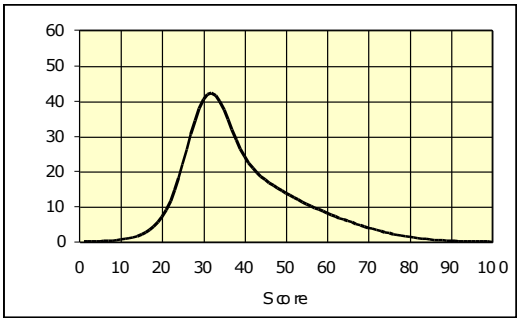
Frequency Graph D



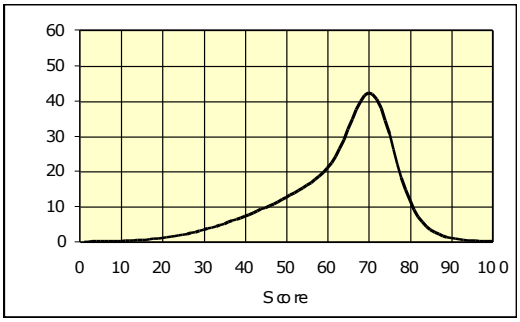
Frequency Graph E



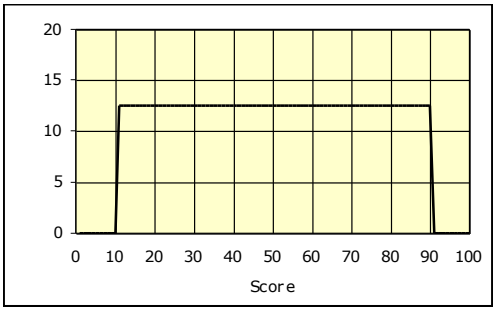
Frequency Graph F



Frequency Graph G

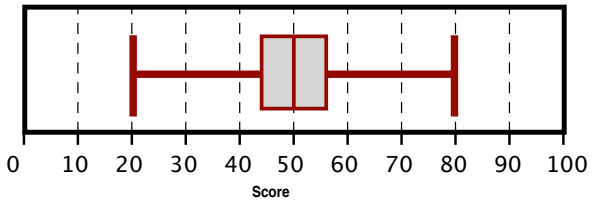


Frequency Graph H

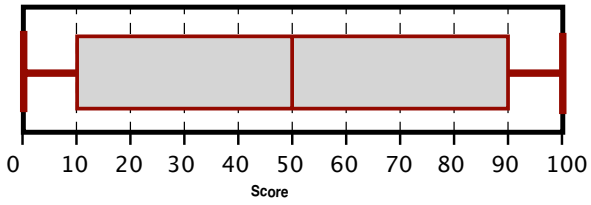


Card Set: Box Plots

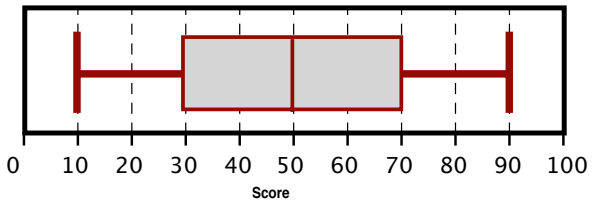
Box Plot 1



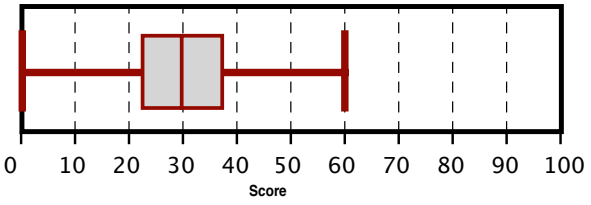
Box Plot 2



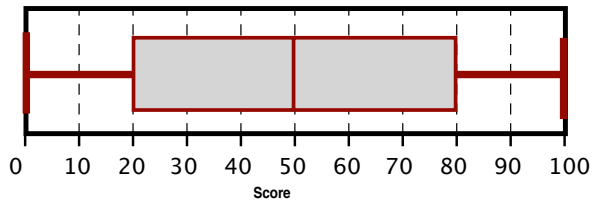
Box Plot 3



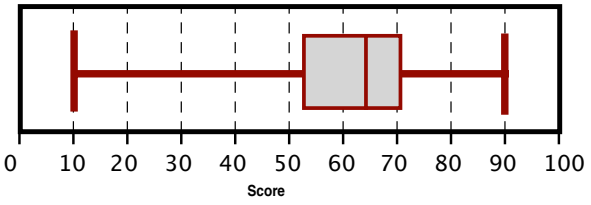
Box Plot 4



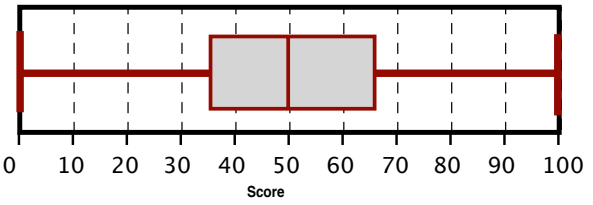
Box Plot 5



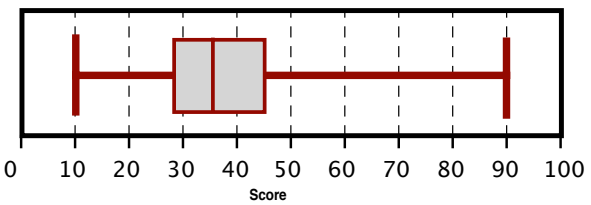
Box Plot 6



Box Plot 7

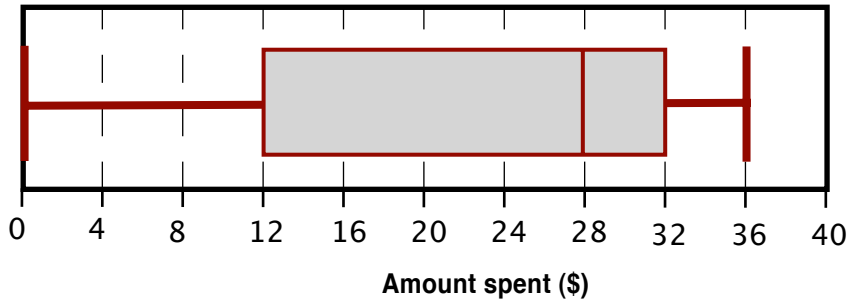


Box Plot 8



Cell Phones 2 (revisited)

1. The box plot shows the monthly spending of a group of 160 students on their cell phones:



What does the box plot tell you about the students' monthly spending?

The quartile that shows the biggest spread in spending is the _____

I know this from the box plot because _____

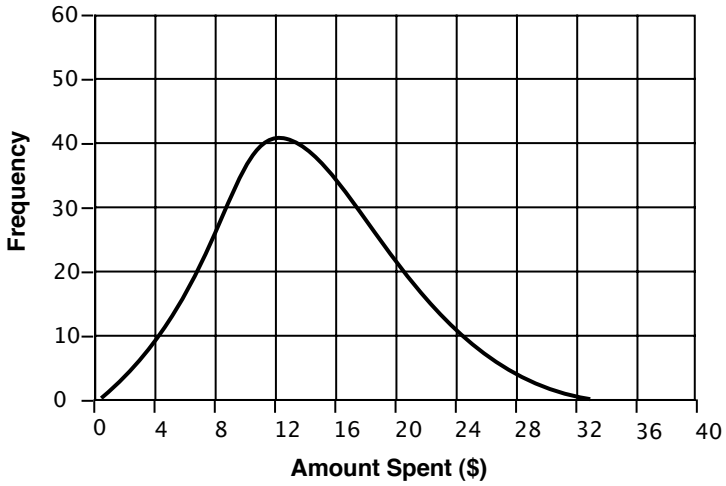
For the 80 students who spent the least, the spread of data is **greater / less** (*circle*) than the spread of data for the 80 students who spent the most each month.

I know this from the box plot because _____

_____ (*add number*) students spend less than $32.

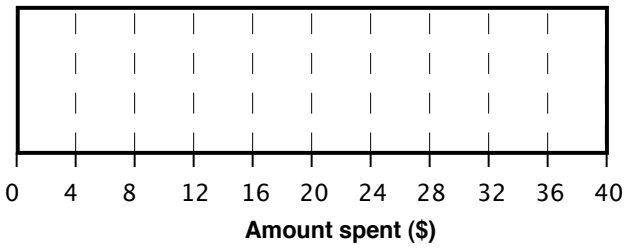
I know this from the box plot because _____

2. Here is a frequency graph of the monthly spending of a group of students on their cell phones:

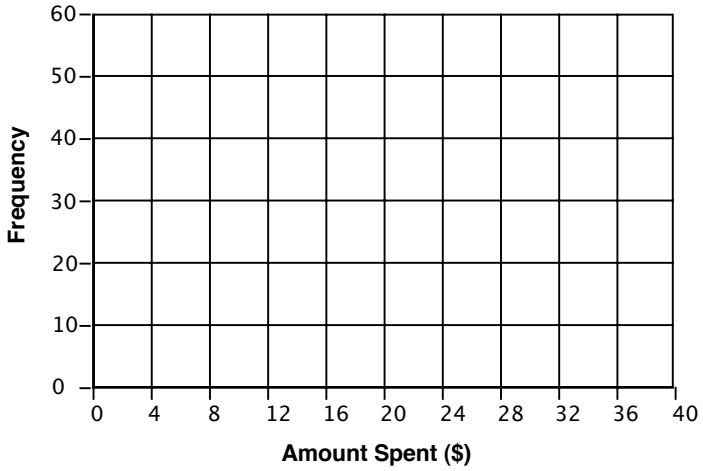


a. Draw a possible box plot for this graph.

Describe your box plot, by adding explanations to the graph or box plot.



b. Sketch another possible frequency graph for your box plot.

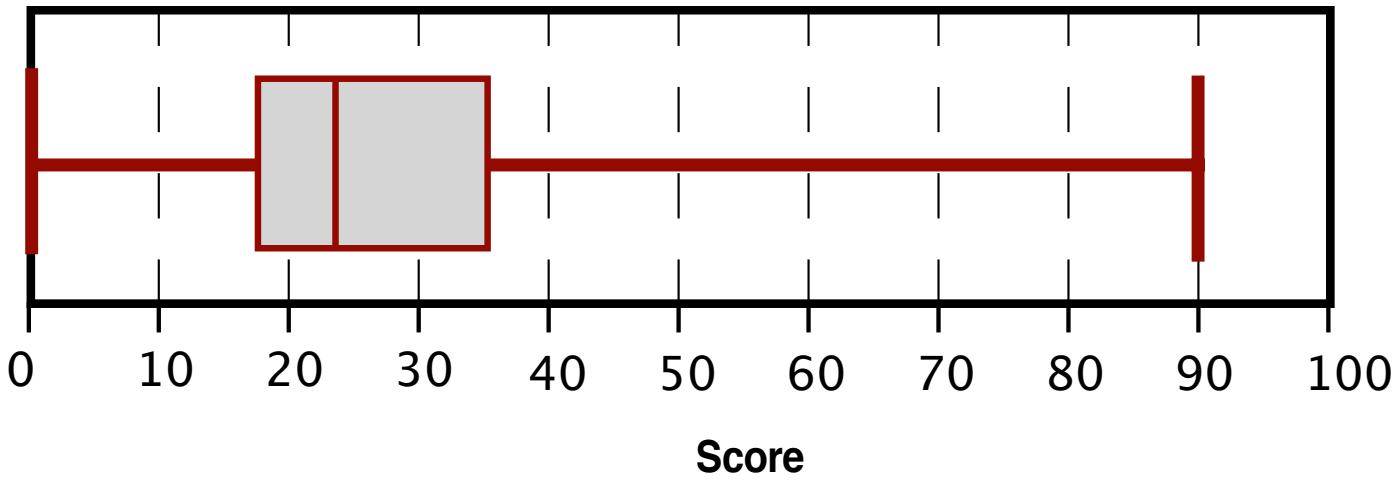


I have drawn the graph this shape because .....

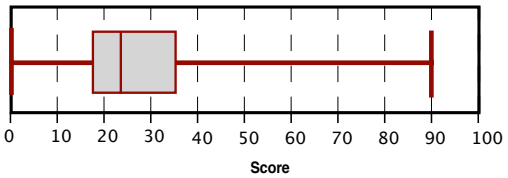
.....

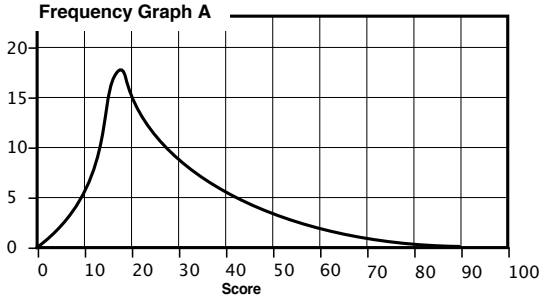
.....

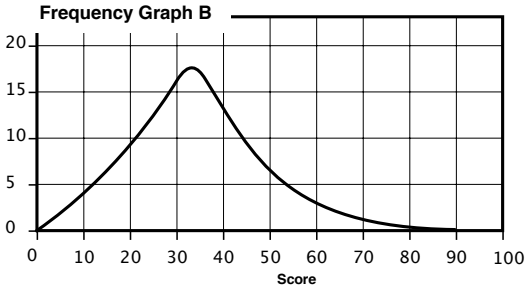
Box Plot of Test Scores

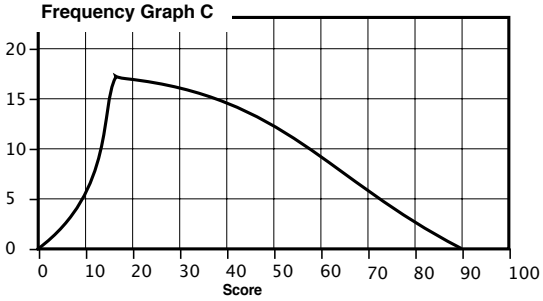


Matching a Box Plot to a Graph









Matching Cards

1. Take turns at matching pairs of cards that you think belong together.

2. Each time you do this, explain your thinking clearly and carefully.

3. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.
4. Write your reasons for each match on the poster.

You both need to be able to agree on and explain the placement of every card.

Sharing Posters

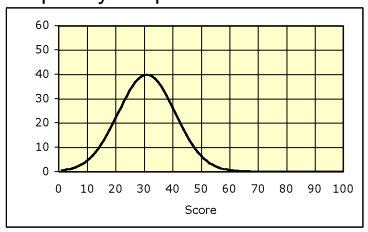
1. If you are staying at your desk, be ready to explain the reasons for your group's matches.

2. If you are visiting another group:
   - Copy your matches onto a piece of paper.
   - Go to another group's desk and check to see which matches are different from your own.
   - If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

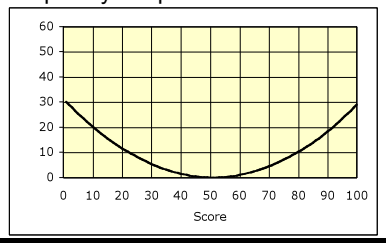
3. When you return to your own desk, you need to consider as a group whether to make any changes to your poster.

Card Sets

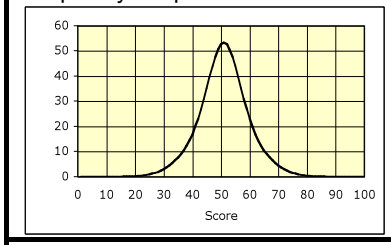
Frequency Graph A



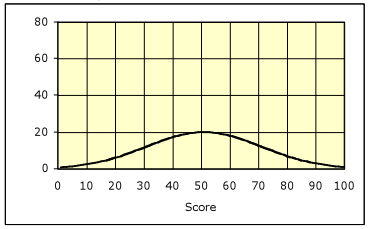
Frequency Graph B



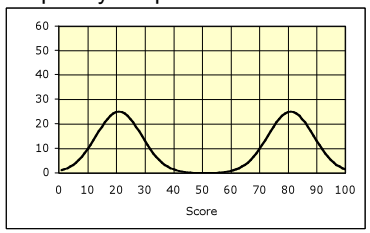
Frequency Graph C



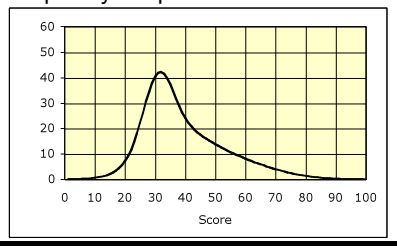
Frequency Graph D



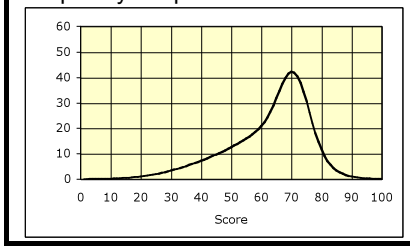
Frequency Graph E



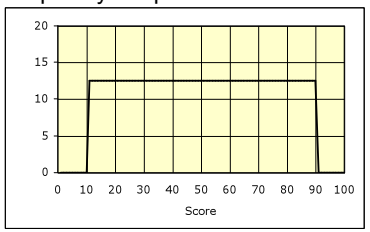
Frequency Graph F



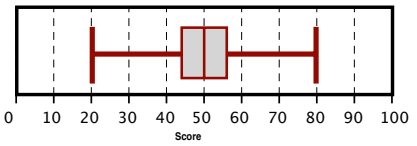
Frequency Graph G



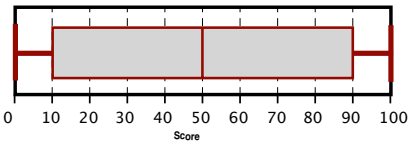
Frequency Graph H



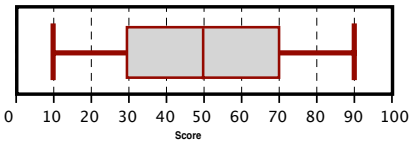
Box Plot 1



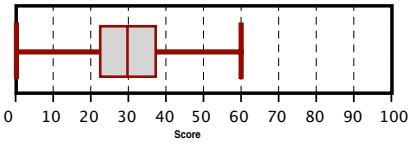
Box Plot 2



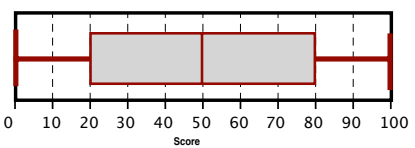
Box Plot 3



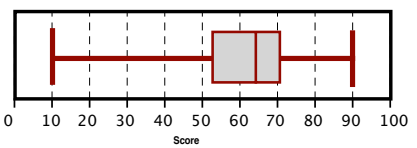
Box Plot 4



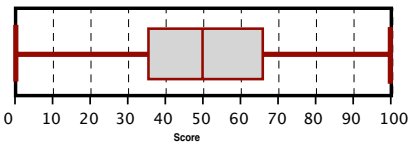
Box Plot 5



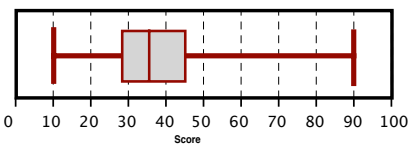
Box Plot 6



Box Plot 7



Box Plot 8



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
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David Foster, Mary Bouck, and **Diane Schaefer**

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Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org