CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Transforming 2D Figures

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Transforming 2D Figures

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Describe in words the transformation that maps an object to a transformed image.
- Given a geometric figure and a rotation, reflection or translation, draw the transformed figure (or the original figure if the image is given.)
- Describe transformations as algebraic functions that take points in the plane as inputs and give other points as outputs.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

G-CO: Experiment with transformations in the plane.
Understand congruence in terms of rigid motions.
A-SSE: Interpret the structure of expressions.

This lesson also relates to **all** the *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 4, 5, 7, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work individually on a task designed to reveal their current understanding and difficulties. You then review their solutions and create questions for students to consider in order to improve their work.
- After a whole-class introduction, students work in pairs or threes on a collaborative task, describing transformations and completing the transformed image/original figure for a given transformation. The lesson ends with a whole-class discussion of the work.
- In a follow-up lesson, students work alone on a new assessment task, or return to the original task and try to improve their responses.

MATERIALS REQUIRED

- Each individual student will need a copy of *Figure to Figure* and *Figure to Figure (revisited)*, a mini-whiteboard, pen, and eraser.
- Each small group of students will need the cut-up *Card Set: Transformations*, a glue stick, a marker pen, and a large sheet of poster paper. Tracing paper should be made available on request.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson, a 90-minute lesson (or two 50-minute lessons), and 15 minutes in a follow-up lesson. Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Figure to Figure* (15 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of *Figure to Figure*. Introduce the task briefly and help the class to understand what they are being asked to do:

> *This task is all about transformations.*
>
> *Each question contains two grids labeled 'Before' and 'After'. 'Before' refers to the original figure before it has been transformed and 'After' refers to the transformed image. For some questions you are asked to draw in either the object or the image. You can draw this on the other grid or on the same grid as the figure given - whichever you prefer.*
>
> *Read through the questions and try to answer them as carefully as you can. Show all your work on the sheet.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in some tasks, which should help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions like these confidently. This is their goal.

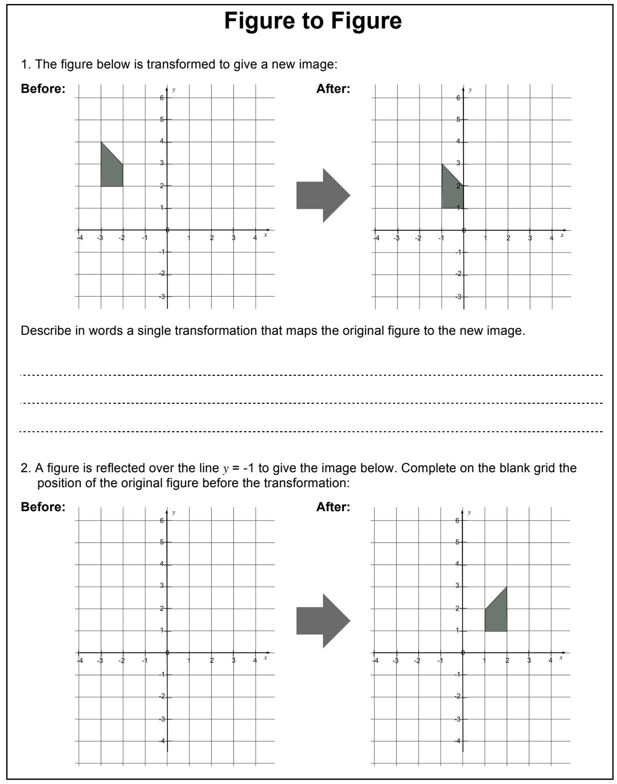


Figure to Figure

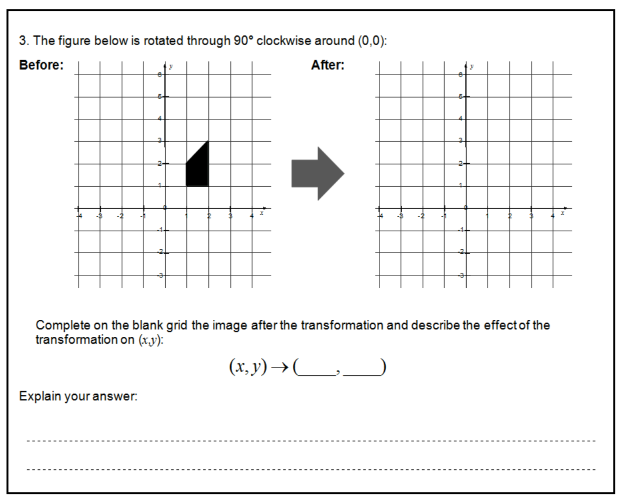
1. The figure below is transformed to give a new image:

Before: After:

Describe in words a single transformation that maps the original figure to the new image.

2. A figure is reflected over the line y = -1 to give the image below. Complete on the blank grid the position of the original figure before the transformation:

Before: After:



3. The figure below is rotated through 90° clockwise around (0,0):

Before: After:

Complete on the blank grid the image after the transformation and describe the effect of the transformation on (x,y):

$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

Explain your answer:

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the page T-4. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students in the follow-up lesson.

Common issues:	Suggested questions and prompts:
Confuses the terms 'horizontally' and 'vertically' For example: The student describes the transformation as a translation +2 units vertically and −1 unit horizontally in Q1.	• Look at the start of the word 'horizontally'. What are we referring to when we talk about the horizon? Which way is this?
Determines the translation by mapping a point in the original figure to a non-corresponding point on the transformed image (Q1) For example: The student describes the transformation as a translation +1 unit horizontally and −1 unit vertically (by mapping the bottom right vertex on the original figure to the bottom left vertex on the transformed image.)	• Choose a vertex of the original figure. If you translate this point +1 unit horizontally and −1 unit vertically where does it end up?
Translates rather than reflects the shape (Q2) For example: The student has translated the figure -5 units horizontally and so omitted to draw the mirror image.	• If you were to place a mirror on line $y = -1$, what would the reflected image look like?
Reflects in the wrong line For example: The student reflects in the line $y = 1$ or $x = -1$ (Q2.)	• Can you name a co-ordinate that lies on the line $y = -1$? And another? What do you notice?
Confuses the terms 'clockwise' and 'counterclockwise' For example: The student rotates the figure counter clockwise (Q3.)	• Think about the direction of the hands on a clock. This direction is 'clockwise'.
Ignores the center of rotation and rotates from a corner of the original figure For example: The student rotates around the point (1, 1) (Q3.)	• Where is the center of rotation? • Mark the center of rotation and draw a line to a corner of the shape. Where will this line be once it has been rotated?
Fails to complete the algebraic notation (Q3)	• Where has the point (1, 1) been mapped to after the transformation? • Can you use the mappings for all four of the figure's vertices to describe the effect on the point (x, y)?
Omits an explanation (Q3) For example: The student correctly completes the algebraic notation but does not explain their answer.	• What does (x, y) -> $(-y, x)$ mean? • Why does this describe a rotation through 90° around (0, 0)?
Correctly answers all the questions The student needs an extension task.	• Can you describe the transformations in Q1 and Q2 algebraically? What would be the effect of the transformation on (x, y)?

SUGGESTED LESSON OUTLINE

Whole-class interactive introduction (15 minutes)

Give each student a mini-whiteboard, pen, and eraser.

Remind the class of the assessment task they have already attempted.

Recall what we were working on previously. What was the task about? [Transformations: Rotations, Reflections and Translations.]

One of the things you were asked to do in the task was describe a transformation in words. Today you are also going to be describing transformations, as well as being given some descriptions of transformations to work with.

Display Slide P-1 of the projector resource:

Describing Transformations

A: Rotation	1. A line
	2. A vertical distance
	3. An angle
B: Reflection	4. An axis
	5. A direction
	6. A center
C: Translation	7. A horizontal distance
	8. A scale factor

When we describe a transformation using words we need to make sure that we include all the information required to describe the transformation exactly.

Some transformations can be described by just one piece of information whereas some transformations need more than one piece of information to describe them.

On your mini-whiteboard write down which piece or pieces of information go with which transformation. For example, if you think that the first piece of information goes with 'Translation', write C1 and so on.
There may be some pieces of information that aren't used for any of the three transformations listed.

Students should complete this task individually and will need to refer to their choices throughout the whole-class discussion that follows.

Once students have had a chance to identify the information required to describe each of the three transformations, ask to see their mini-whiteboards to check that they each have an answer and to get an overview of any contrasting opinions. Then spend a few minutes going through each of the three transformations and checking that the students understand what information is required to fully describe each.

Amy, which information did you match to 'Rotation'? Did anyone match up different information to this?

Once students are happy with the information required for each of the three transformations (A: 3, 5, & 6, B: 1 and C: 2, 5, & 7 (2 and 7 not always both given if one of the distances is zero)) you may want to briefly discuss the information that wasn't used (4 & 8) and identify when these pieces of information are needed:

When do we use an axis to describe a transformation? [When describing a reflection over the x-axis, for example.]

When do we use scale factor to describe a transformation? [When describing an enlargement or one-way stretch, for example.]

When else might we use a center to describe a transformation? [When describing an enlargement.]

Students may be confused over whether a rotation is around a point or around an axis. When working solely in 2 dimensions, it is convenient to envisage rotation of a figure around a point in the plane, but in our 3-dimensional world we would imagine an axis (e.g. a spike) passing perpendicularly through the point and the shape turning around this axis. When a 3-dimensional shape is rotated, this must be around an axis.

Display Slide P-2 of the projector resource:

Algebraic Notation Scenario (1)

If the x co-ordinates for all the points are increased by three how could we describe this?

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)?$$

When a figure is transformed, some or all of the points on the shape move position.

If the x-coordinates for all the points were increased by three, how could we describe this for a general point (x, y)? [(x, y) -> (x+3, y).]

On your mini-whiteboard copy and complete the algebraic description.

Students may not be familiar with this notation, so take time to ensure that everyone knows what is being represented.

Display Slide P-3 of the projector resource:

Algebraic Notation Scenario (2)

If both the x and y co-ordinates are doubled how could we describe this?

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)?$$

What about if both the x- and y-coordinates were doubled, how could we describe this algebraically? [(x, y) -> (2x, 2y).]

Students may well come up with a variety of answers for these. Do not discuss this at this stage but note any common misconceptions that arise. Explain to students that during this lesson they will be given some transformations that have been described using this notation.

Collaborative work (35 minutes)

Ask students to work in groups of two or three. Give each group *Card Set: Transformations* (already cut-up), a large sheet of poster paper, and a glue stick. If poster paper is not available, students could use three separate sheets, one for each type of transformation.

Introduce the activity:

You've got eight Transformations cards, each showing two out of a possible four pieces of information:

(1) *A transformation described using the algebraic notation we have just discussed.*

(2) *A transformation described in words.*

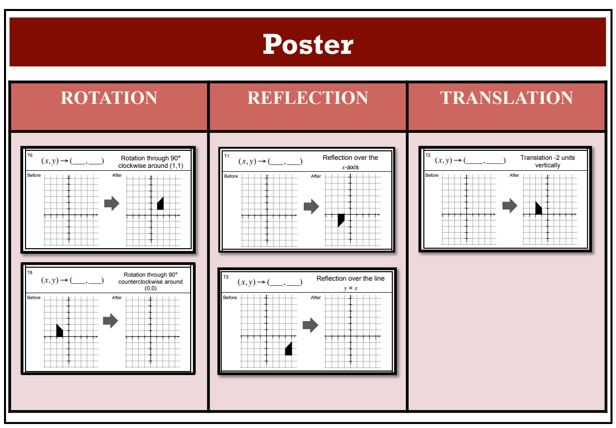
(3) *The original figure before the transformation has been applied.*

(4) *The transformed image.*

Your job is going to be to complete the cards with the missing information and group the cards depending on whether they are describing rotations, reflections or translations.

Divide your large sheet of paper into three columns, one for each type of transformation: Rotation, Reflection, Translation.

Show Slide P-4 to illustrate this:



Describe how the students are to work in their groups:

Take turns to complete a card. Each time you do this; explain your thinking clearly and carefully.

Your partner should then either explain your reasoning again in his or her own words, or challenge the reasons you gave.

It is important that everyone in the group understands the transformation descriptions and figure positions.

Once you are all agreed on the completion of a card, glue it in the appropriate column on your poster paper.

Leave the summary of these instructions (Slide P-5 of the projector resource) displayed so that students know what to do.

You have two tasks during the paired work: to make a note of student approaches to the task and to support students working as a group.

Make a note of student approaches to the task

Listen and watch students carefully. In particular, notice how students attempt the task, where they get stuck and how they overcome any difficulties.

Do they sort the cards in any way before they begin to complete them? If so, how?

Do students start with the cards that have a written description completed and draw the missing figure before completing the algebraic notation? If so, how do they find the missing expressions in terms of x and y?

How do they use the algebraic description to complete the card?

Do they complete the missing figure first or use cards that they have completed already to interpret the transformation in words?

Notice whether students are addressing the difficulties they experienced in the assessment. For example, are students having difficulty rotating a shape around (1, 1) or reflecting a shape over the line $y = x$?

Support student reasoning

As students work on the task, support them in working together. Encourage them to take turns and if you notice that only one partner is matching cards or that they are not working collaboratively on the task, ask students in the group to explain a card completed by someone else in the group.

Liam has completed this card. Laura, can you explain how he knew this was a rotation?

Encourage students to explain carefully how they have made each connection. If students in the group take different approaches when completing cards, encourage them to clearly explain their chosen approach.

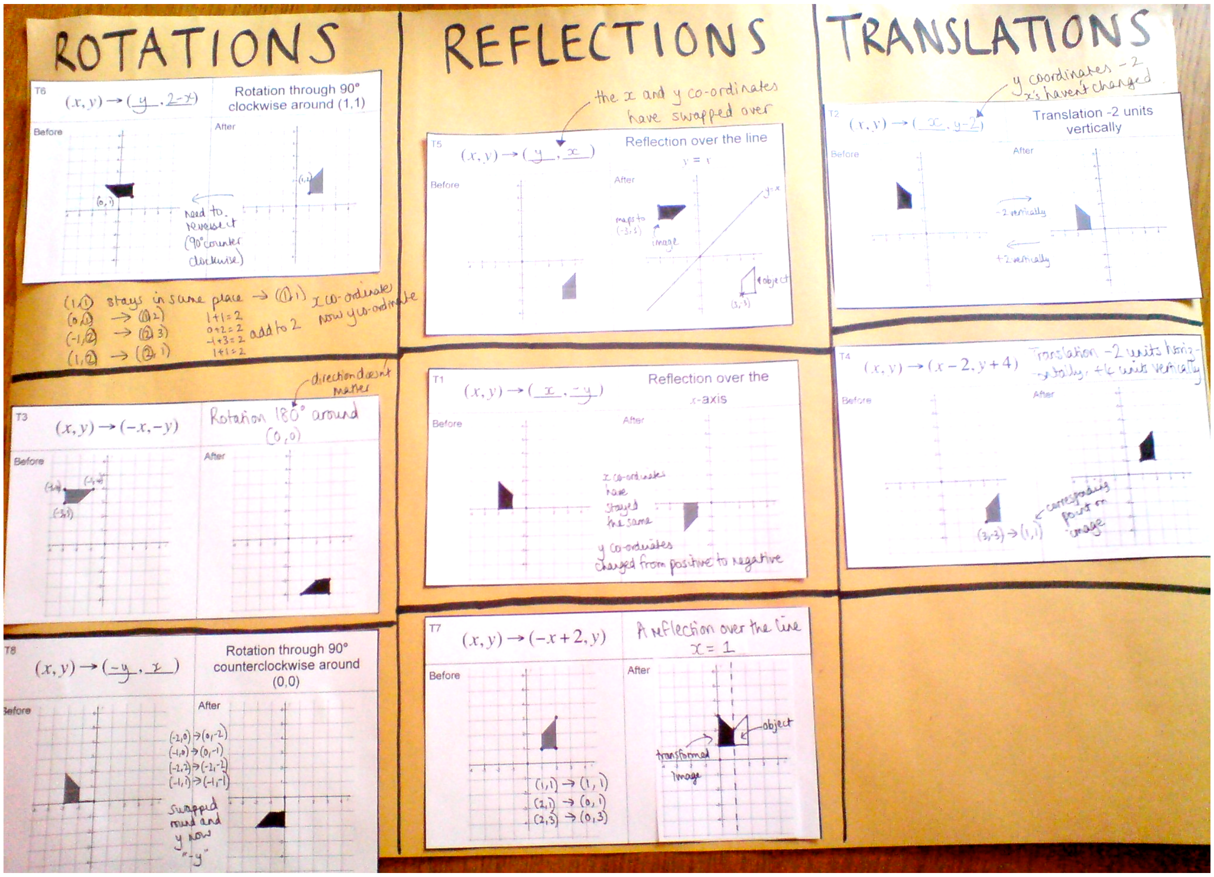
Try to avoid giving students the information they need to complete a card. If students are struggling to get started, encourage them to concentrate on trying to complete T1 and/or T8 first. You could suggest using tracing paper if that might help.

Some students may not complete the algebraic notation on the cards. Check that they have all four pieces of information complete before they glue a card down.

It is not essential that students complete all of the cards but rather that they are able to develop effective strategies for completing the cards.

If students do successfully complete all of the cards, encourage them to generalize regarding the algebraic descriptions. They know that these descriptions work for these particular cases, but will they always work? How could they check? What do they notice?

The finished poster may look something like this:



Extending the lesson over two days

If you are taking two days to complete the unit you might want to end the first lesson here. Make sure that students have stuck down all of their completed cards onto their poster. Then, at the start of the second day, allow the students time to familiarize themselves with their work, before asking them to share their work with another group.

Sharing work (20 minutes)

When students have completed the task, ask them to check their work against that of a neighboring group:

Check to see if there are cards that have been placed in different columns to your own categorizations. If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

You need to compare the completed information on the cards as well as which column they have been placed in.

Once you have compared the cards, you then need to consider whether to make any changes to your own poster.

Whole-class discussion (20 minutes)

It is likely that some groups will not have completed all of the cards. The aim of this discussion is not to check answers but to explore the different strategies used by students when completing the cards, as well as identifying areas in which students struggled and discussing any links they may have made.

First select a card that most groups completed correctly. Once one group has described how they completed the card and how they categorized the card as a Rotation, Reflection, or Translation (if it wasn't already given), ask other students to contribute any alternative strategies used.

Did anyone complete this card differently?

Aim to discuss at least one of each of the three transformations. Use your knowledge of the students' individual and group work to call on a wide range of students for contributions.

Susan, please describe how you completed a Rotation/Reflection/Translation card.

Once the completion of at least one of each of the three types of transformation card has been discussed, explore further the different strategies used when completing the cards and discuss how students overcame any difficulties they had when completing the cards.

Which cards were the easiest to complete? Why was this?

Which cards were difficult to do? Why was this?

When completing the cards, did you always start by drawing in the missing object/image? Why was this? Did anyone use a different strategy?

Did anything surprise you about transformations in today's lesson? What was it? Why was it surprising?

Did you change your mind about completing any of the cards? Why?

What do you think are the big things you have learned from today's lesson?

You may want to draw on the questions in the *Common issues* table to support your own questioning. Slides P-6 to P-13 (printed on transparency film if desired) may be used to support this discussion.

Follow-up lesson: reviewing the assessment task (15 minutes)

Give each student a copy of the assessment task *Figure to Figure (revisited)* and their original solutions to the assessment task *Figure to Figure*. If you have not added questions to individual pieces of work then write your list of questions on the board. Students then select from this list only those questions they think are appropriate to their own work.

Look at your original responses to the Figure to Figure task and the questions [on the board/written on your paper.] Answer these questions and revise your response.

Now look at the new task sheet, Figure to Figure (revisited). Use what you have learned to answer these questions.

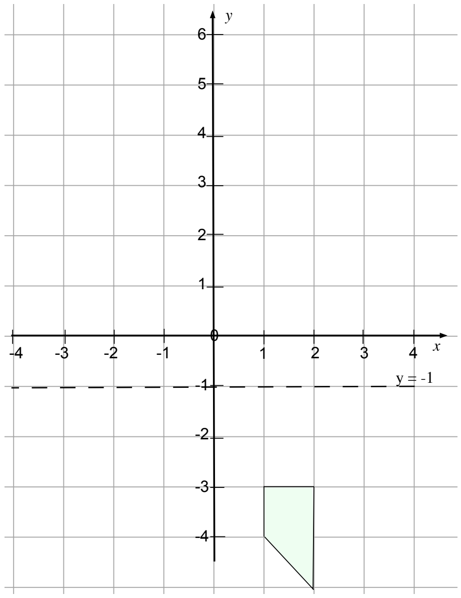
If students struggled with the original assessment task, you may feel it more appropriate for them to just spend their time revisiting *Figure to Figure*, rather than attempting *Figure to Figure (revisited)* as well. If this is the case, give them another copy of the original assessment task instead.

SOLUTIONS

Assessment task: *Figure to Figure*

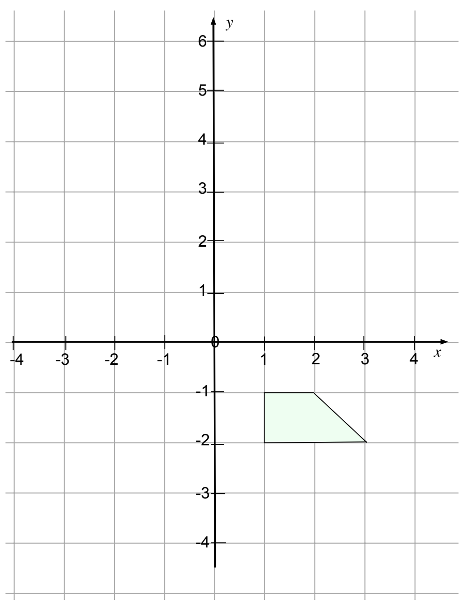
1. The transformation is a translation +2 units horizontally and –1 unit vertically.

2. The completed grid should be as follows:



The student may or may not have drawn in the mirror line $y = -1$. Some students may choose to draw the original object on the same grid as the transformed image.

3. The completed grid should be as follows:



A rotation through 90° clockwise around (0,0) can be described as $(x, y) \to (y, -x)$.

When explaining the algebraic notation for this transformation, students may find it helpful to think of this single transformation as a combination of reflections. The figure is first reflected over the line

$y = x$ (which is $(x, y) \to (y, x)$) and then the resulting image is reflected in the x-axis (which is $(x, y) \to (x, -y)$), giving the result of $(x, y) \to (y, -x)$.

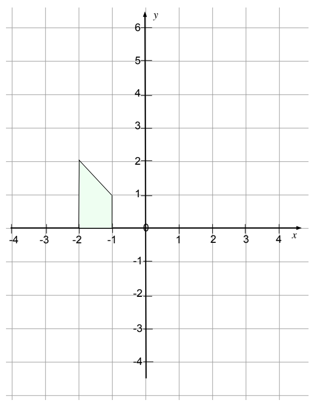
Collaborative task:

The completed cards are as follows:

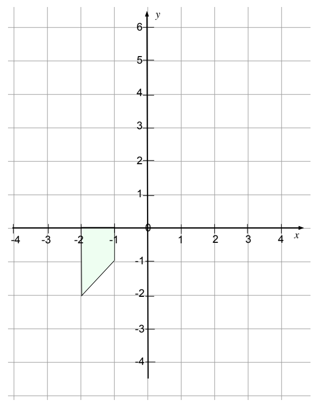
(Words in *italics* represent the information given.)

T1 $(x, y) \to (x, -y)$ *Reflection over the x-axis*

Before

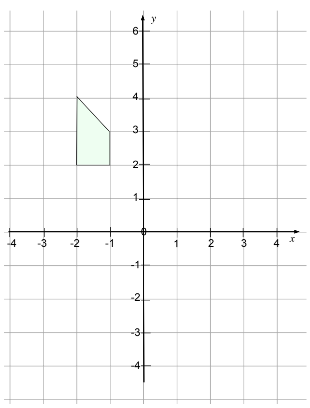


After

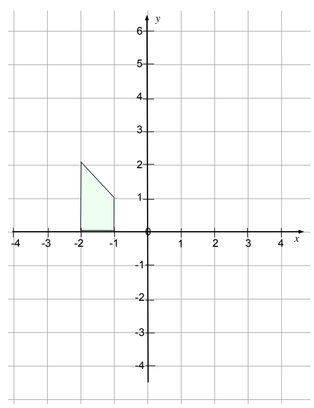


T2 $(x, y) \to (x, y - 2)$ *Translation -2 units vertically*

Before

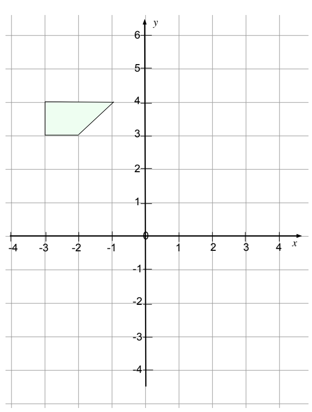


After

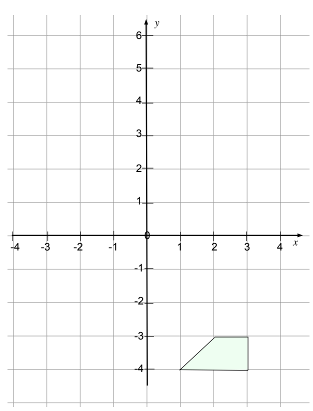


T3 $(x, y) \to (-x, -y)$ Rotation through 180° around (0,0)

Before

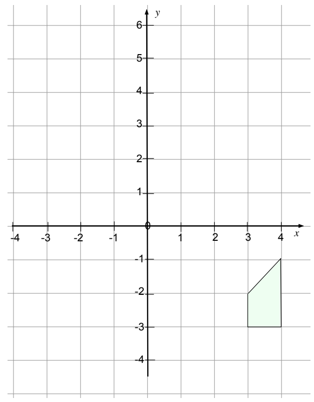


After

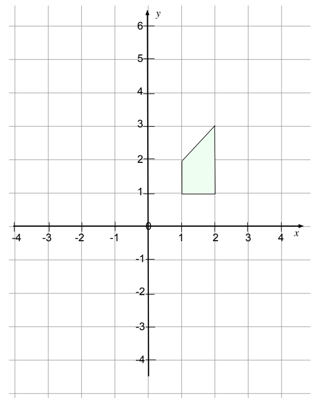


T4 $(x,y) \to (x-2, y+4)$ Translation -2 units horizontally and +4 units vertically

Before

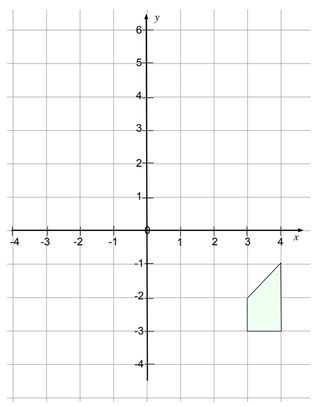


After

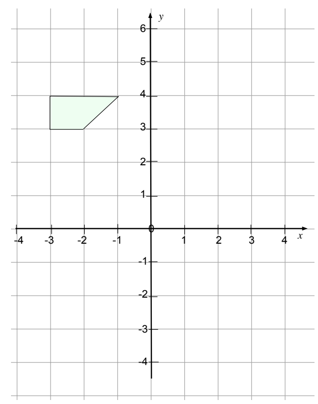


T5 $(x,y) \to (y,x)$ *Reflection over the line* $y = x$

Before

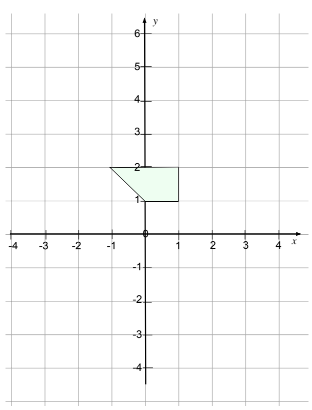


After

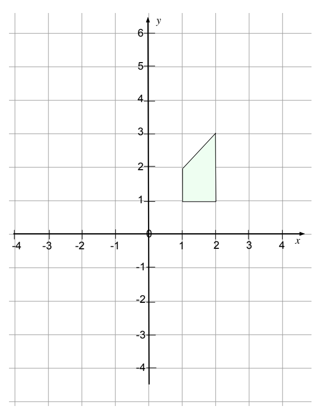


T6 $(x,y) \to (y, 2-x)$ *Rotation through 90° clockwise around (1,1)*

Before

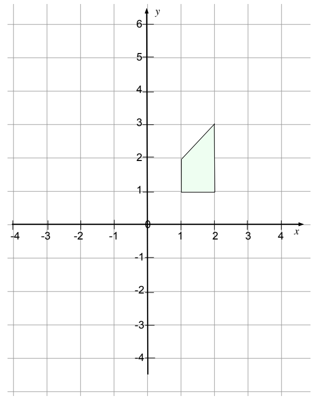


After

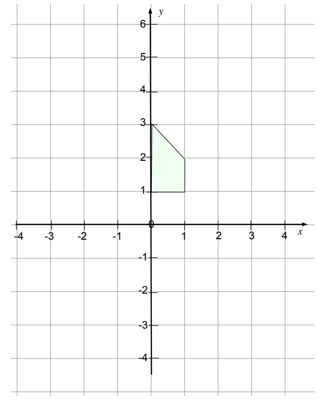


T7	$(x,y) \to (-x+2, y)$	Reflection over the line x = 1

Before

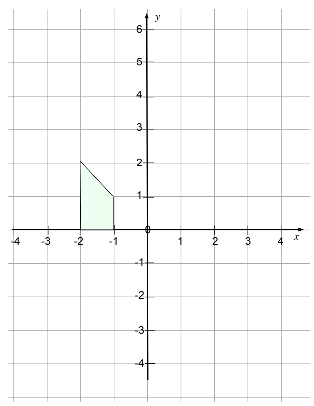


After

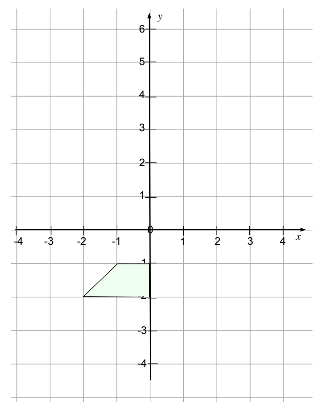


T8	$(x,y) \to (-y, x)$	*Rotation through 90° counterclockwise around (0,0)*

Before

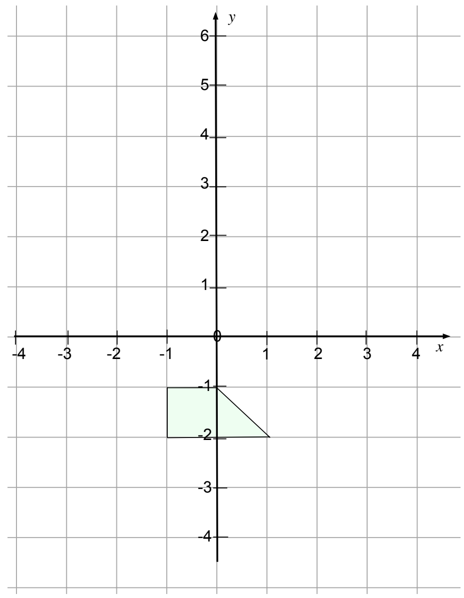


After



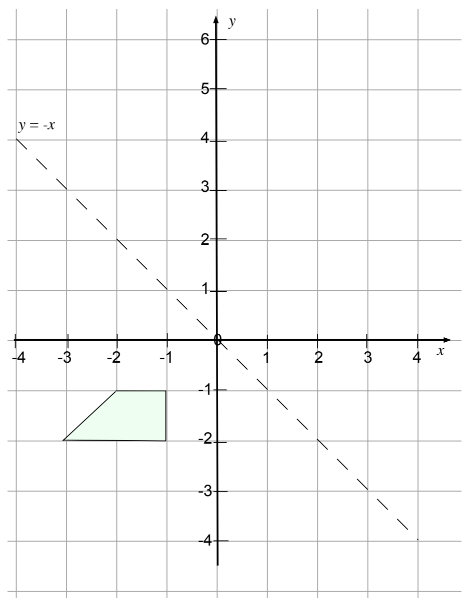
Assessment task: *Figure to Figure (revisited)*

1. The transformation is a rotation through 90° clockwise around (–1, 1). The completed grid should be as follows:



(Some students may choose to draw the original object on the same grid as the transformed image.)

2. The completed grid should be as follows:



The student may or may not have drawn in the mirror line $y = -x$. Some students may choose to draw the original object on the same grid as the transformed image.

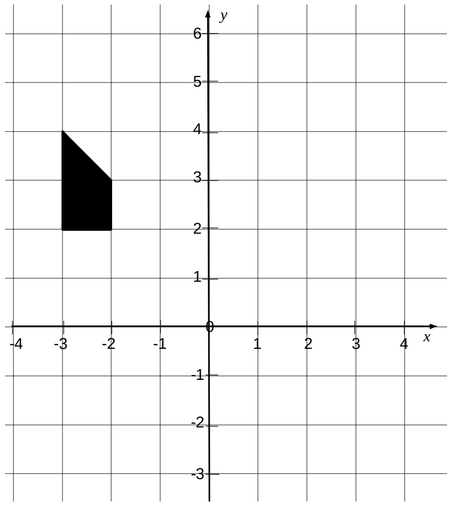
3. The transformation is a translation and can be described as $(x, y) \rightarrow (x + 2, y - 1)$.

Students could describe this as a translation by +2 units horizontally and -1 unit vertically.

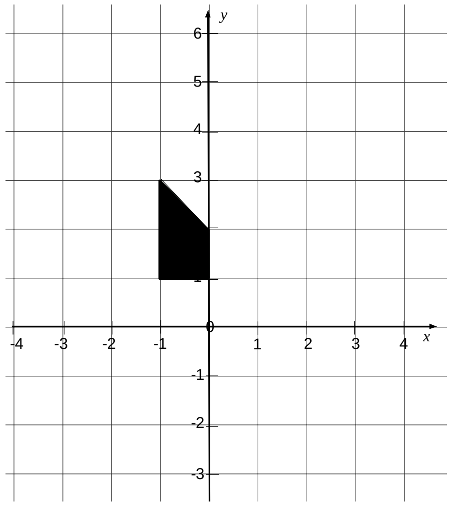
Figure to Figure

1. The figure below is transformed to give a new image:

Before:



After:

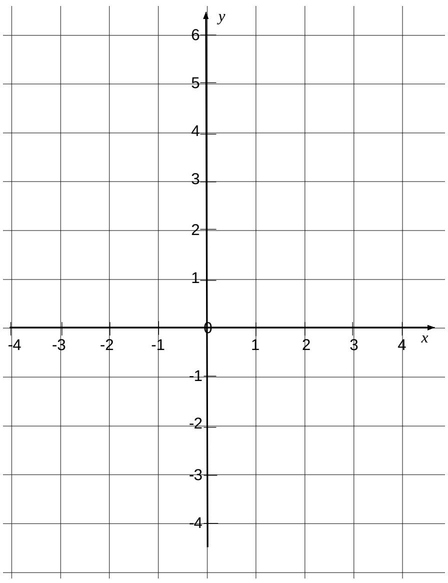


Describe in words a single transformation that maps the original figure to the new image.

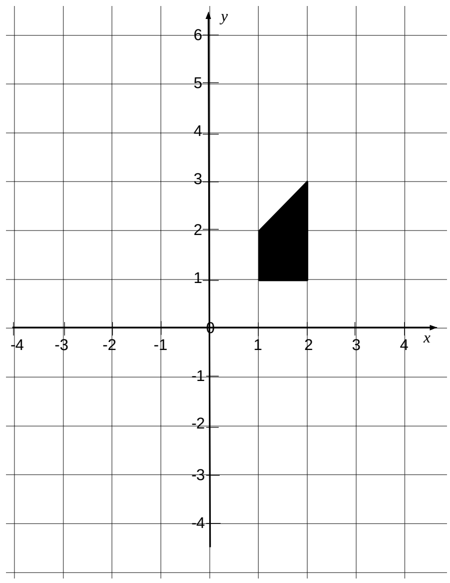
----

2. A figure is reflected over the line y = -1 to give the image below. Complete on the blank grid the position of the original figure before the transformation:

Before:

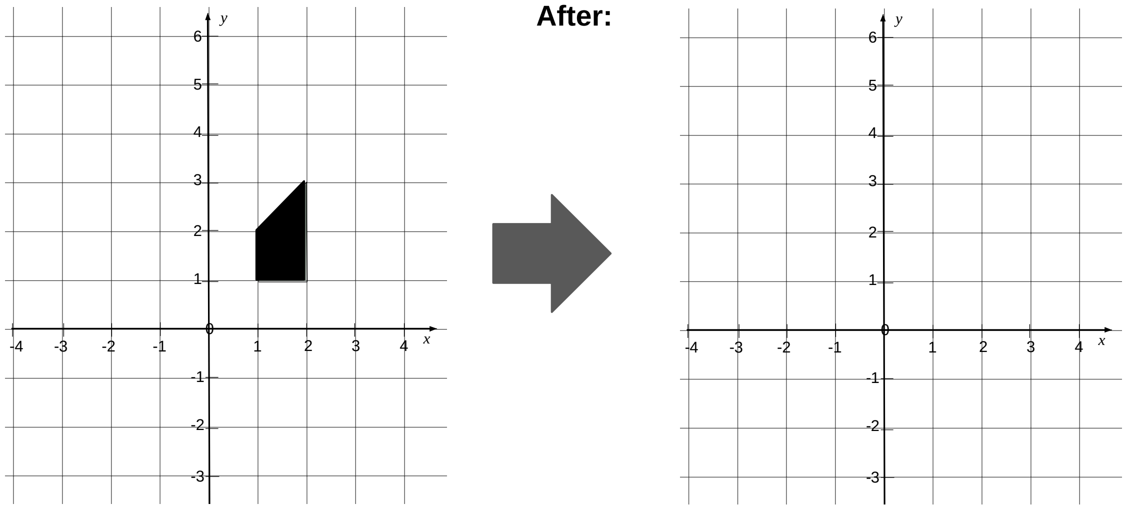


After:



3. The figure below is rotated through 90° clockwise around (0,0):

Before: **After:**



Complete on the blank grid the image after the transformation and describe the effect of the transformation on (x,y):

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

Explain your answer:

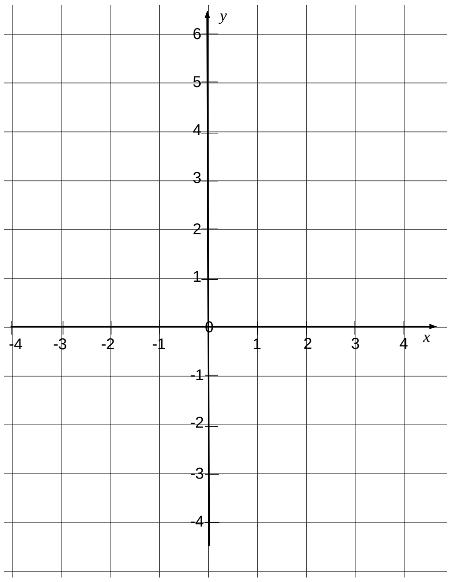
Card Set: Transformations (1)

T1

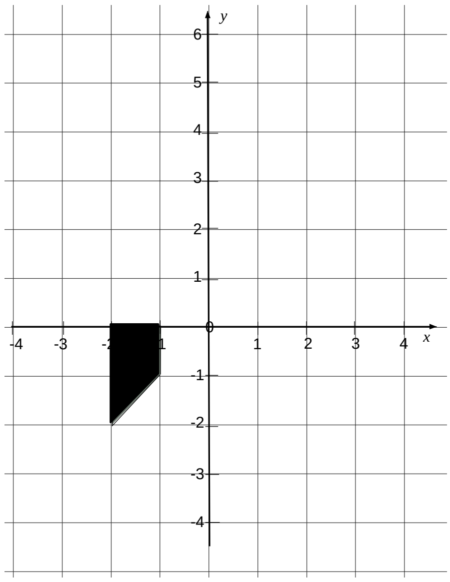
$(x, y) \rightarrow (\_\_\_, \_\_\_)$

Reflection over the *x*-axis

Before



After

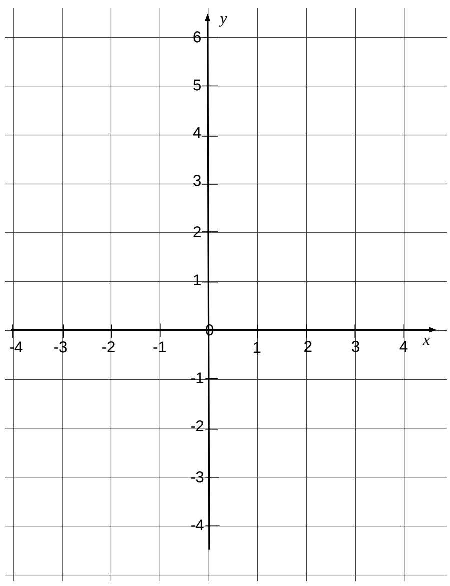


T2

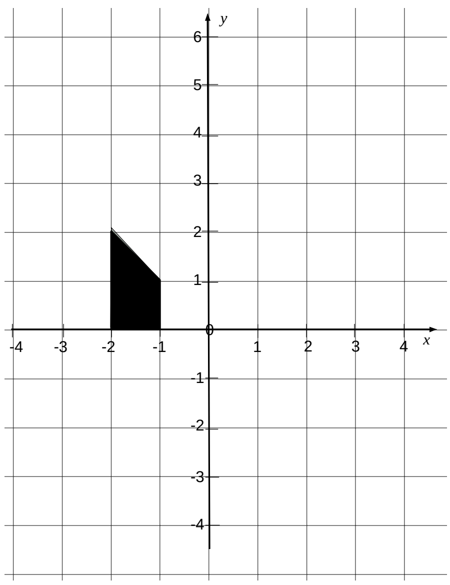
$(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$

Translation -2 units vertically

Before



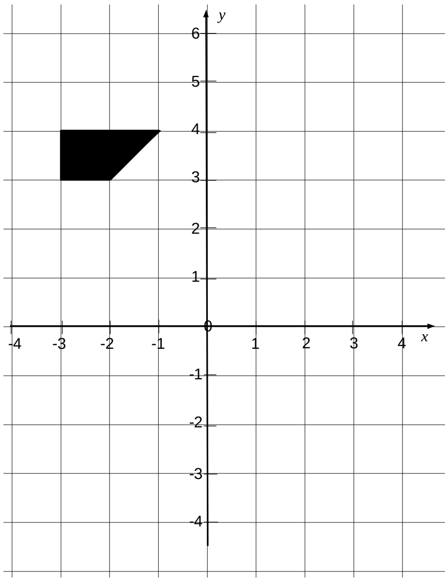
After



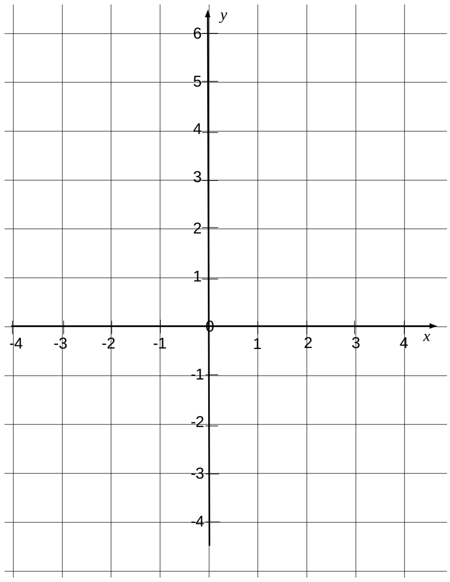
Card Set: Transformations (2)

T3 $$(x, y) \rightarrow (-x, -y)$$

Before

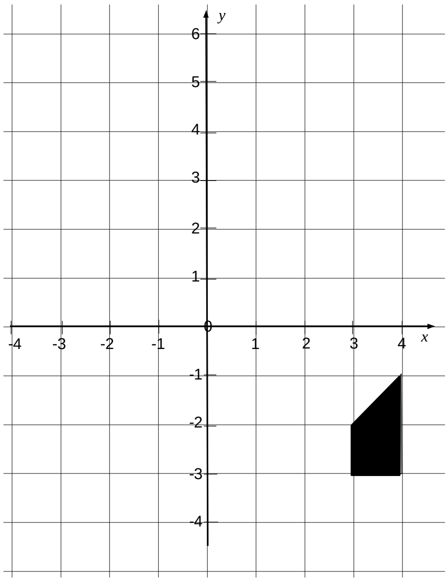


After

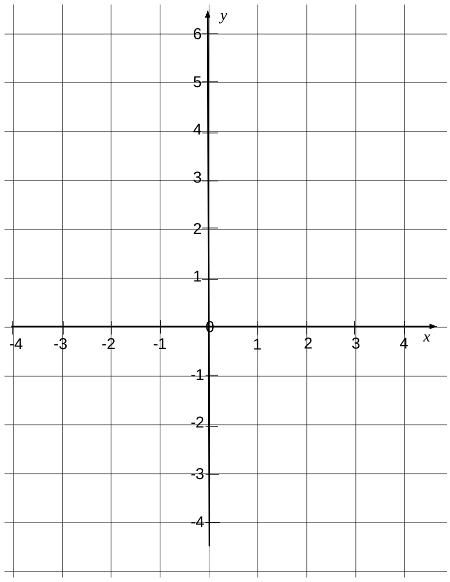


T4 $$(x, y) \rightarrow (x-2, y+4)$$

Before



After



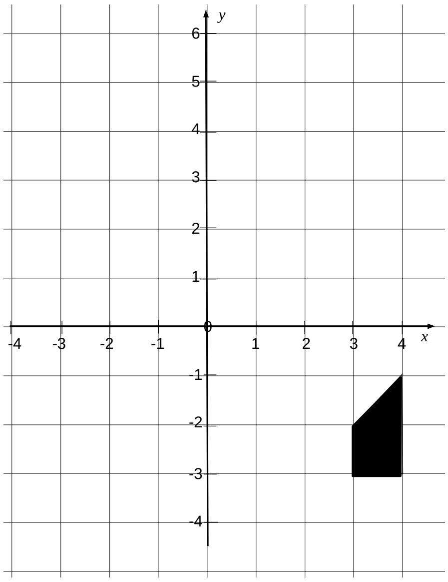
Card Set: Transformations (3)

T5

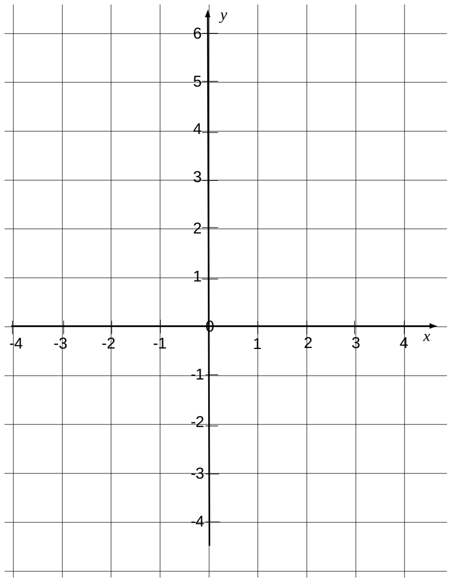
$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

Reflection over the line $y = x$

Before



After

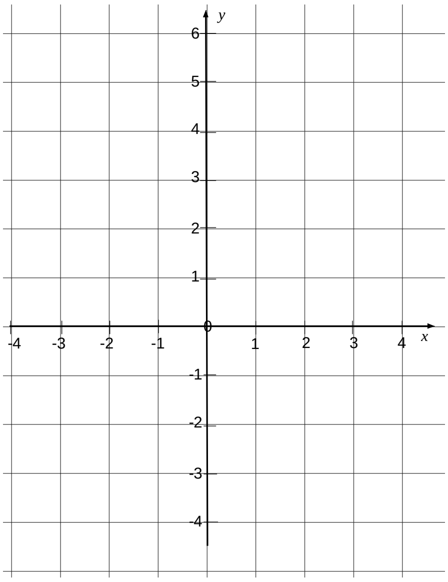


T6

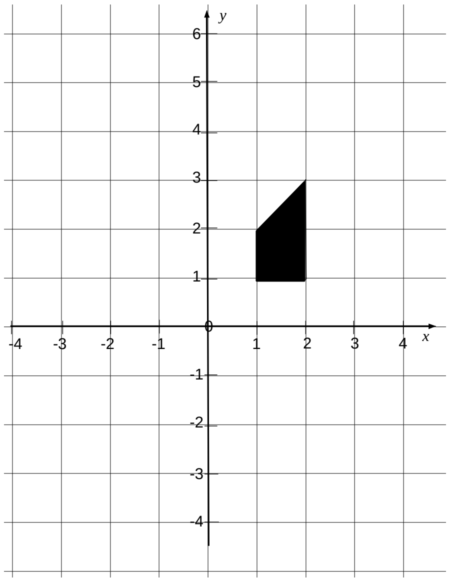
$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

Rotation through 90° clockwise around (1,1)

Before



After

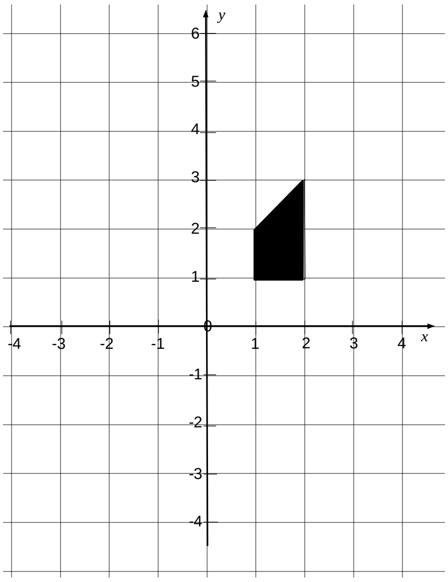


Card Set: Transformations (4)

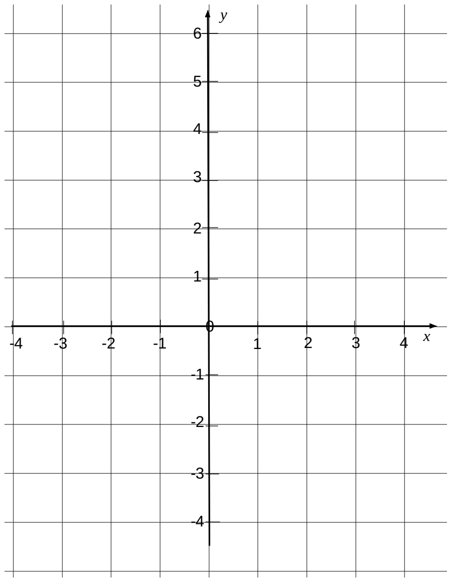
T7

$$(x, y) \rightarrow (-x + 2, y)$$

Before



After

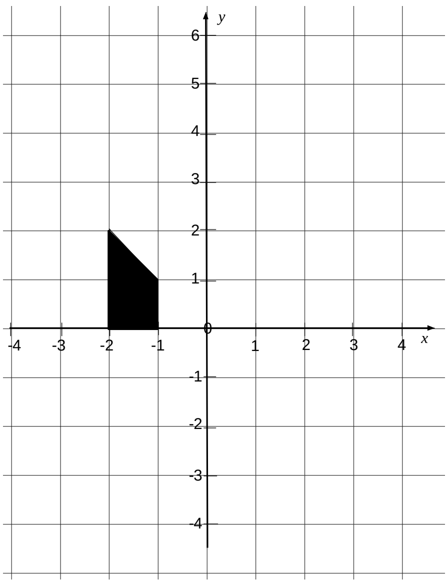


T8

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

Rotation through 90° counterclockwise around (0,0)

Before



After

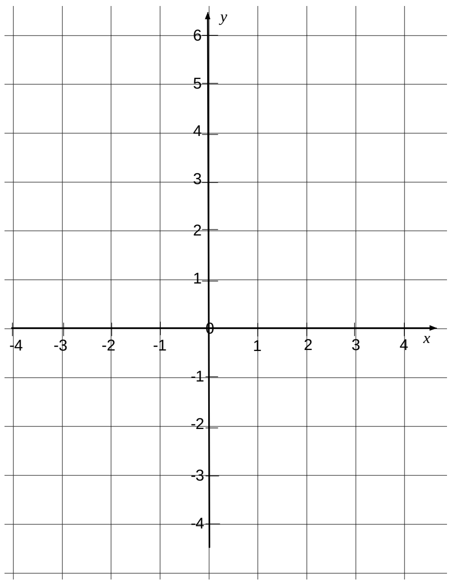
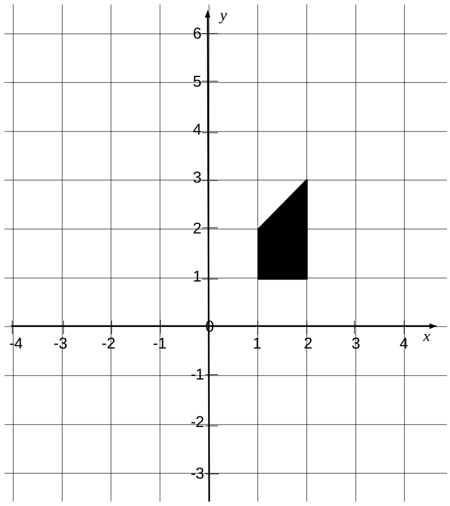
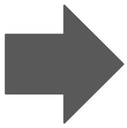


Figure to Figure (revisited)

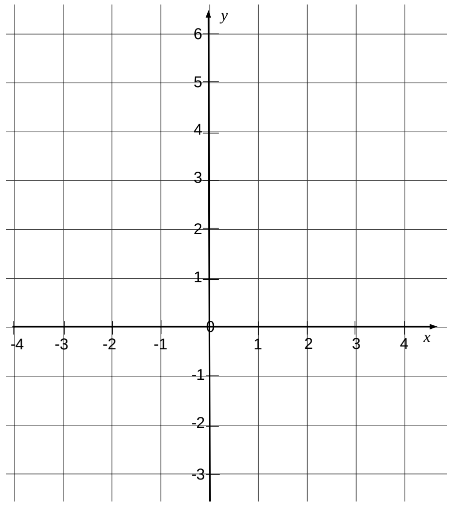
1. A transformation is described as $(x, y) \to (-2 + y, -x)$

Before:





After:

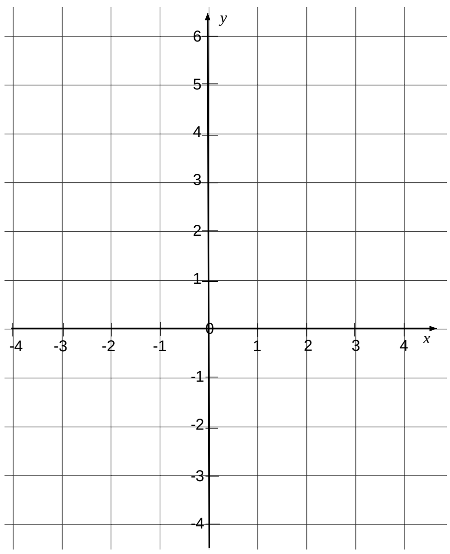


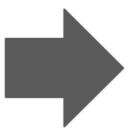
Complete on the blank grid the image after the transformation and describe the single transformation in words.

----

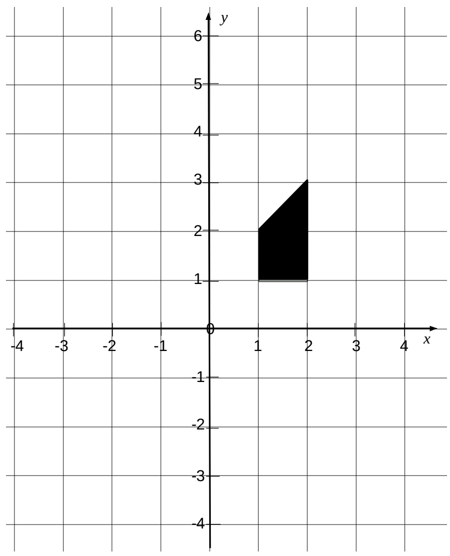
2. A figure is reflected over the line $y = -x$ to give the image below. Complete on the blank grid the position of the original figure before the transformation:

Before:



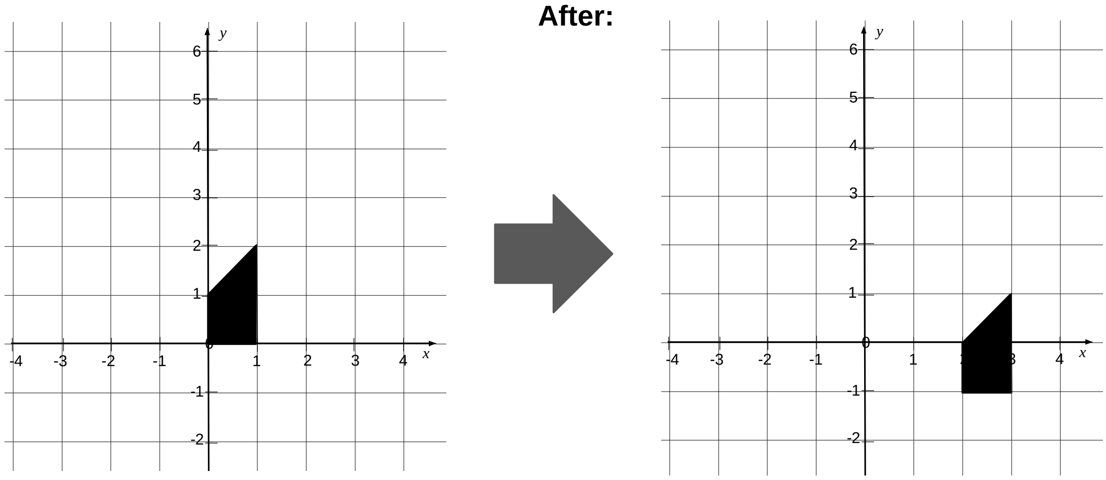


After:



3. The figure below is transformed to give the following image:

Before: **After:**



Describe the effect of the transformation on (x,y) and describe the transformation in words:

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

Describing Transformations

A: Rotation

B: Reflection

C: Translation

1. A line
2. A vertical distance
3. An angle
4. An axis
5. A direction
6. A center
7. A horizontal distance
8. A scale factor

Algebraic Notation Scenario (1)

If the x co-ordinates for all the points are increased by three how could we describe this?

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)?$$

Algebraic Notation Scenario (2)

If both the x and y co-ordinates are doubled how could we describe this?

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)?$$

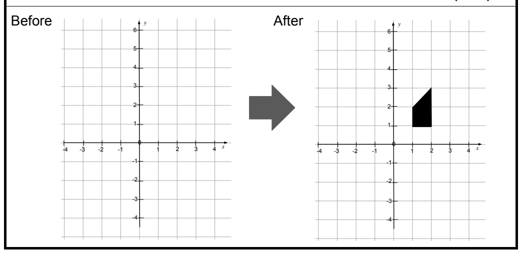
Poster

ROTATION

T6

$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

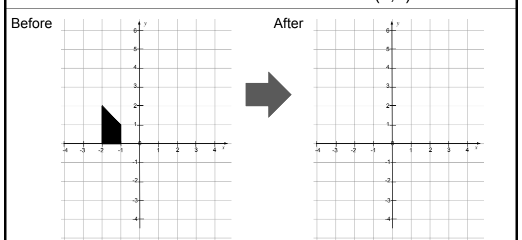
Rotation through 90° clockwise around (1,1)



T8

$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

Rotation through 90° counterclockwise around (0,0)

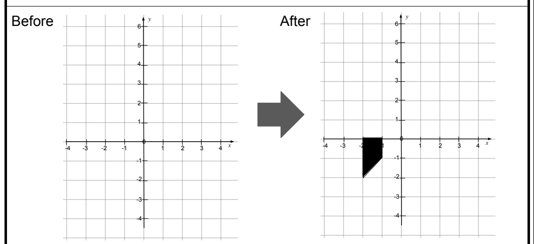


REFLECTION

T1

$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

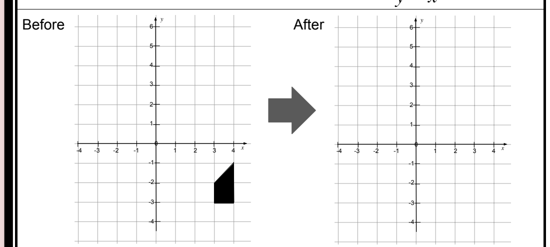
Reflection over the x-axis



T5

$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

Reflection over the line $y = x$

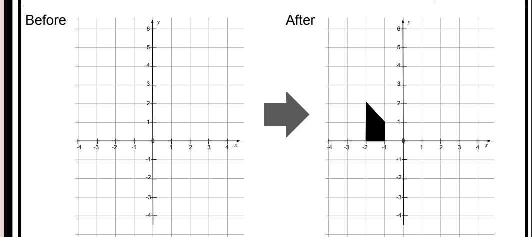


TRANSLATION

T2

$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$

Translation -2 units vertically



Working Together

1. Take turns to complete a card. Explain your thinking clearly and carefully.
2. Partners either explain that reasoning again using their own words or challenge the reasons given.
3. It is important that everyone in the group understands the transformation descriptions and figure positions.
4. Once agreed, glue the card in the appropriate column on your poster paper.

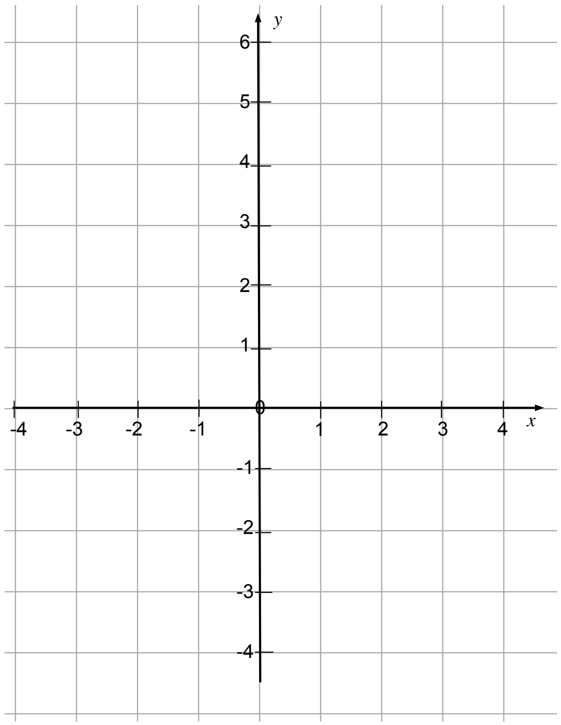
Transformation Card T1

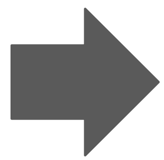
T1

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

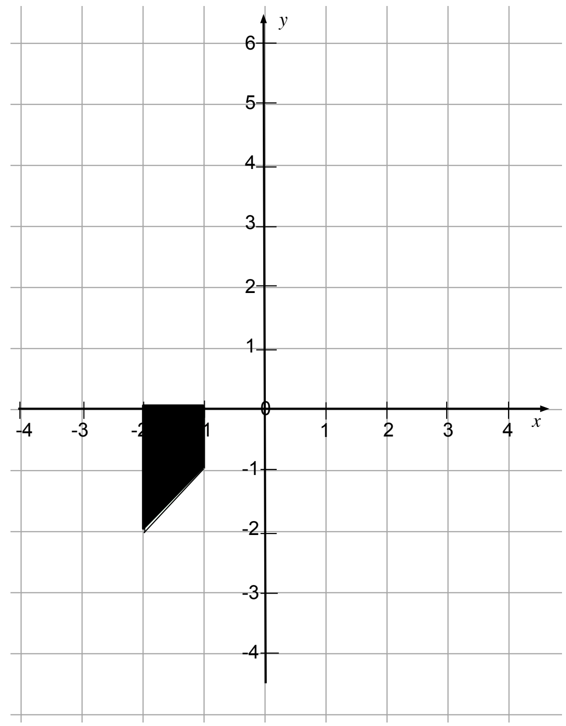
Reflection over the x-axis

Before





After

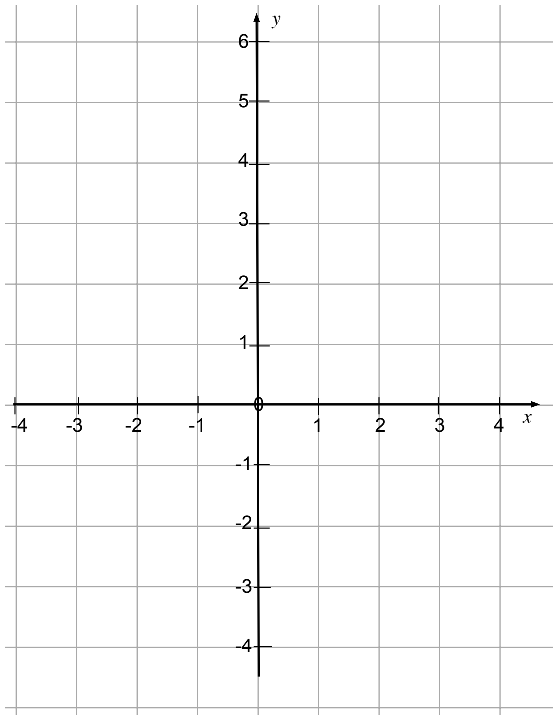


Transformation Card T2

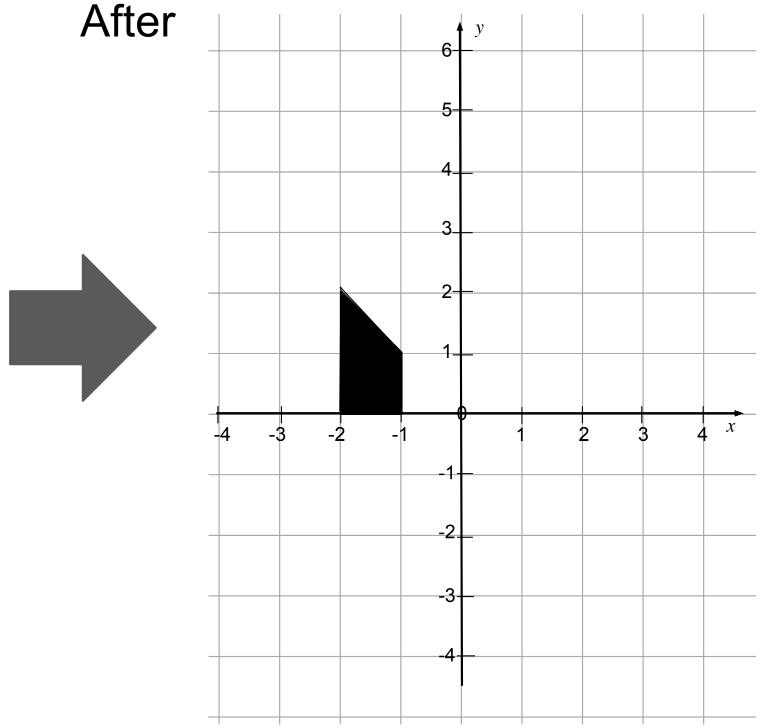
T2 $(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$

Translation -2 units vertically

Before



After



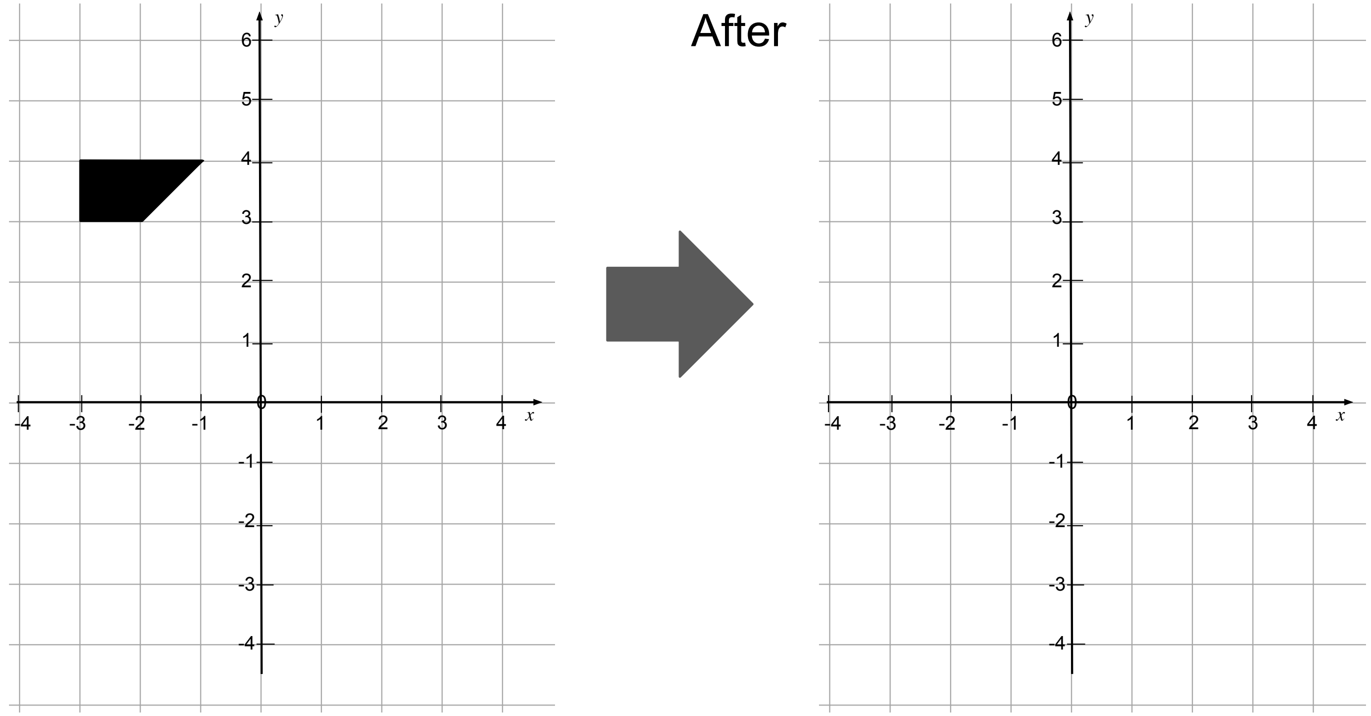
Transformation Card T3

T3

$$(x, y) \rightarrow (-x, -y)$$

Before

After

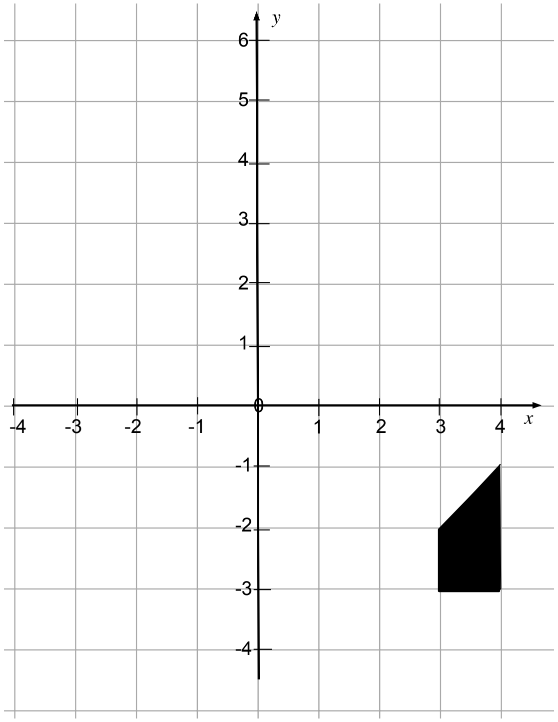


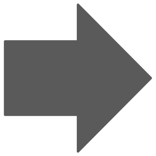
Transformation Card T4

T4

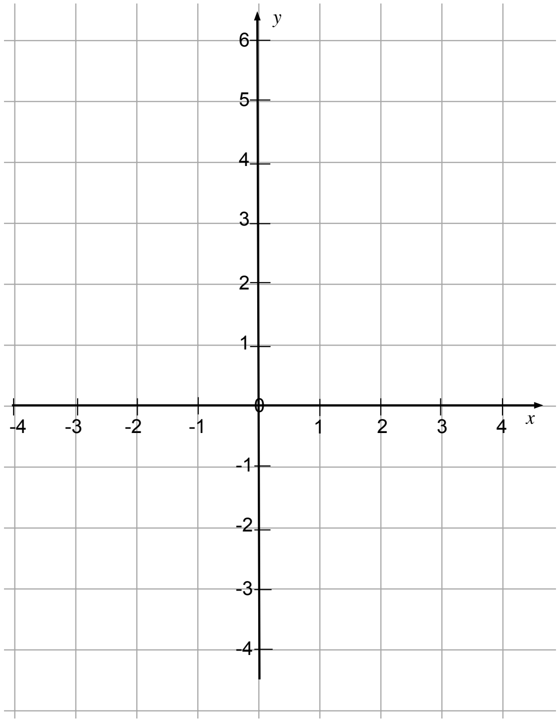
$$(x, y) \rightarrow (x - 2, y + 4)$$

Before





After



Transformation Card T5

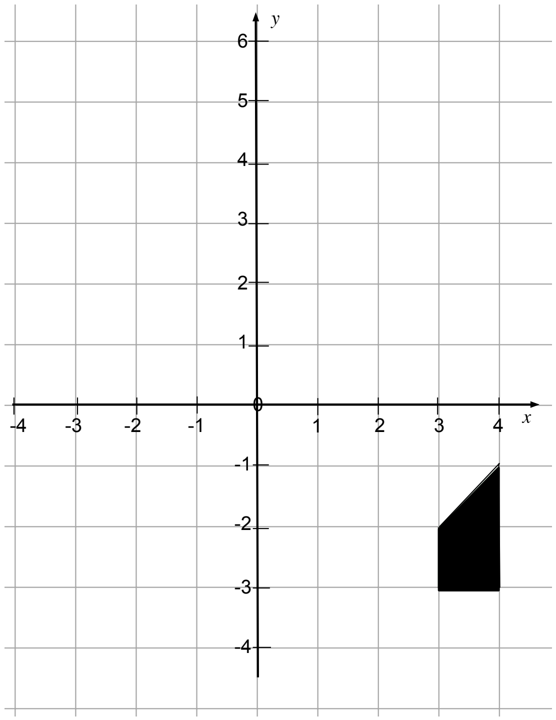
T5

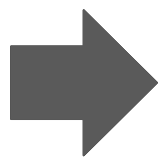
$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

Reflection over the line

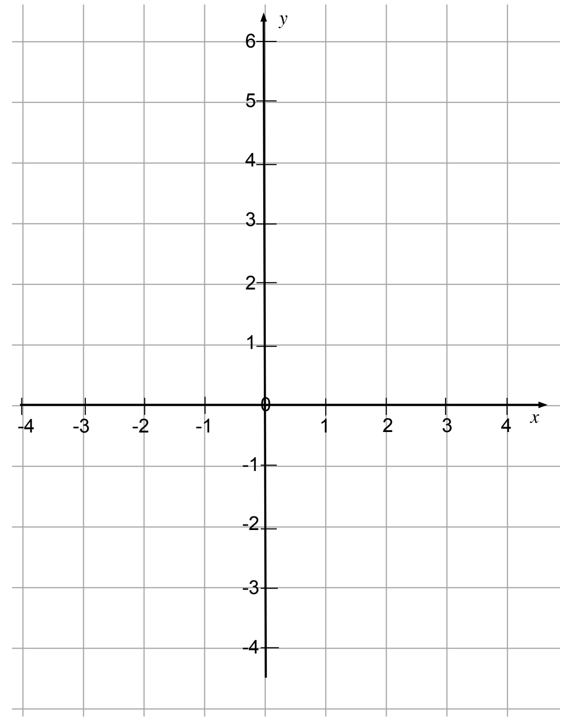
$y = x$

Before





After



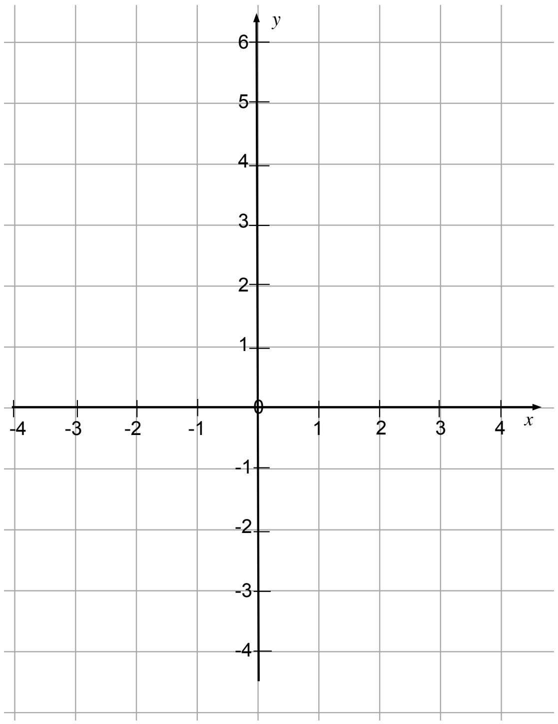
Transformation Card T6

T6

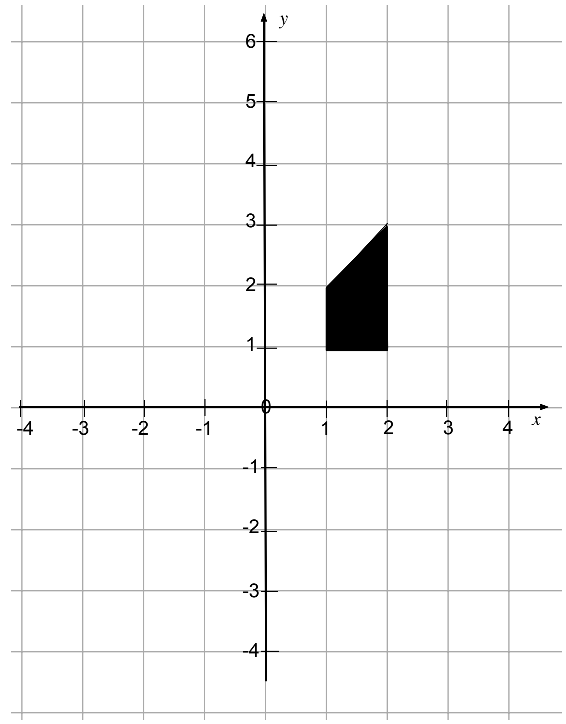
$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

Rotation through 90° clockwise around (1,1)

Before



After

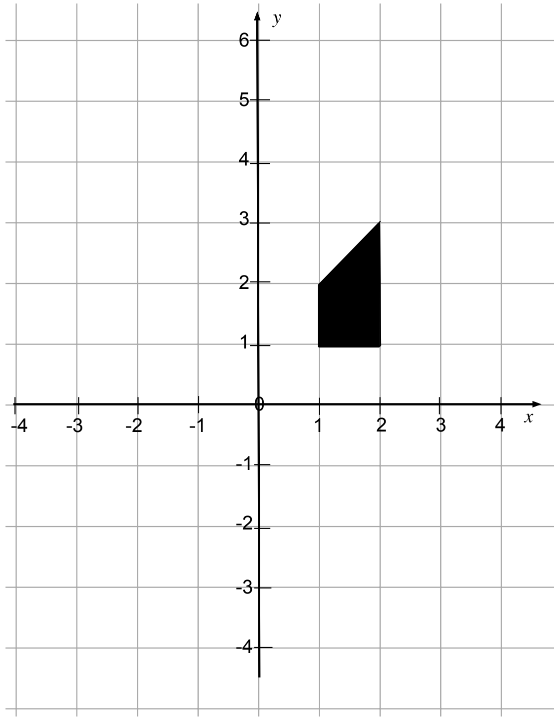


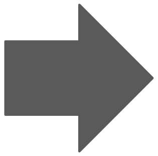
Transformation Card T7

T7

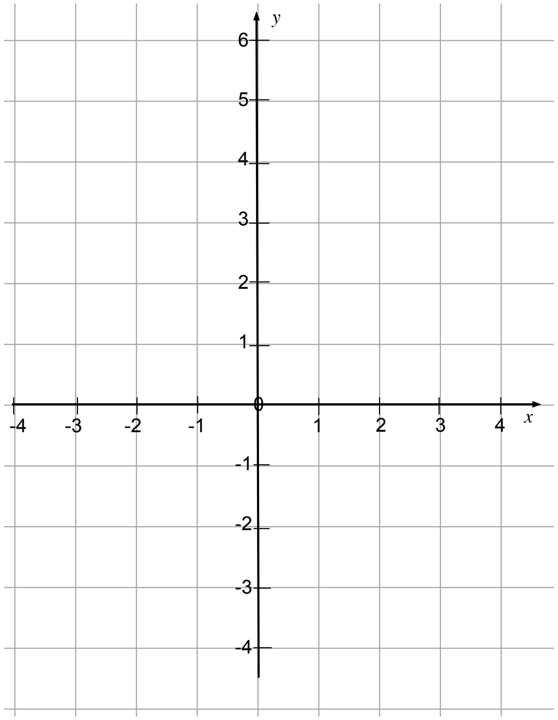
$$(x,y) \rightarrow (-x+2,y)$$

Before





After



Transformation Card T8

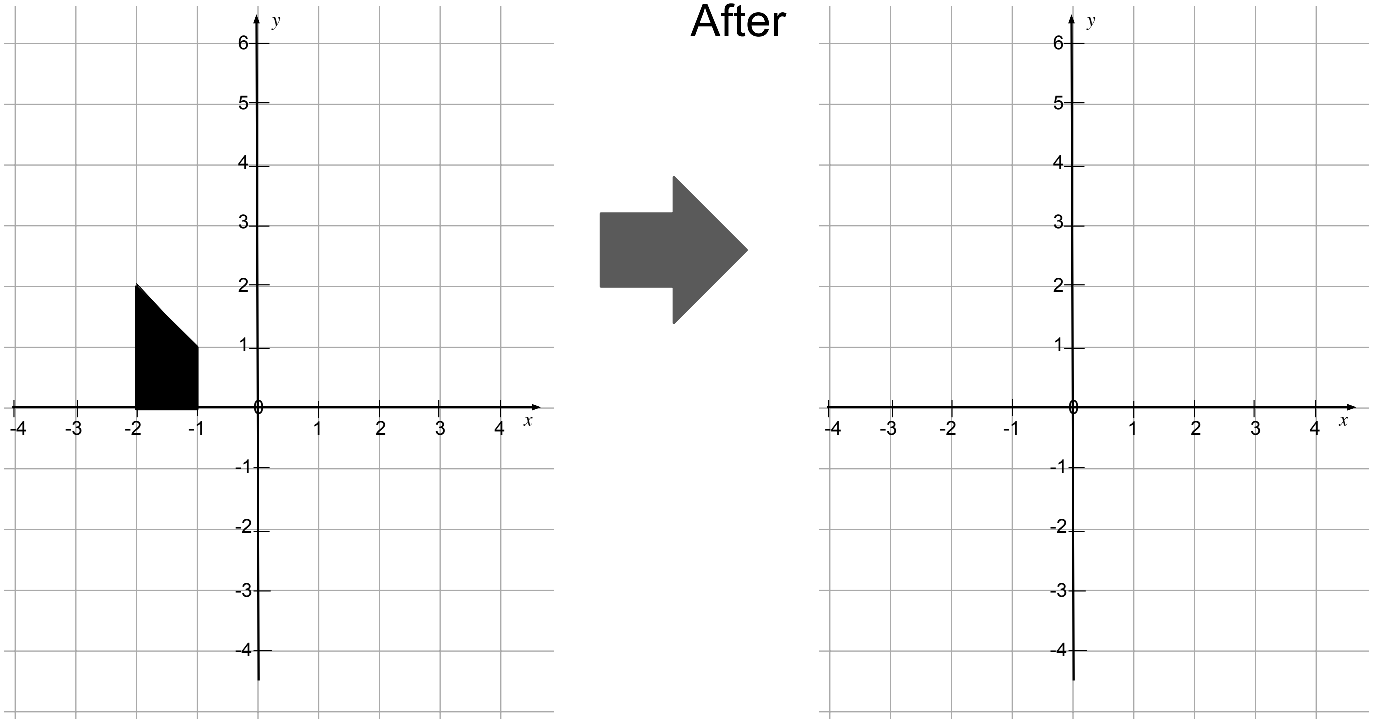
T8

$$(x, y) \rightarrow (\_\_\_\_, \_\_\_\_)$$

Rotation through 90° counterclockwise around (0,0)

Before

After



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and **Daniel Pead**

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The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

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Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org