CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Solving Problems with Circles and Triangles

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Solving Problems with Circles and Triangles

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to use geometric properties to solve problems. In particular, the lesson will help you identify and help students who have difficulty:

- Solving problems by determining the lengths of the sides in right triangles.
- Finding the measurements of shapes by decomposing complex shapes into simpler ones.

The lesson unit will also help students to recognize that there may be different approaches to geometrical problems and to understand the relative strengths and weaknesses of those approaches.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics:*

G-SRT: Prove theorems involving similarity.
Define trigonometric ratios and solve problems involving right triangles.
G-C: Understand and apply theorems about circles.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2, 3, and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

INTRODUCTION

- Before the lesson, students attempt the problem individually. You then review their work and create questions for students to answer in order to improve their solutions.
- During the lesson, students review their individual solutions before working collaboratively in small groups to produce an improved solution to the problem. They then comment on and evaluate some solutions to the same problem, produced by students in another class.
- In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, students review and write about what they learned.

MATERIALS REQUIRED

- Each individual student will need the *Circles and Triangles* task, the *How Did You Work?* questionnaire, a ruler, calculator, pencil, mini-whiteboard, pen, and eraser.
- Each small group of students will need a copy of the *Sample Responses to Discuss,* a large sheet of paper for making a poster, and a felt-tipped pen.
- There is a projector resource to help with instructions and to support whole-class discussion.

TIME NEEDED

15 minutes before the lesson, an 80-minute lesson (or two 45-minute lessons), and 15 minutes in a follow-up lesson. Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Circles and Triangles* (15 minutes)

Have students do this task in class or for homework a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

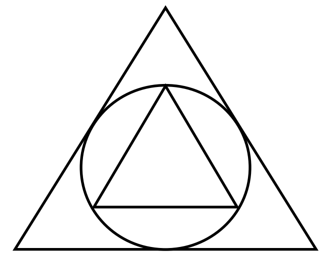
Give each student a copy of the *Circles and Triangles* task, a pencil, and a ruler. Issue calculators if students ask for them.

> *Read through the task and try to answer the questions as carefully as you can. Show all your work so that I can understand your reasoning.*
>
> *Don't worry too much if you don't understand everything, because there will be a lesson [tomorrow] using this task.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. If students are struggling to get started, ask questions that help them understand what is required, but make sure you do not do the task for them.

Circles and Triangles

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.



1. Calculate the *exact* ratio of the areas of the two triangles.
   Show all your work.

2. Draw a second circle inscribed inside the small triangle.
   Find the *exact* ratio of the areas of the two circles.

Students who sit together often produce similar answers and then when they come to compare their work, they have little to discuss. For this reason we suggest that, when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

Assessing students' responses

We suggest that you do not write scores on students' work. The research shows that this is counterproductive, as it encourages students to compare scores and distracts their attention from what they might do to improve their mathematical work.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on page T-3. These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest that you make a list of your own questions, based on your students' work, using the ideas on page T-3. We recommend that you:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight appropriate questions for each student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students at the beginning of the lesson.

Common issues:	**Suggested questions and prompts:**
Has difficulty getting started	• What do you know about the angles or lines in the diagram? What do you need to find out? • Can you add some helpful labels to the lengths? • Can you add any helpful construction lines to your diagram? What do you know about them? • Can you find relationships between the lengths from what you know about geometry?
Works out the ratio by measuring the dimensions of the triangles	• What are the advantages/disadvantages of your method? • Are your measurements accurate enough? How do you know?
Does not explain the method clearly For example: The student does not explain why triangles are similar. Or: The student does not explain why triangles are congruent.	• Would someone unfamiliar with your type of solution easily understand your work? • How do you know these triangles are similar/congruent? • It may help to label points and lengths in the diagram.
Has problems recalling standard ratios The student makes an error using the ratios for a $30°, 60°, 90°$ triangle $(1, \sqrt{3}, 2)$.	• What do you know about cos 30° and sin 30°? How can you use this information? • How might the Pythagorean Theorem help you?
Uses perception alone to calculate the ratio For example: The student rotates the small triangle about the center of the circle, assuming the diagram alone is enough to show the ratio of areas is 4:1.	• What math can you use to justify your answer?
Makes a technical error For example: The student makes an error manipulating an equation.	• Check to see if you have made any algebraic errors.
Uses ratios of lengths rather than ratios of areas For example: When finding the ratio of the areas of the two circles, the student finds the ratio of the radii, rather than the *squares* of the radii.	• What is the formula for the area of the circle? How can you use it to find the ratio of the areas of the circles?
Produces correct solutions	• Can you solve the problem using a different method? Which method do you prefer? Why?

SUGGESTED LESSON OUTLINE

Improve individual solutions to *Circles and Triangles* (10 minutes)

Return students' papers and give each student a mini-whiteboard, pen, and eraser.

Recall what we were looking at in a previous lesson. What was the task?

I read your solutions and I have some questions about your work.

If you have not added questions to individual pieces of work, write your list of questions on the board and ask students to select questions appropriate to their own work.

Ask students to spend a few minutes answering your questions. It is helpful if they do this using mini-whiteboards, so that you can see what they are writing.

I would like you to work on your own for about ten minutes to answer my questions.

Collaborative small-group work on *Circles and Triangles* (30 minutes)

When students have made a reasonable attempt at answering your questions, organize them into groups of two or three. Give each group a large piece of paper and a felt-tipped pen. Ask students to have another go at the task, but this time ask them to combine their ideas and make a poster to show their solutions.

Put your own work aside until later in the lesson. I want you to work in groups now.

Your task is to work together to produce a solution that is better than your individual solutions.

While students work in small groups you have two tasks, to note their different approaches to the task and to support their reasoning.

Note different student approaches to the task

What mathematics do students choose to use? Have they moved on from the mathematical choices made in their individual work? Do they measure the lengths of the sides of the triangles? Do they draw construction lines? Do they use similar triangles? Do they use algebra? Do they use proportion?

Do students attempt to use the special ratios for 30°, 60°, 90° triangles ($1 : \sqrt{3} : 2$)? If so, how do they do this?

When finding the ratio of the areas of the two triangles, do they find the ratio of the squares of the bases or do they use an alternative method? When finding the ratio of the areas of the two circles, do students find the ratio of the squares of the radii or do they use an alternative method?

Do students fully explain their solutions?

Note any errors and think about your understanding of students' strengths and weaknesses from the assessment task. You can use this information to focus the whole-class discussion at the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking. Focus on supporting students' strategies rather than finding the numerical solution. You may find the questions on the previous page helpful.

If the whole-class is struggling on the same issue, write relevant questions on the board.

You may find that some students think the empirical approach (measuring the diagram) is best.

Will your answer change if you measure in inches rather than millimeters?

This question may focus students' attention on the lack of units of measure in the solution and the problem of accuracy.

What are the strengths/weaknesses of this approach?

Are your measurements exact?

Do you think that, if we asked another group that used this same method, they would come up with exactly the same answer as you?

Extending the lesson over two days

If you are taking two days to complete the lesson unit then you may want to end the first lesson here. At the start of the second day, allow students a few minutes to familiarize themselves with their posters before moving on to the collaborative analysis of sample responses.

Collaborative small-group analysis of *Sample Responses to Discuss* (25 minutes)

When students have had sufficient time to attempt the problem in their group, give each group copies of the *Sample Responses to Discuss*. This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

You may decide there is not enough time for each group to work through all four pieces of work. In that case, be selective about what you hand out. For example, groups that have successfully completed the task using one method will benefit from looking at different approaches. Other groups, that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

Here are some different solutions to the problem.

Compare these solutions with your own.

Imagine you are the teacher. Describe how the student approached the problem.

Write your explanation on each solution.

What do you like/dislike about the work?

What isn't clear about the work?

What questions would you like to ask this student?

To encourage students to do more than check to see if the answer is correct, you may wish to use Slide P-1 of the projector resource *Analyzing Sample Responses to Discuss*. During the group work, check to see which of the explanations students find more difficult to understand.

Whole-class discussion: comparing different solution methods (15 minutes)

Organize a whole-class discussion comparing the four given solutions. Collect comments and ask for explanations.

We are going to look at and compare the four solutions.

Can you explain Bill's method?

Why does Carla draw another triangle in the inner circle?

Encourage students to challenge explanations while keeping your own interventions to a minimum.

Do you agree with Tyler's explanation?
[If yes] Explain again, in your own words.
[If no] Explain what you think.

Finally, ask students to evaluate and compare methods:

Which one did you like best? Why?

Which approach did you find most difficult to understand? Why?

Did anyone come up with a method different from these?

Some issues that might be discussed, with suggested questions and prompts, are given below:

Anya uses measurement

Strengths: It is easy to do. It gives you a feeling for the answer. Anya's calculations are correct. She has rounded to two decimal places.

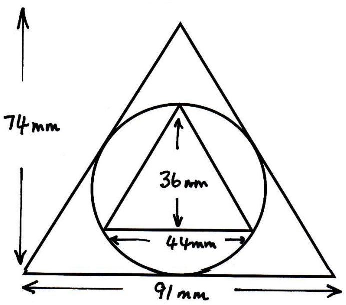
Weaknesses: You only know it is true for the particular case you measure. It's not exact. It doesn't tell you why it's true. Anya does not calculate the areas of the circles, nor their ratio (about: $25^2:11^2 = 5:1$).

Do you think Anya's answer is accurate?

Would an answer rounded to four decimal places be better?

What do you think the answer should be?

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.



1. Calculate the ratio of the areas of the two triangles. Show all your work.

$\Delta = \frac{1}{2} \cdot 74 \cdot 91 = 3367$

$\Delta = \frac{1}{2} \cdot 36 \cdot 44 = 792$

Ratio $= \frac{3367}{792} = 4.25$

Bill uses algebra and ratios

Strengths: Bill's method does not depend on the size of the diagram. You can use this method for all sorts of problems.

Weaknesses: Bill's work is difficult to follow. There are gaps in his explanation, and it is quite difficult work. Bill does not answer the question, as he does not calculate the ratio of the areas of the triangles. He does not explain why the side lengths in the triangle are in the ratios he writes down, which is based on these trigonometric ratios:

$$\sin 30° = \frac{c}{r} = \frac{1}{2}$$

$$\cos 30° = \frac{b}{r} = \frac{\sqrt{3}}{2}$$

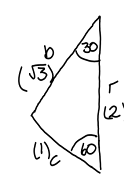
$$\tan 60° = \frac{a}{r} = \sqrt{3}$$

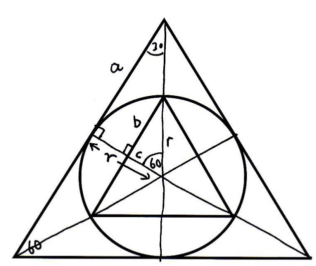
This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.

$\frac{c}{r} = \frac{1}{2}$ $\quad c = \frac{r}{2}$

$\frac{b}{r} = \frac{\sqrt{3}}{2}$ $\quad b = \frac{r\sqrt{3}}{2}$

$a = r\sqrt{3}$





1. Calculate the ratio of the areas of the two triangles. Show all your work.

Area of small triangle $= 6 \times \frac{1}{2} \times b \times c = 6 \times \frac{1}{2} \times \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \frac{3r^2\sqrt{3}}{4}$

Area of large triangle $= 6 \times \frac{1}{2} \times a \times r = 3 \times r\sqrt{3} \times r = 3r^2\sqrt{3}$

You could ask students to explain where the ratios in Bill's solution come from and then to use the lengths to complete the solution.

Why does $\frac{c}{r} = \frac{1}{2}$ *?* *Why does* $\frac{b}{r} = \frac{\sqrt{3}}{2}$ *?* *Why does* $a = r\sqrt{3}$ *?*

Why does Bill multiply by 6? What is the ratio of the areas of the two triangles?

Carla uses transformations – rotation and enlargement

Strengths: It is simple. It is clear, even elegant. It is easy to do.

Weaknesses: You have to see it! There are some gaps in the explanation that need to be completed.

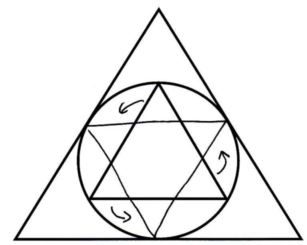
How do you know that, if you rotate the small triangle, it hits the midpoints of the large triangle?

How do you know the four small triangles are congruent?

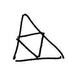
How do you know the four small triangles are equilateral?

How do you know the circle has been enlarged in the same ratio as the triangle?

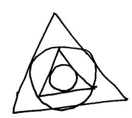
This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.



1. Calculate the ratio of the areas of the two triangles. Show all your work.

Spin the little triangle.
You get  So big area : small area = 4 : 1

2. Draw a second circle inscribed inside the small triangle. Find the ratio of the areas of the two circles.



Big triangle + big circle is enlargement of small triangle + small circle.
So ratio of big circle to small circle = 4 : 1

Darren uses algebra and similar triangles

Strengths: Darren's method does not depend on the size of the diagram. Darren has labeled the diagram; this makes his work easier to understand.

Weaknesses: Darren's work is difficult to follow at times. He has failed to explain part of his work.

Are triangles OBC and OEF similar? How do you know?

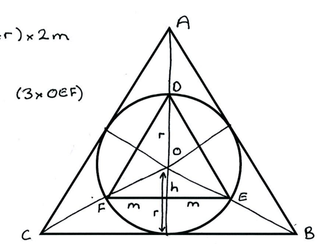
What does Darren mean by "double × double"?

Can you use math to show Darren's answer is correct?

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.

Area DEF = $\frac{1}{2} \times (h+r) \times 2m$
$= m \times (h+r)$
$= 3 \times \frac{1}{2} \times h \times 2m$ (3 × OEF)

$h + r = 3h$
$h = \frac{r}{2}$



1. Calculate the ratio of the areas of the two triangles. Show all your work.

Triangle OEF is similar to triangle OBC.
The height of OBC is double the height of OEF, so CB is double EF.
It follows that the area of OBC is double × double - four times bigger than OEF.
Area ABC : Area DEF = 3 × OBC : 3 × OEF = 4 : 1

Through comparing different methods, students may come to realize the power of using different methods to solve the same problem.

Follow-up lesson: individual review (15 minutes)

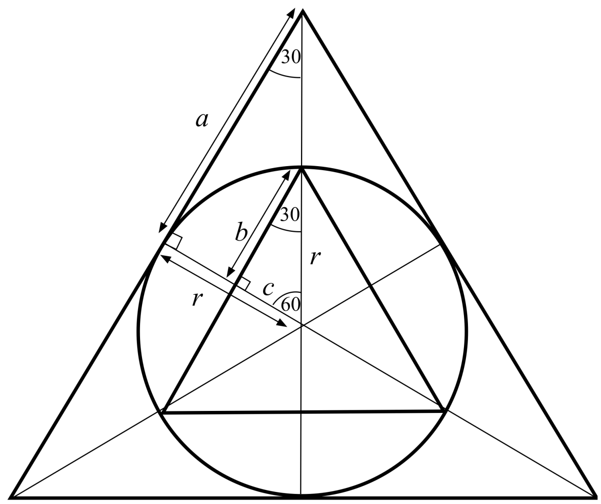
Give each student a copy of the *How Did You Work?* questionnaire. This is intended to provide some individual reflection time that involves them in making comparisons of the work they have seen.

> *Think carefully about your work on this task. On your own, answer the review questions as carefully as you can.*

Some teachers give this as a homework task.

SOLUTIONS

Bill's method



1. $\tan 60° = \dfrac{a}{r} = \sqrt{3} \Rightarrow a = r\sqrt{3}$

$\cos 30° = \dfrac{b}{r} = \dfrac{\sqrt{3}}{2} \Rightarrow b = \dfrac{r\sqrt{3}}{2}$

$\sin 30° = \dfrac{c}{r} = \dfrac{1}{2} \Rightarrow c = \dfrac{r}{2}$

Area of small equilateral triangle:

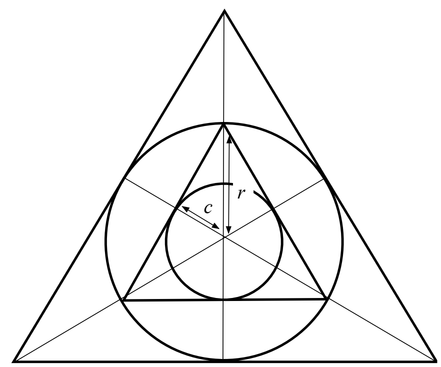
$$6 \times \frac{1}{2} \times b \times c = 3 \times \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \frac{3\sqrt{3}r^2}{4}$$

Area of large equilateral triangle:

$$6 \times \frac{1}{2} \times a \times r = 3 \times r\sqrt{3} \times r = 3\sqrt{3}r^2$$

Ratio of area of the outer to the area of the inner triangle = $3\sqrt{3}r^2 : \dfrac{3\sqrt{3}}{4} r^2 = 4:1$.

2. c is the radius of the inscribed circle. $c = \dfrac{r}{2}$



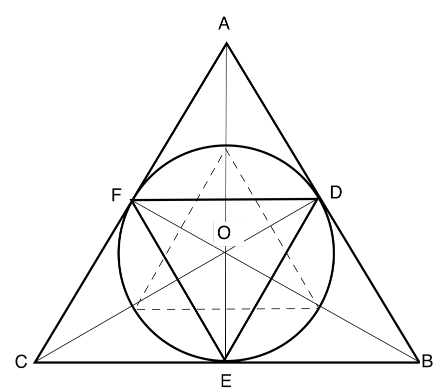
Ratio of area of circles:

$= \pi r^2 : \pi c^2$

$= \pi r^2 : \pi \dfrac{r^2}{4}$

$= 4:1.$

Carla's method



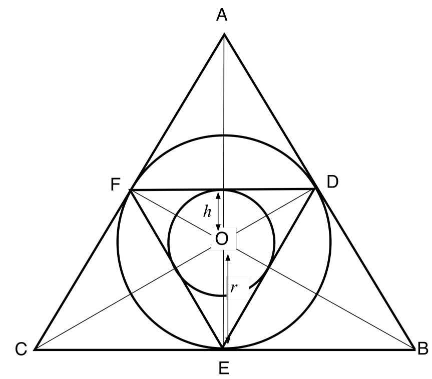
1. The small equilateral triangle is rotated through 60° about O, the center of the circle. The arm of the rotation is the radius of the circle. Therefore points D, E, and F are all points on the circumference of the circle. These points bisect the sides of Δ ABC.

Δ CFE is isosceles (CF = CE because the lengths of two tangents to a circle from a point are equal), so
∠CFE = ∠FEC = (180 – 60) ÷ 2 = 60°.
Therefore Δ CFE is equilateral.

It follows by symmetry that all four small triangles are equilateral and congruent.

Hence the ratio of the area of the outer to the area of the inner triangle = 4 : 1.

2. Ratio of the area of the outer to the area of the inner triangle:



= (3 × area Δ OCB) : (3 × area Δ FDO)

$$= \left(3 \times \frac{1}{2} \times r \times CB\right) : \left(3 \times \frac{1}{2} \times h \times \frac{1}{2} \times CB\right)$$

$= 2r : h.$

Since we know from Q1 that this ratio is 4 : 1 $\Rightarrow h = \dfrac{r}{2}$.

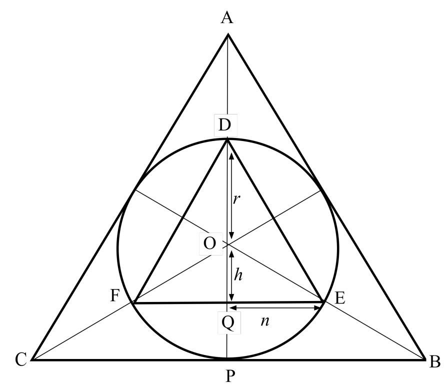
Ratio of area of circles

$= \pi r^2 : \pi h^2$

$= \pi r^2 : \pi \dfrac{r^2}{4}$

$= 4 : 1.$

Darren's method



1. Area of Δ DEF = 3 × area of Δ OEF

$$\Rightarrow \frac{1}{2} \times 2n \times (h + r) = 3 \times \frac{1}{2} \times 2n \times h$$

$\Rightarrow n(h + r) = 3nh$

$\Rightarrow h + r = 3h$

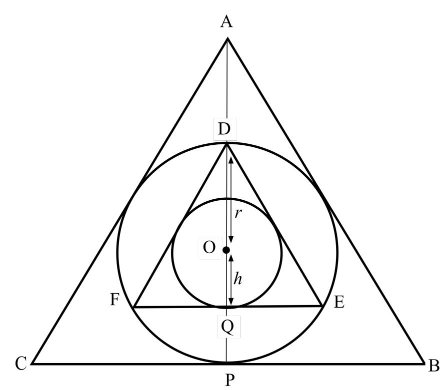
$\Rightarrow h = \dfrac{r}{2}$

Triangle OQE is similar to triangle OPB:
∠POB is common to both triangles and OQE = OPB = 90° (altitudes of an equilateral triangle).
Therefore PB = $2n$ and so CB = $4n$ (altitudes of an equilateral triangle bisect a side).

Area Δ ABC = 3 x area Δ OBC $= 3 \times \dfrac{1}{2} \times 4n \times r = 6nr.$

Area Δ DEF = 3 x area Δ OEF $= 3 \times \dfrac{1}{2} \times 2n \times \dfrac{r}{2} = \dfrac{3nr}{2}.$

Ratio of areas of triangles $= 6nr : \dfrac{3nr}{2} = 4 : 1.$



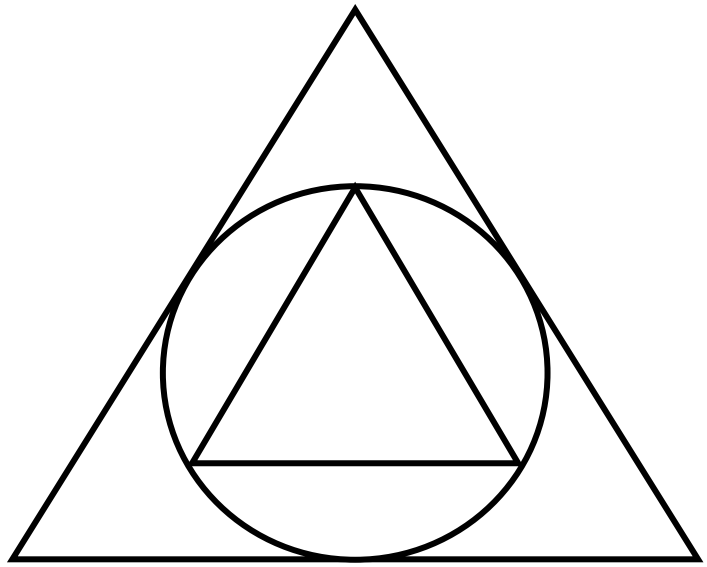
2. h is the radius of the inscribed circle.

$h = \dfrac{r}{2}.$

The ratio of the area of the outer circle to the area of the inner circle $= \pi r^2 : \pi h^2 = \pi r^2 : \pi \dfrac{r^2}{4} = 4 : 1.$

Circles and Triangles

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.



1. Calculate the *exact* ratio of the areas of the two triangles.
   Show all your work.

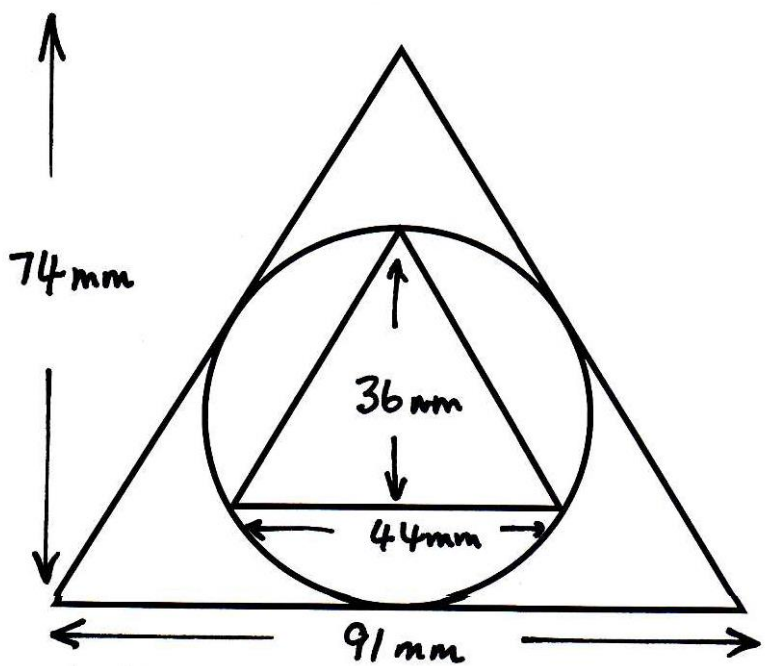
2. Draw a second circle inscribed inside the small triangle.
   Find the *exact* ratio of the areas of the two circles.

Sample Responses to Discuss: Anya

Imagine you are Anya's teacher. Describe how Anya approached the problem.

Write your explanation on a separate sheet.
What do you like/dislike about her work?
What is unclear about her work?
What questions would you like to ask Anya?

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.



1. Calculate the ratio of the areas of the two triangles.
   Show all your work.

$$\Delta = \frac{1}{2} \cdot 74 \cdot 91 = 3367$$

$$\Delta = \frac{1}{2} \cdot 36 \cdot 44 = 792$$

$$\text{Ratio} = \frac{3367}{792} = 4.25$$

Sample Responses to Discuss: Bill

Imagine you are Bill's teacher. Describe how Bill approached the problem.

Write your explanation on a separate sheet.
What do you like/dislike about his work?
What is unclear about his work?
What questions would you like to ask Bill?

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.

$\frac{c}{r} = \frac{1}{2}$ $\quad c = \frac{r}{2}$

$\frac{b}{r} = \frac{\sqrt{3}}{2}$ $\quad b = \frac{r\sqrt{3}}{2}$

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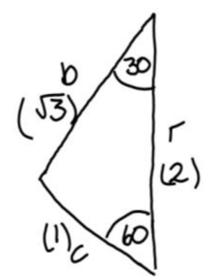
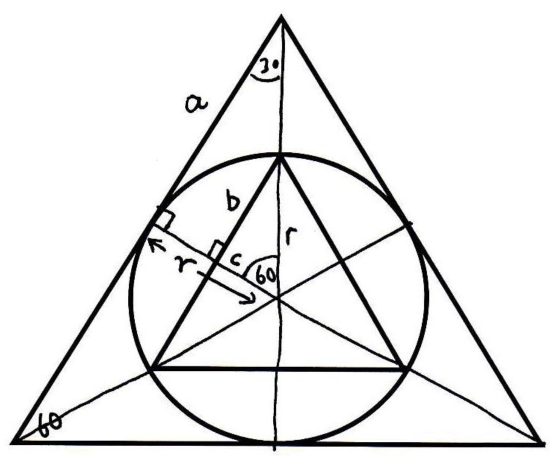




1. Calculate the ratio of the areas of the two triangles.
   Show all your work.

Area of small triangle $= 6 \times \frac{1}{2} \times b \times c = 6 \times \frac{1}{2} \times \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \frac{3r^2\sqrt{3}}{4}$

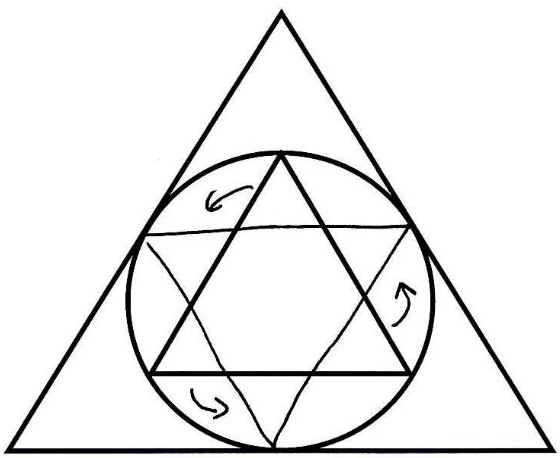
Area of large triangle $= 6 \times \frac{1}{2} \times a \times r = 3 \times r\sqrt{3} \times r = 3r^2\sqrt{3}$

Sample Responses to Discuss: Carla

Imagine you are Carla's teacher. Describe how Carla approached the problem.

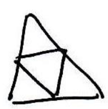
Write your explanation on a separate sheet.
What do you like/dislike about her work?
What isn't clear about her work?
What questions would you like to ask Carla?

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.

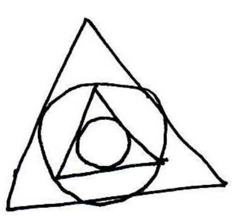


1. Calculate the ratio of the areas of the two triangles.
   Show all your work.

Spin the little triangle.

You get  So big area : small area = 4 : 1

2. Draw a second circle inscribed inside the small triangle.
   Find the ratio of the areas of the two circles.



Big triangle + big circle is enlargement of small triangle + small circle.

So ratio of big circle to small circle = 4 : 1

Sample Responses to Discuss: Darren

Imagine you are Darren's teacher. Describe how Darren approached the problem.

Write your explanation on a separate sheet.
What do you like/dislike about his work?
What is unclear about his work?
What questions would you like to ask Darren?

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.

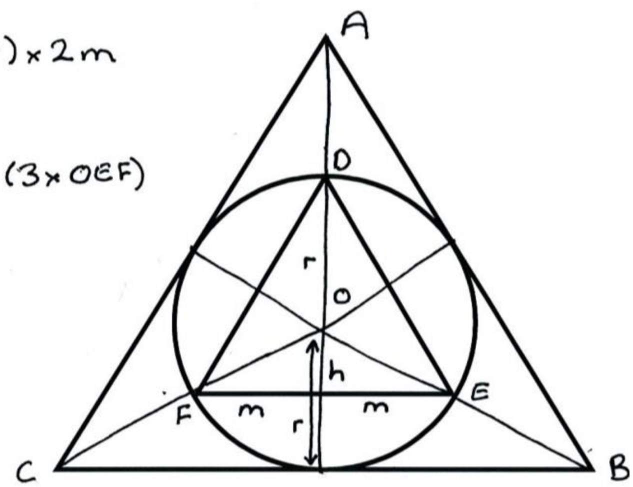
Area $DEF = \frac{1}{2} \times (h+r) \times 2m$

$= m \times (h+r)$

$= 3 \times \frac{1}{2} \times h \times 2m \quad (3 \times OEF)$

$h + r = 3h$

$h = \frac{r}{2}$



1. Calculate the ratio of the areas of the two triangles.
   Show all your work.

Triangle OEF is similar to triangle OBC.
The height of OBC is double the height of OEF,
so CB is double EF.
It follows that the area of OBC is
double × double - four times bigger than OEF.
Area ABC : Area DEF = 3 × OBC : 3 × OEF = 4 : 1

How Did You Work?

1. My own method was similar to one of the sample responses ☐ OR My own method was different from **all** of the sample responses ☐

   ☐ *(add name of sample response)*

   Because: .... Because: ....

2. Our joint method was similar to one of the sample responses ☐ OR Our joint method was different from **all** of the sample responses ☐

   ☐ *(add name of sample response)*

   Because: .... Because: ....

3. Which of the sample methods provides the clearest and most complete solution to the problem? Explain your choice.

   ....

   ....

   ....

   ....

4. Which of the sample methods provides the most elegant solution to the problem? Explain your choice.

   ....

   ....

   ....

   ....

5. On a separate piece of paper, rework Bill's sample response so that it is clearer, easier to understand, and accurate.

Analyzing Sample Responses to Discuss

- Explain what the student has done.
- What do you like/dislike about the work?
- What is unclear about the work?
- What questions would you like to ask the student?

Sample Responses to Discuss: Anya

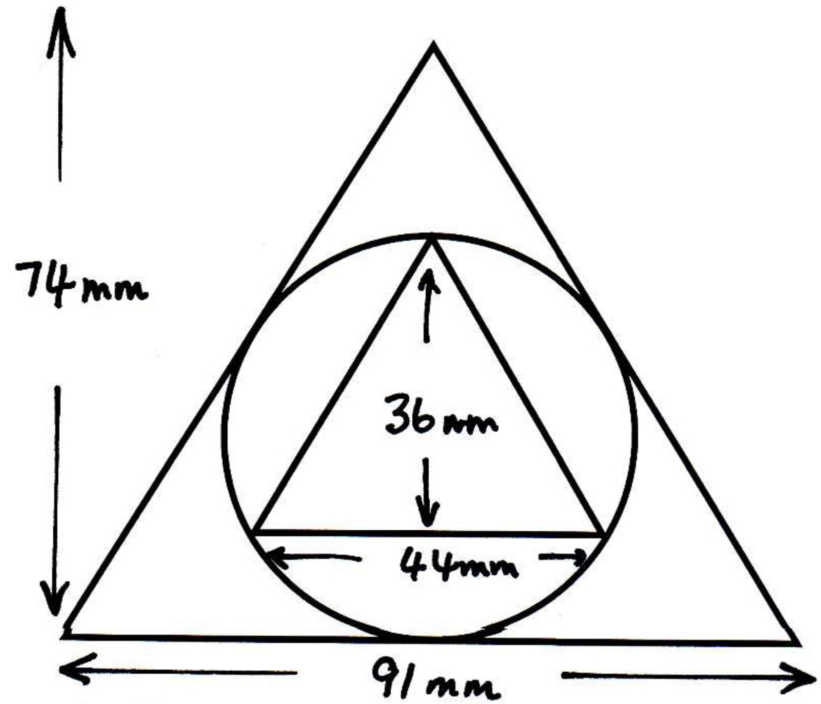
This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.

1. Calculate the ratio of the areas of the two triangles. Show all your work.

$$\Delta = \frac{1}{2} \cdot 74 \cdot 91 = 3367$$

$$\Delta = \frac{1}{2} \cdot 36 \cdot 44 = 792$$

$$\text{Ratio} = \frac{3367}{792} = 4{\cdot}25$$



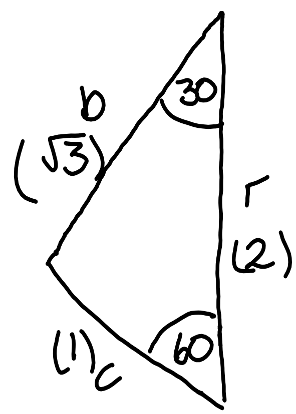
Sample Responses to Discuss: Bill

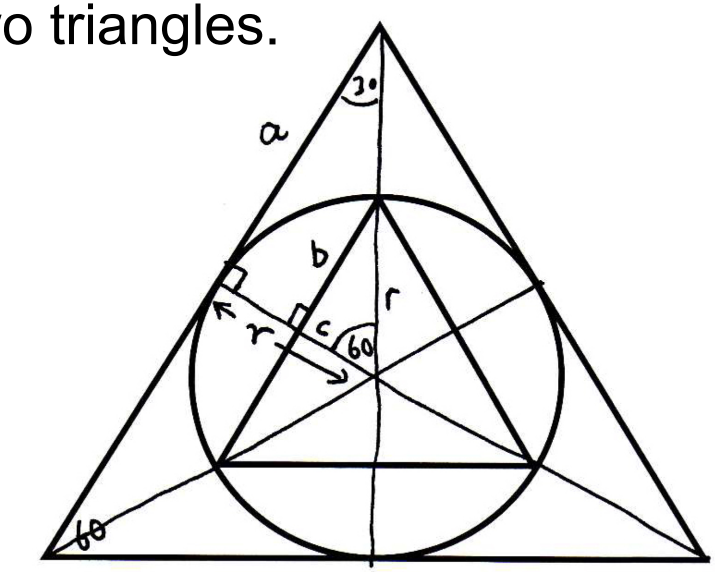
1. Calculate the ratio of the areas of the two triangles.
   Show all your work.

$\frac{c}{r} = \frac{1}{2}$ $\quad$ $c = \frac{r}{2}$

$\frac{b}{r} = \frac{\sqrt{3}}{2}$ $\quad$ $b = \frac{r\sqrt{3}}{2}$

$a = r\sqrt{3}$



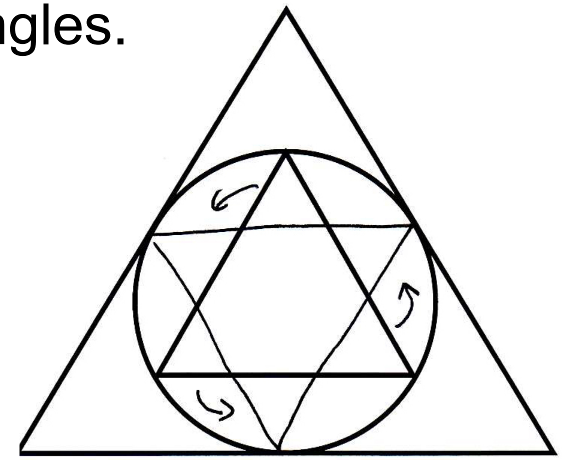


Area of small triangle $= 6 \times \frac{1}{2} \times b \times c = 6 \times \frac{1}{2} \times \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \frac{3r^2\sqrt{3}}{4}$

Area of large triangle $= 6 \times \frac{1}{2} \times a \times r = 3 \times r\sqrt{3} \times r = 3r^2\sqrt{3}$

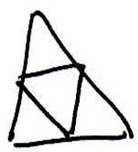
Sample Responses to Discuss: Carla

1. Calculate the ratio of the areas of the two triangles.
   Show all your work.



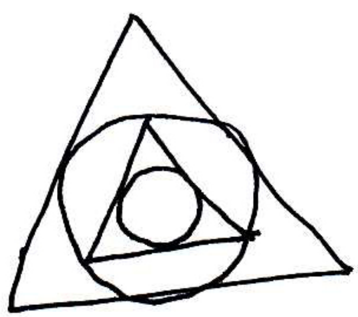
Spin the little triangle.

You get



So big area : small area = 4 : 1

2. Draw a second circle inscribed inside the small triangle.
   Find the ratio of the areas of the two circles.



Big triangle + big circle is enlargement of small triangle + small circle.

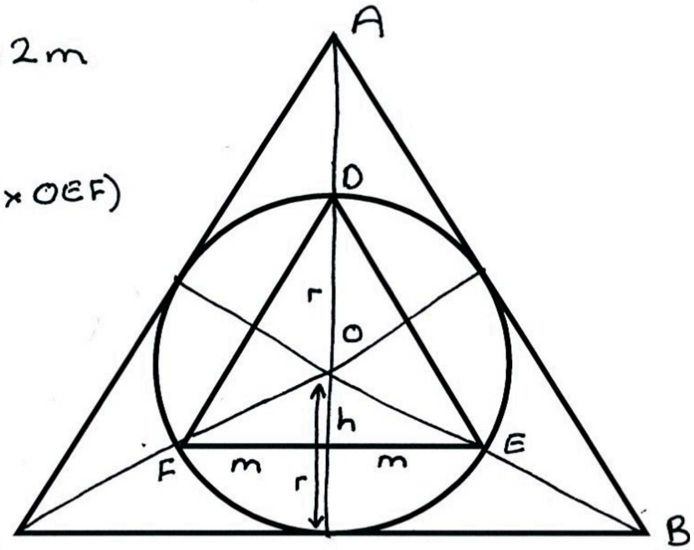
So ratio of big circle to small circle = 4 : 1

Sample Responses to Discuss: Darren

1. Calculate the ratio of the areas of the two triangles. Show all your work.

$$\text{Area } DEF = \frac{1}{2} \times (h+r) \times 2m$$
$$= m \times (h+r)$$
$$= 3 \times \frac{1}{2} \times h \times 2m \quad (3 \times OEF)$$

$$h + r = 3h$$
$$h = \frac{r}{2}$$



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Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
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Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
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Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org