CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

# Proving the Pythagorean Theorem

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

For more details, visit: http://map.mathshell.org
© 2015 MARS, Shell Center, University of Nottingham
May be reproduced, unmodified, for non-commercial purposes under the Creative Commons license detailed at http://creativecommons.org/licenses/by-nc-nd/3.0/ - all other rights reserved

# Proving the Pythagorean Theorem

## MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to produce and evaluate geometrical proofs. In particular, it is intended to help you identify and assist students who have difficulties in:

- Interpreting diagrams.
- Identifying mathematical knowledge relevant to an argument.
- Linking visual and algebraic representations.
- Producing and evaluating mathematical arguments.

## COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

G-CO: Prove geometric theorems.
G-SRT: Prove theorems involving similarity.
A-SSE: Interpret the structure or expressions.
A-CED: Create equations that describe numbers or relationships.

This lesson also relates to **all** the *Standards for Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2, 3, 6, 7, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students attempt the assessment task individually. You then review the work and formulate questions that will help them to improve.
- The lesson begins with a whole-class discussion of the geometric diagram from the assessment task. Students then work collaboratively on the task, in pairs or threes, to produce a better collective solution than those produced individually, justifying their work within their groups. In the same small groups, students critique examples of other students' work.
- In a whole-class discussion, students explain and evaluate the arguments they have seen and used.
- Finally, students work alone on a new task, similar to the original assessment task.

## MATERIALS REQUIRED

- Each student will need some *Grid Paper* and a copy of *Proving the Pythagorean Theorem* and *Proving the Pythagorean Theorem (revisited)*.
- Each small group of students will need a large sheet of paper, copies of the *Sample Methods to Discuss*, and the *Comparing Methods of Proof* sheet.
- Provide copies of *Extension: Proving the Pythagorean Theorem using Similar Triangles* as necessary. There is a projector resource to help with whole-class discussions.

## TIME NEEDED

Approximately 20 minutes before the lesson, an 80-minute lesson (or two 45 minute lessons), and 20 minutes in a follow-up lesson. Exact timings will depend on the needs of the class.

## BEFORE THE LESSON

### Assessment task: *Proving the Pythagorean Theorem* (20 minutes)

Have the students do this task a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and identify students who have difficulty. You will then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the *Proving the Pythagorean Theorem* task and a sheet of grid paper.

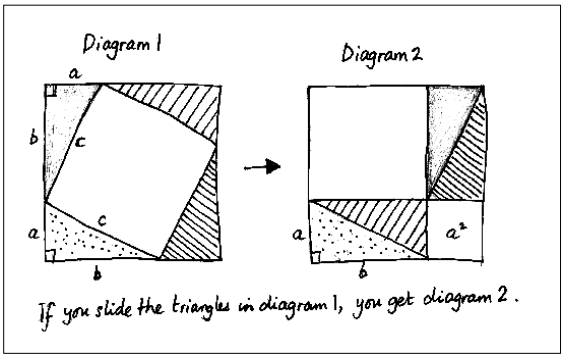
*Spend twenty minutes working individually and answering these questions.*

*Write all your reasoning on the sheet, explaining what you are thinking.*

**Proving the Pythagorean Theorem**

Marty is trying to prove the Pythagorean theorem.

He draws a sketch:




1. Re-draw Marty's diagram so it is more accurate, using pencil and rule. Label your diagram clearly.
2. Write down what you know about all the lengths, angles, shapes, and areas on the diagram. Give reasons for your statements.
3. Using the diagram, construct a proof to show that in any right triangle with sides $a$, $b$ and hypotenuse $c$, $a^2+b^2=c^2$.

Students who sit together often produce similar answers so that, when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually you ask them to move to different seats. At the beginning of the formative assessment lesson allow them to return to their usual places. Experience has shown that this produces more profitable discussions.

It is important that students try to answer the questions without assistance, as far as possible.

### Assessing students' responses

Collect students' responses to the task. Read through their scripts and make some notes on what their work reveals about their current levels of understanding and their different approaches to producing a proof.

We strongly suggest that you do not score students' work. Research shows that this is counterproductive as it encourages students to compare scores and distracts their attention from what they might do to improve their mathematics.

Instead, help students to make further progress by asking questions that focus their attention on aspects of the work. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the start of the collaborative group work.

| Common issues: | Suggested questions and prompts: |
|---|---|
| **Constructs an inaccurate diagram**<br>For example: The student's diagram does not show a square because they have not drawn all lengths labeled $a$ equal in length. | • Look carefully at the diagram Marty drew. Which lengths are equal?<br>• Label the sides of each triangle in Marty's diagram. Use this to help you draw accurately. |
| **Does not identify and use relevant mathematical knowledge**<br>For example: The student does not deduce that the angle between the sides marked $c$ is a right angle.<br>Or: The student does not recognize that the side length of the large square is $a + b$. | • What can you say about the angles in this diagram?<br>• What different geometrical figures can you see?<br>• What do you know about the areas of these figures?<br>• How could you write this side length using algebra? |
| **Writes an incomplete solution**<br>For example: The student writes some relevant theorems and notices some relevant structure but has lost direction. | • What do you already know?<br>• What do you want to find out?<br>• Try working backwards: what will the end result be?<br>• Your rearrangement shows that $c^2 = a^2+b^2$, but can you explain why this is true using words and algebra? |
| **Relies on perceptual properties of diagram**<br>For example: The student writes that the area of the square side $c$ on the first diagram is equal to $a^2 + b^2$ on the second diagram, but doesn't justify that claim.<br>Or: The student assumes that two conjoining triangles form a rectangle. | • You have assumed that the area is $a^2 + b^2$. How do you know that this is correct?<br>• You have assumed that the two triangles form a rectangle. They do look like a rectangle, but how do you know you this is not a different type of parallelogram? |
| **Reasons empirically**<br>For example: The student measures the sides of the triangle and uses those measures in length/area calculations. | • Think of your triangle as representing **any** right triangle with sides $a, b, c$.<br>• Think of what is needed for a proof. Is your argument a proof? |
| **Relies on visual reasoning**<br>For example: The student draws arrows to show how to move from one diagram to another, but does not provide a written, algebraic argument. | • How can you make your argument clearer?<br>• How can you turn your visual argument into a proof? |
| **Writes a complete and correct solution**<br>Provide a copy of the *Proving the Pythagorean Theorem using Similar Triangles* extension task. | • Here is a new diagram. Use it to prove the Pythagorean Theorem. |

## SUGGESTED LESSON OUTLINE

### Whole-class introduction (15 minutes)

Remind students of the *Proving the Pythagorean Theorem* task they completed prior to this lesson:

*What is the Pythagorean Theorem?*

*Are there any other ways of stating the theorem?*

Prompt students for both length and area expressions.

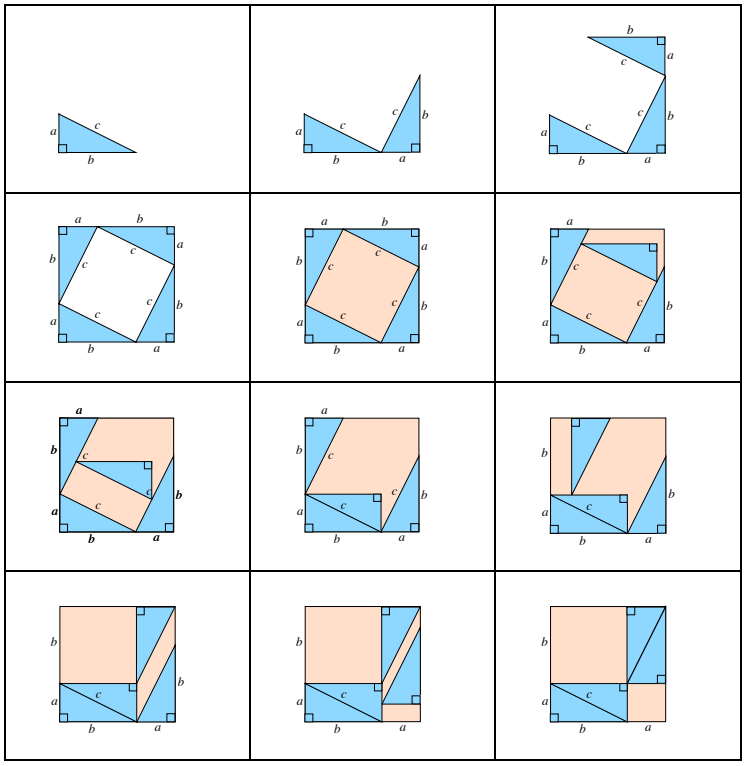
Check that students understand when the Pythagorean Theorem can be applied:

*For what kind of triangles is the Pythagorean Theorem true?*

*Is it true for any right triangle? Even one that looks like this? [Draw some extreme cases.]*

*There are many different ways of proving the Theorem. We are going to look at some this lesson.*

Re-introduce Marty's diagram from the assessment task, using the projectable resource (Slides P-1 to P-12). Work through the construction processes step-by-step.



Don't try to complete the proof at this stage. Instead, help students to understand the construction of the diagram. Research shows that students benefit from reconstructing and analyzing diagrams.

Prompt students to use clear mathematical language to describe the construction processes, such as 'congruent', 'perpendicular', 'reflect in a horizontal/vertical line', 'translate', and 'rotate by 90° (counter) clockwise'.

*Describe how the blue triangle has moved.*

*How do you know that the sides of the square are straight?*

*How do you know that the center shape is a square?*

*The sides of the triangle are a and b. So what is the total length of this line?*

*How do you know that the orange area never changes?*

### Collaborative group work (25 minutes)

Organize students into groups of two or three. Give each group a large sheet of paper.

*Remember the task I gave you [last lesson]? I read your solutions to the task and have some questions about your work.*

*Spend a few minutes on your own, reading my questions. Use my questions to review your work.*

*Then, take turns to share your ideas with the rest of the group.*
*After you have all shared, work together in your group to come up with a better solution.*

Explain that when proving, it is useful to gather knowledge of the mathematical structure:

*When thinking about your joint solution, you may first want to discuss how the diagram is constructed.*

*Think about what you already know about lengths, areas and angles and think about what you know of this diagram in particular.*

*Think about how you can use the diagram to figure out the statement of the Pythagorean Theorem.*

*Make sure you explain everything really clearly on your poster paper.*

You have two tasks during small group work: to note different student approaches to the task and support student problem solving.

### Note different student approaches to the task

Notice how students make a start on the task, if they get stuck, and how they respond if they do come to a halt. Note which properties they identify and how they choose which information may be useful. Notice when students look for different ways of writing the same information. Do they use visual/geometrical as well as algebraic language? Do they produce a general (algebraic or visual) solution, or do they introduce measures and work empirically?

### Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, write a relevant question on the board. You might also ask a student who has performed well on a particular part of the task to help a struggling student.

The following questions and prompts have been found most helpful:

*Which side(s) of the triangle form(s) the side of the square?*

*What is the length of this line segment?*

*How could you calculate the area of this square? Is that the only way to figure out the area?*

*How do you know this is always true? Maybe it only works for these numbers.*

*You have assumed that … Why do you believe that is true?*

### Extending the lesson over two days

If you are taking two days to complete the lesson unit then you may want to end the first lesson here. At the start of the second day, allow students time to familiarize themselves with their joint solution before moving on to the collaborative analysis of sample methods.

## Collaborative analysis of *Sample Methods to Discuss* (25 minutes)

Give each small group of students a copy of the three *Sample Methods to Discuss* and a copy of *Comparing the Sample Methods*. Use Slide P-13 to draw students' attention to the questions on these sheets.

*Read through these proofs. None of them are perfect.*

*You need to describe what each student has done and decide whether his or her approach will lead to a proof. You also need to consider how the work can be improved.*

*Answer the questions underneath each piece of work and once you have done this, evaluate the completed arguments.*

*Which approach did you find most convincing? Why?*
*Produce a complete correct solution using your preferred method. I have some Grid Paper if you want to use it.*

This analysis task will give students the opportunity to engage with and evaluate different types of argument without providing a complete solution strategy. It also raises questions about what counts as a good mathematical argument for discussion at the end of the lesson.

We have included methods here that reflect some common issues in student reasoning. Penelope and Sophie have only considered specific cases. In Penelope's case, her reasoning can be generalized, using algebra, but Sophie's diagram will only work for isosceles right triangles, so her method cannot be generalized. Nadia's approach shows the beginnings of an algebraic method, but this is incomplete and contains an error.

During small group work, support student thinking as before. Also, listen to see what students find difficult. Identify one or two aspects of these approaches to discuss in a whole-class discussion. Note similarities and differences between the sample approaches and those the students took in small-group work.

If students require more work, ask them to produce a proof using Penelope's diagram and compare this with their improved version of Marty's (visual and algebraic) solution method.

### Whole-class discussion: comparing the *Sample Methods to Discuss* (15 minutes)

Organize a whole-class discussion to analyze the different *Sample Methods to Discuss* (you may want to use Slides P-14 to P-16 to support this discussion). The intention is that you focus on getting students to explain the methods of working and compare different styles of argument, rather than just checking numerical or algebraic solutions.

The three approaches shown in the *Sample Methods to Discuss* are described in detail below. You may want to use some of this information to support the whole-class discussion.

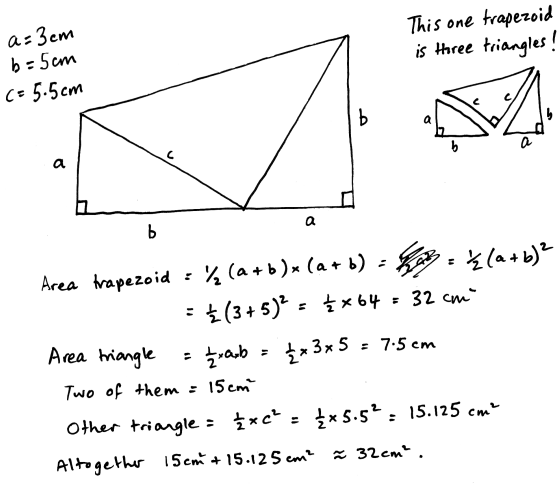
**Penelope** has measured the diagram and found the area of the figure in two different ways. She finds that the whole trapezoid has an area that is roughly equal to the three triangles added up. This does not amount to a proof of the Pythagorean theorem, because it only considers a special case. She has some material here for developing a proof, notably the (implied) equation:

$$\frac{1}{2}(a+b)^2 = \frac{1}{2}ab \times 2 + \frac{1}{2}c^2 .$$

If this were simplified, it could be made into a proof.

*Penelope assumes the shape is a trapezoid. How do you know she is correct?*

*How can we develop her ideas into a proof?*



**Nadia's** method is rather like Marty's: she has not tried to transform the diagram, but has tried to find the area in two different ways. Some students may notice that Nadia's diagram is like two of Penelope's placed on top of one another.

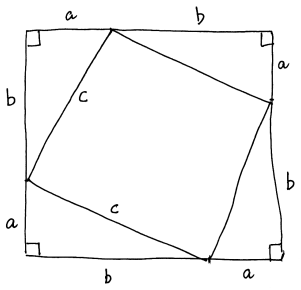
There is an algebraic error in the last line (a common one), but if this were corrected the proof could be completed by equating

$(a+b)^2 = 2ab + c^2$

*Nadia assumes the angle in the inner quadrilateral is a right angle. Is she correct? How do you know?*

*Your group used a similar solution method to Nadia. Can you explain the solution?*

*How can we develop her ideas into a proof?*




Area of middle square = $c^2$

Area of four triangles = $\frac{1}{2} \times a \times b \times 4 = 2 \times a \times b$.

Total area = $(a+b)^2 = a^2 + b^2$

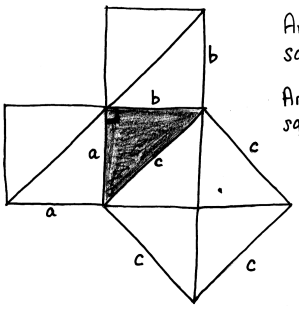
**Sophie's** method does not generalize. It can be used only for isosceles right triangles, in which case $a = b$. Emphasize the need to check that diagrams work for all possible cases.

*Will Sophie's method work for all right angled triangles?*

*What would happen if you tried to draw Sophie's diagram making the sides of the right triangle unequal?*

*How could Penelope or Nadia improve her solution?*

*Was it hard to understand these approaches?*




Area of small squares = $a^2 + b^2$

Area of big square = $c^2$

$c^2 = a^2 + b^2$

(4 triangles)

Once students have explained all three methods, ask them to compare them.

*Which did you think was the most convincing proof? Why?*

## Follow-up lesson: *Proving the Pythagorean Theorem (revisited)* (20 minutes)

Invite students to work individually on the *Proving the Pythagorean Theorem* (*revisited*) task. This is intended to help you, and the students, ascertain whether students can apply what they have learned from the lesson. Some teachers might ask students to complete this task for homework.

### Optional extension task

A further method for proving the Pythagorean Theorem is begun on the sheet *Extension: Proving the Pythagorean Theorem using Similar Triangles*. This may be given to students that have been successful with the other approaches shown in this lesson, as a further challenge.

## SOLUTIONS

Students may choose visual, empirical, or algebraic approaches to proof with any of the diagrams. We provide sample algebraic proofs in the *Sample Methods to Discuss* (below).

### Assessment task: *Proving the Pythagorean Theorem*

Marty relies on a visual transformation of the area. This visual transformation is a powerful mathematical tool, but it leaves too much for the reader to do; it does not constitute a full proof. Marty provides no explanation of his approach. He moves the four congruent triangles to form two rectangles of side lengths $a$ and $b$, with two squares of sides length $a$, $b$ respectively making up the rest of the large square area.

Marty could strengthen his solution by showing the connection between the variables he uses to label the side lengths and the areas of the constituent parts of the figures more explicitly. In particular, he needs to describe how the algebra he uses links the lengths $a$, $b$, $c$ to the transformed areas in his second diagram. He needs to provide much more explanation of his work to make it clear for the reader.

For example, he could write:

- The side length of the large square is $a+b$ so the area of the large square is $(a+b)^2 = a^2+b^2+2ab$. Now I can find the area of the individual pieces of the large square.
- The inner square has side length $c$ and area $c^2$.
- Each of the right triangles has area $\frac{1}{2}ab$. Two of these triangles form a rectangle $ab$. There are four of them. So this gives an area of $2ab$ from the triangles.
- Adding together all the pieces gives the area of the large square. So the area of the large square is $c^2+2ab$.
- I now have two ways of writing the area of the large square. So $c^2+2ab= a^2+b^2+2ab$.
- Subtract $2ab$ from each side to see that $c^2= a^2+b^2$.

### Analysis of *Sample Methods to Discuss*

#### Penelope's method

Penelope gives a clear explanation of her approach at the beginning of her solution. She is finding the area of the trapezoid as a whole and as the sum of three triangle areas. She provides more information for the reader than the others, so her solution is a better (but still incomplete) **explanation**. However, she takes an empirical approach, using the measures of the sides of the triangles to find that the area is approximately equal whether found as a trapezoid or as the sum of the areas of three triangles. This is not a **proof** of the Pythagorean Theorem because it is not a general argument.

Penelope's solution can be improved using some of the formulas she identified. However, she first needs to show that the quadrilateral is a trapezoid. The two (apparently) vertical sides are parallel, since they meet the line segment $ab$ at the same (right) angle.

The area of the trapezoid = $\frac{1}{2}h(a+b) = \frac{1}{2}(a+b)(a+b) = \frac{1}{2}(a+b)^2 = \frac{1}{2}(a^2+2ab+b^2)$

Penelope can also find the area of the trapezoid as the sum of three triangles. Area of triangle with side lengths $a$, $b$ = $\frac{1}{2}ab$. There are two of these, with total area $ab$.

Penelope needs to establish that the angle between the segments of length $c$ is right before using these to calculate the area of the triangle with side lengths $c$. Since the triangle with sides $a$, $b$ is right, the

two unknown angles sum to 90°. The angle in the triangle with side lengths $c$ forms a straight line with these two angles. So the missing angle in the triangle with side lengths $c$ is 90°.

The area of triangle with perpendicular side lengths $c$ is $\frac{1}{2}c^2$. So the total area of the trapezoid is $\frac{1}{2}c^2 + ab$

Since both methods give the total area of the trapezoid,

$$\frac{1}{2}c^2 + ab = \frac{1}{2}(a^2 + 2ab + b^2) = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2$$

Cancelling $ab$ from both sides, $\frac{1}{2}c^2 = \frac{1}{2}a^2 + \frac{1}{2}b^2$, so $c^2 = a^2 + b^2$ as required.

### Nadia's method

Students may recognize the figure in Nadia's method. Nadia has drawn the first diagram from Marty's method. Nadia only draws one diagram and therefore does not show any transformations of the geometric shapes. Instead, she adopts an algebraic method of proof.

Nadia makes a mistake: she writes $(a+b)^2 = a^2 + b^2$. To improve her solution, Nadia could correct that error. To complete her proof, she could link the algebraic expansion of the parentheses with the areas of the shapes making up the diagram.

Expanding the parentheses correctly gives:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Writing the area of the square in terms of its constituent parts:

$$(a+b)^2 = c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$$

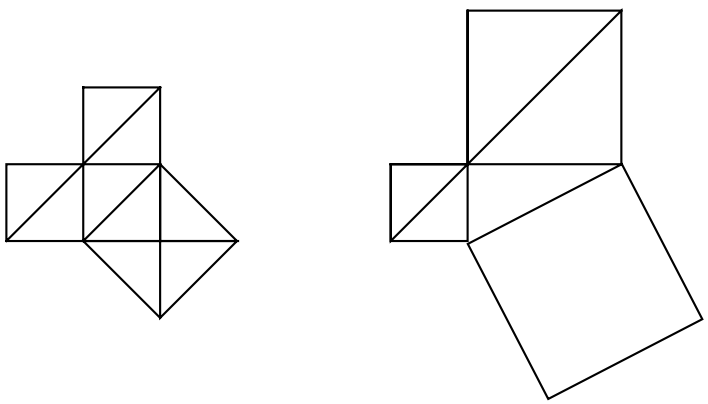
Thus $a^2 + 2ab + b^2 = c^2 + 2ab$

And $a^2 + 2ab + b^2 = c^2 + 2ab$

Subtracting $2ab$ from each side gives the required result.

### Sophie's method

The triangle in Sophie's solution is right, but also isosceles. She represents a general right isosceles triangle. However, the special properties of the right isosceles triangle are an important part of the structure of the diagram. Changing to the general right triangle by choosing $a \neq b$ makes the relationship between the areas of the large square and isosceles triangles of side lengths *a, b* unclear.



So Sophie's is a proof of the Pythagorean Theorem **only** for right isosceles triangles. Check that students recognize the reason for this restricted result. To improve her work, Sophie might redraw the diagram and work on a proof for all right triangles.

### Assessment task: *Proving the Pythagorean Theorem* (*revisited*)

This assessment task is another dissection proof. The sides $a$, $b$ are perpendicular and so can be used to calculate the area of each right triangle. The hypotenuse of the right triangle, $c$, forms the side of the large square.

Area of large square = $c^2$.

Area of large square = area of four triangles + area of small square.

Area of one triangle = $\frac{1}{2} base \times perp.height = \frac{1}{2}ab$.

Area of four triangles = $4 \times \frac{1}{2}ab = 2ab$.

Side length of inner square = $a - b$.

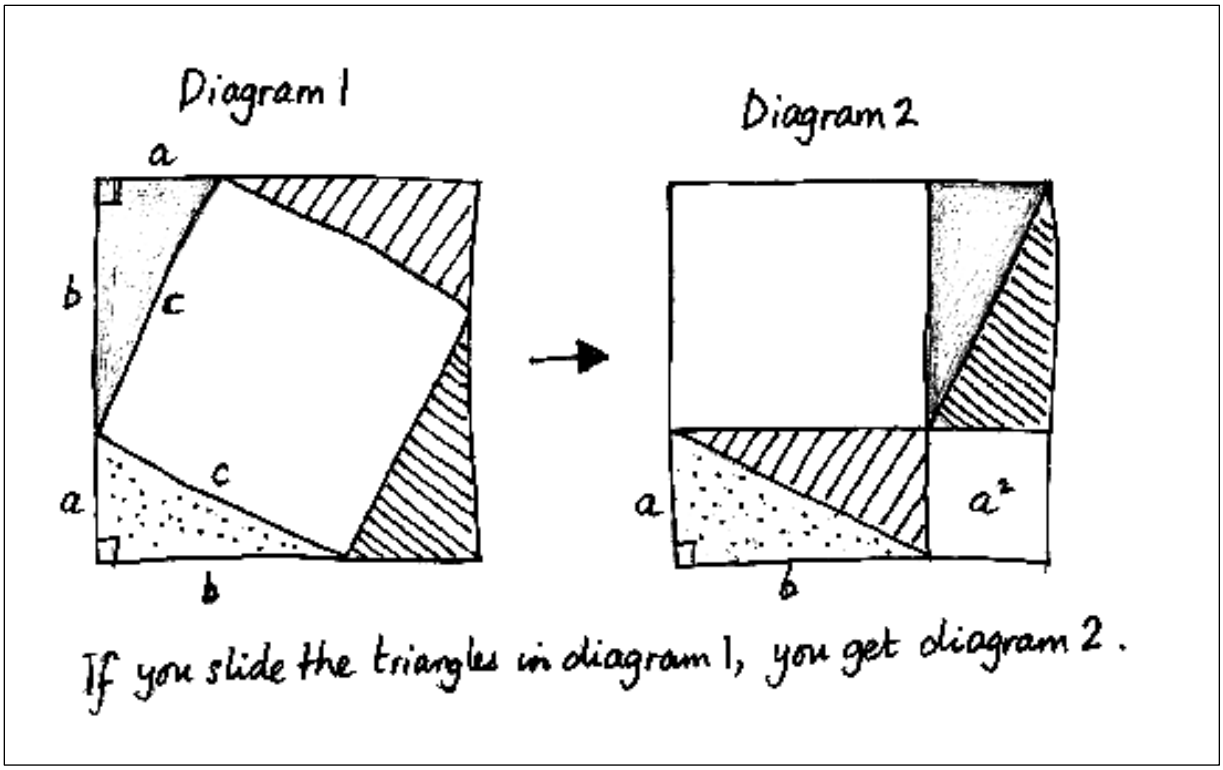
Area of inner square = $(a-b)^2 = a^2 - 2ab + b^2$.

So area of the large square is $c^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$ as required.

# Proving the Pythagorean Theorem

Marty is trying to prove the Pythagorean theorem.

He draws a sketch:



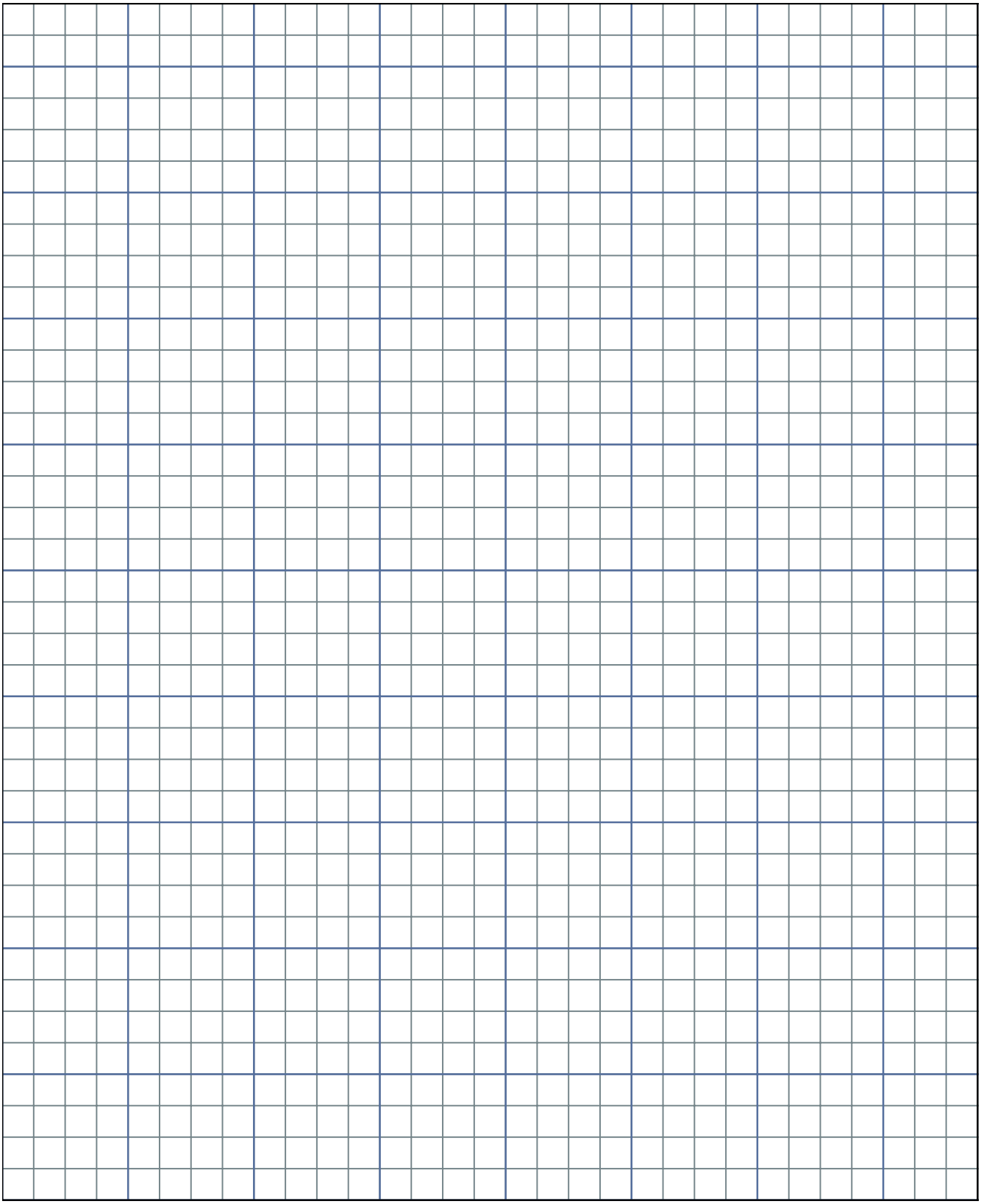

1. Re-draw Marty's diagram so it is more accurate, using pencil and rule.
   Label your diagram clearly.

2. Write down what you know about all the lengths, angles, shapes, and areas on the diagram.
   Give reasons for your statements.

3. Using the diagram, construct a proof to show that in any right triangle with sides $a$, $b$ and hypotenuse $c$, $a^2+b^2=c^2$.

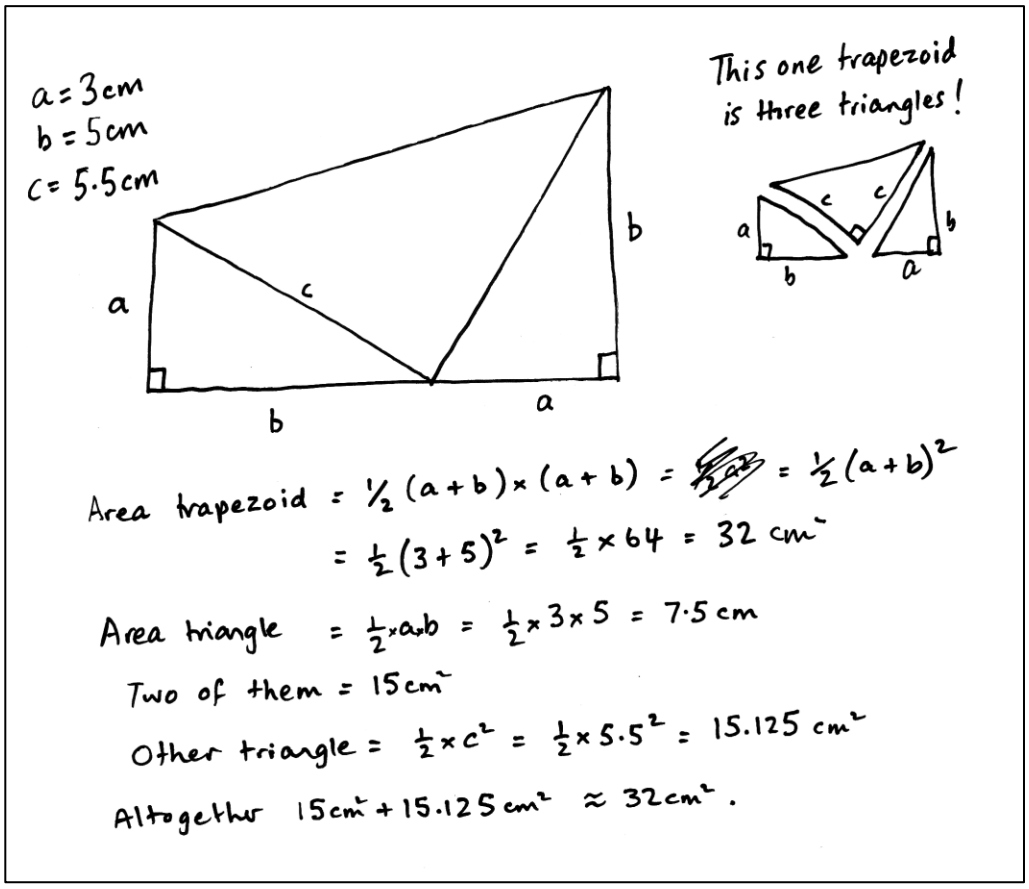

© 2015 MARS, Shell Center, University of Nottingham

# Grid Paper




© 2015 MARS, Shell Center, University of Nottingham

# Sample Methods to Discuss: Penelope



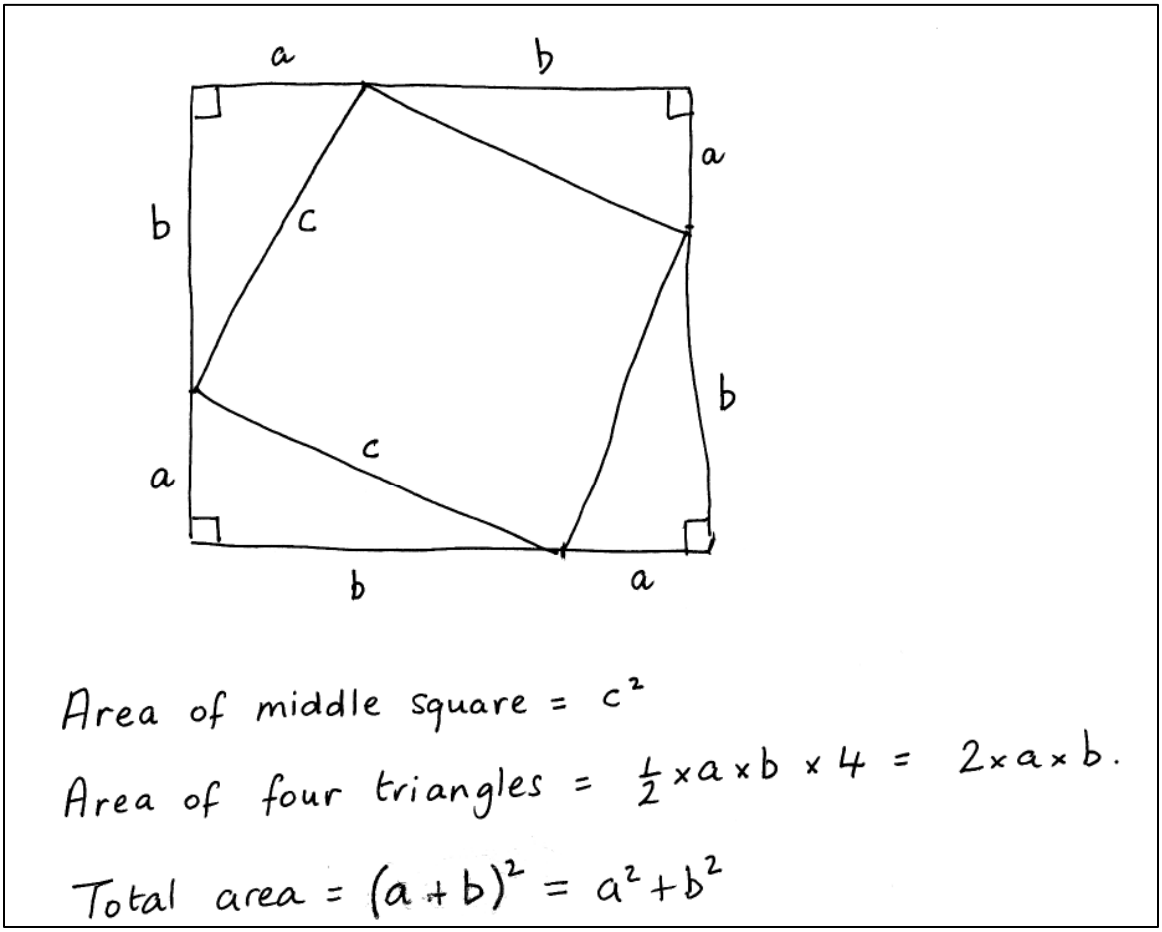

1. What connects Penelope's diagram to the Pythagorean Theorem?

2. Is Penelope's reasoning convincing? Explain your answer.

3. Explain how Penelope could improve her proof.


© 2015 MARS, Shell Center, University of Nottingham

# Sample Methods to Discuss: Nadia




Area of middle square = $c^2$

Area of four triangles = $\frac{1}{2} \times a \times b \times 4 = 2 \times a \times b$.

Total area = $(a+b)^2 = a^2 + b^2$

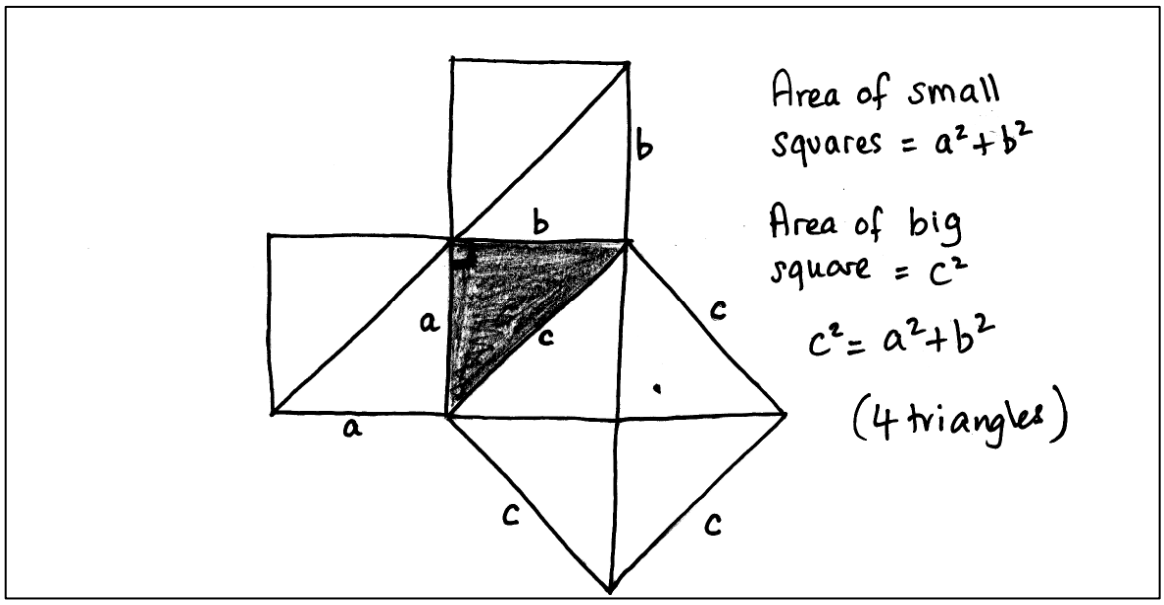
1. Describe the method Nadia used in her proof.

2. Is Nadia's reasoning convincing? Explain your answer.

3. Explain how Nadia could improve her proof.


© 2015 MARS, Shell Center, University of Nottingham

# Sample Methods to Discuss: Sophie




1. What type of triangle does Sophie use in her proof?

2. Is Sophie's reasoning convincing? Explain your answer.

3. Explain how Sophie could improve her proof.


© 2015 MARS, Shell Center, University of Nottingham

# Comparing the Sample Methods

1. Compare the work written by Penelope, Nadia, and Sophie.

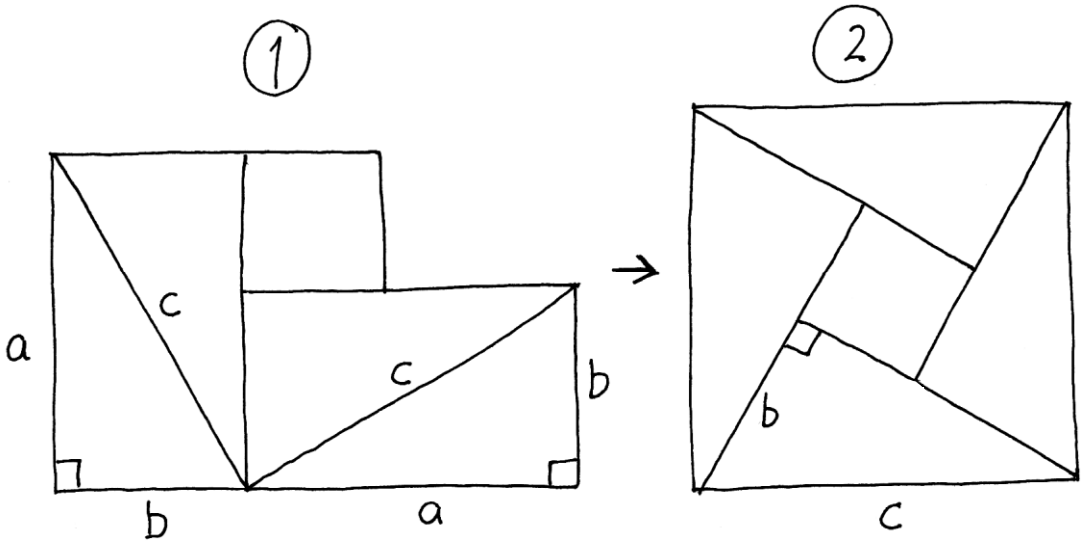
Whose solution method do you find most convincing? Why?

2. Produce a complete and correct proof using your preferred method.

Your teacher has *Grid Paper* if you want to use it.


© 2015 MARS, Shell Center, University of Nottingham

# Proving the Pythagorean Theorem (revisited)



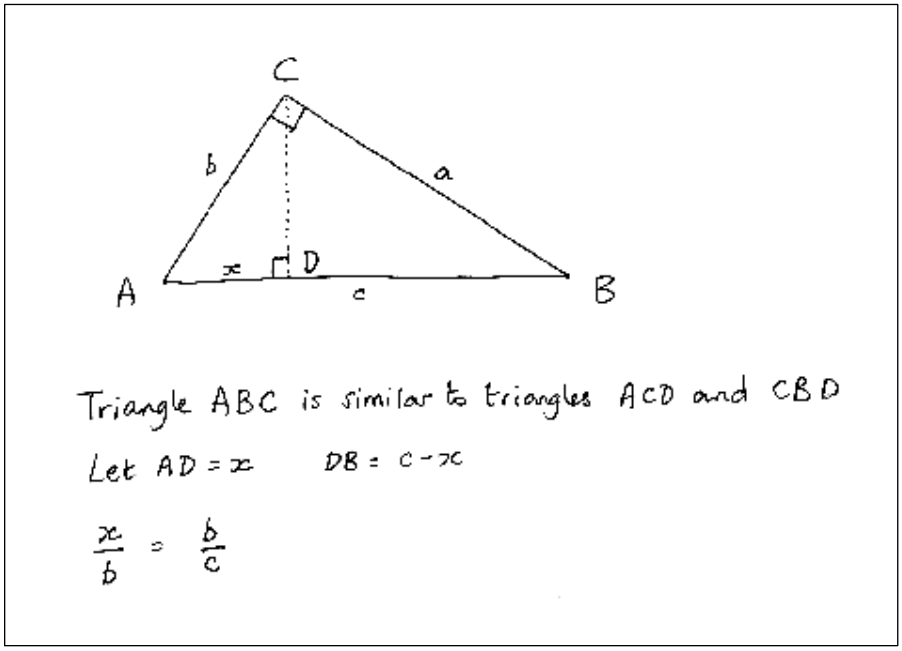

Four congruent right triangles and a square can be rearranged to get from Diagram 1 to Diagram 2.

1. Using a pencil and a rule, re-draw the diagrams accurately.
   Label your diagrams clearly.
2. Write down what you know about the lengths, angles, shapes and areas on the diagrams.
   Give reasons for your statements.
3. Use the diagrams to prove that in any right triangle with sides $a$, $b$ and hypotenuse $c$, $a^2+b^2=c^2$.


© 2015 MARS, Shell Center, University of Nottingham

# Extension: Proving the Pythagorean Theorem using Similar Triangles.

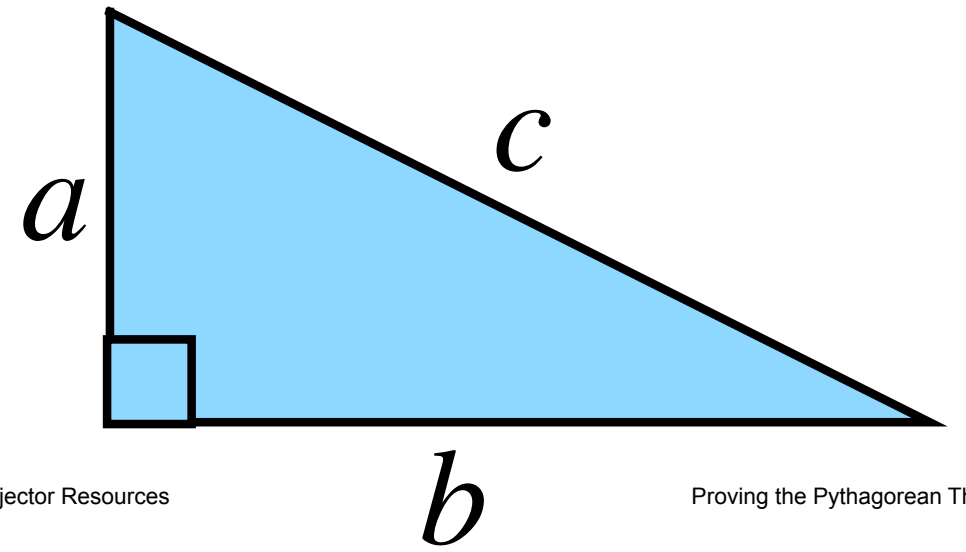
Max has started to prove the Pythagorean Theorem using similar triangles:

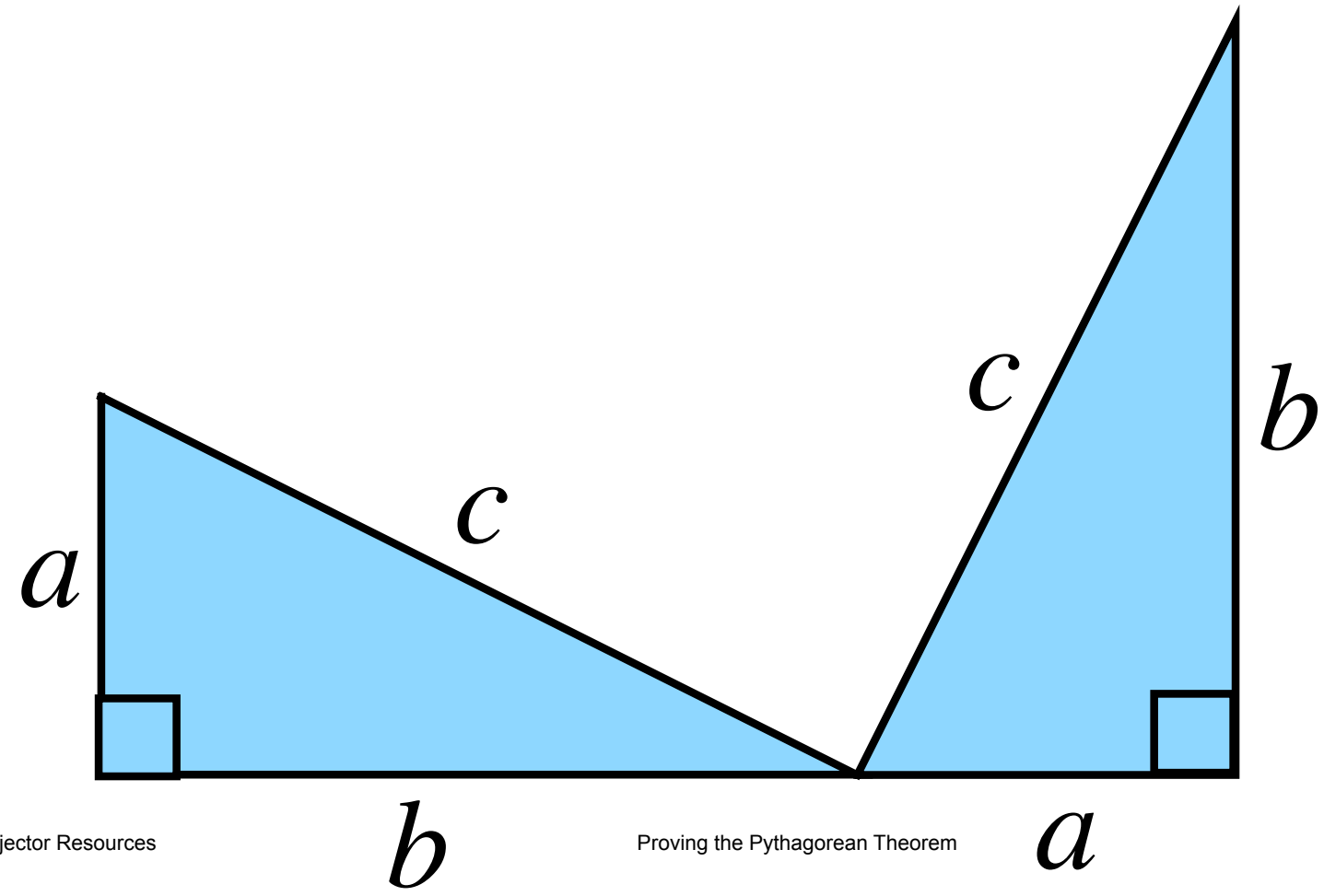


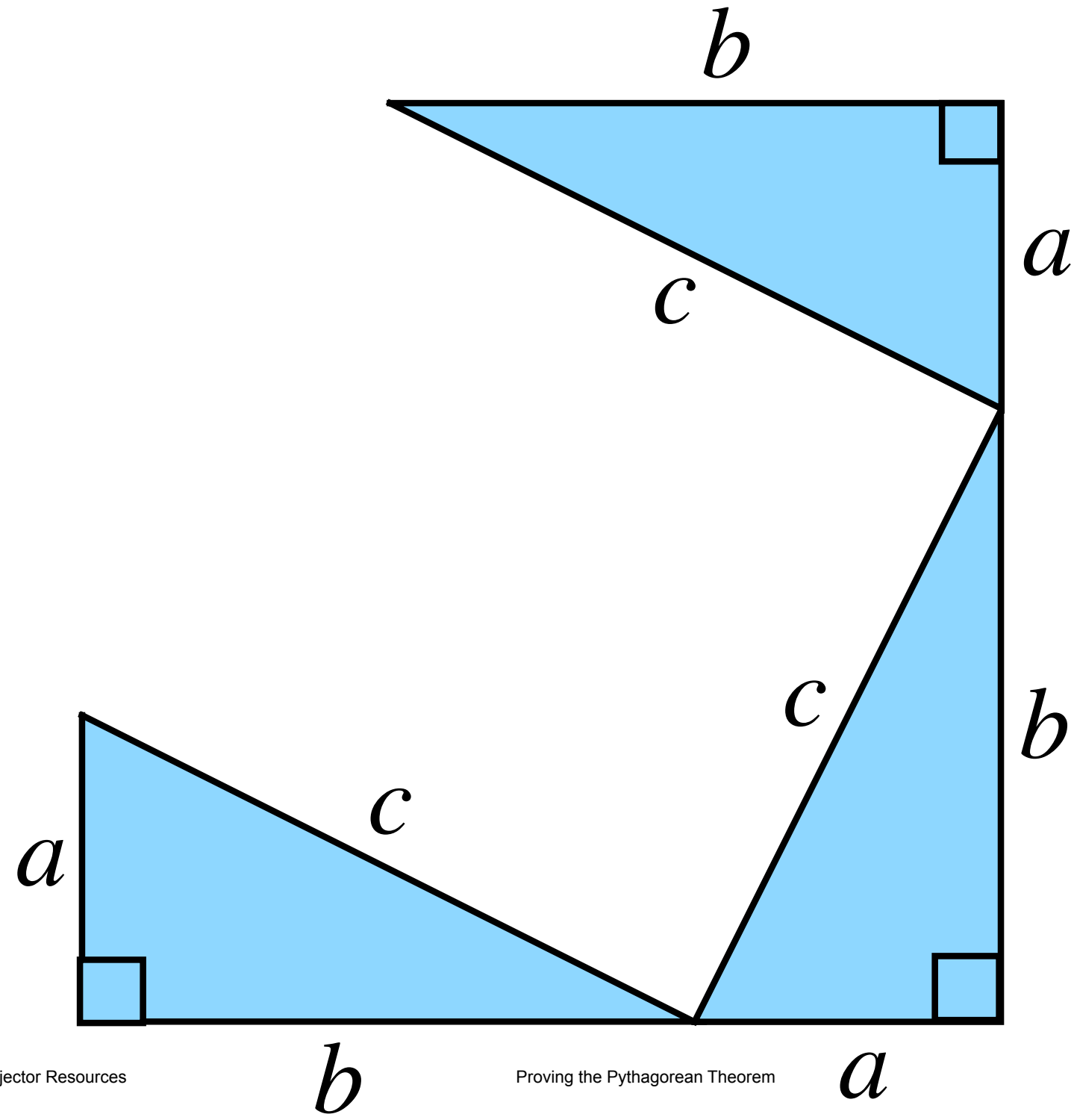

1. Explain why the three triangles are similar.

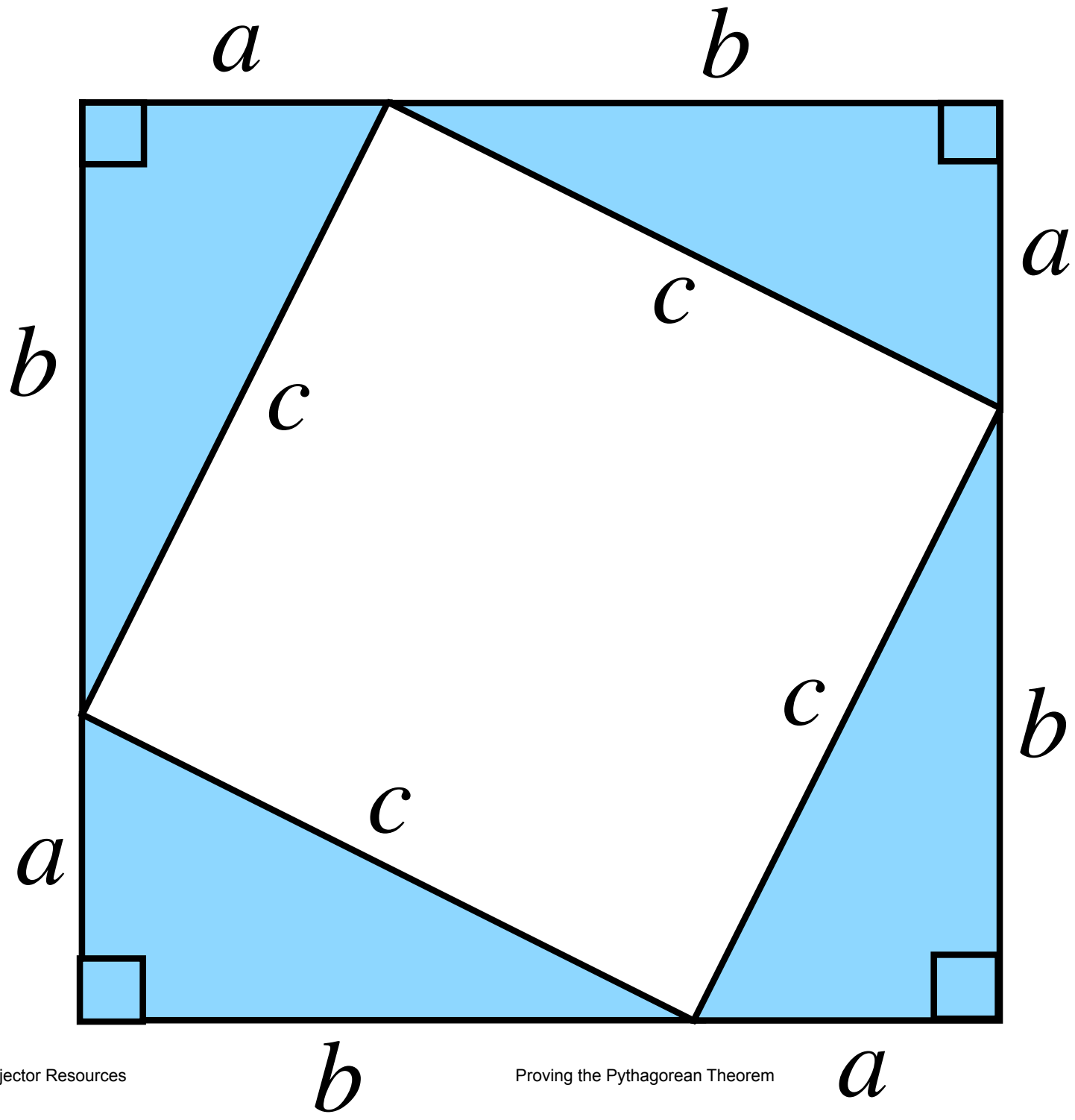
2. Try to complete Max's proof. (Use *Grid Paper* if you need it.)

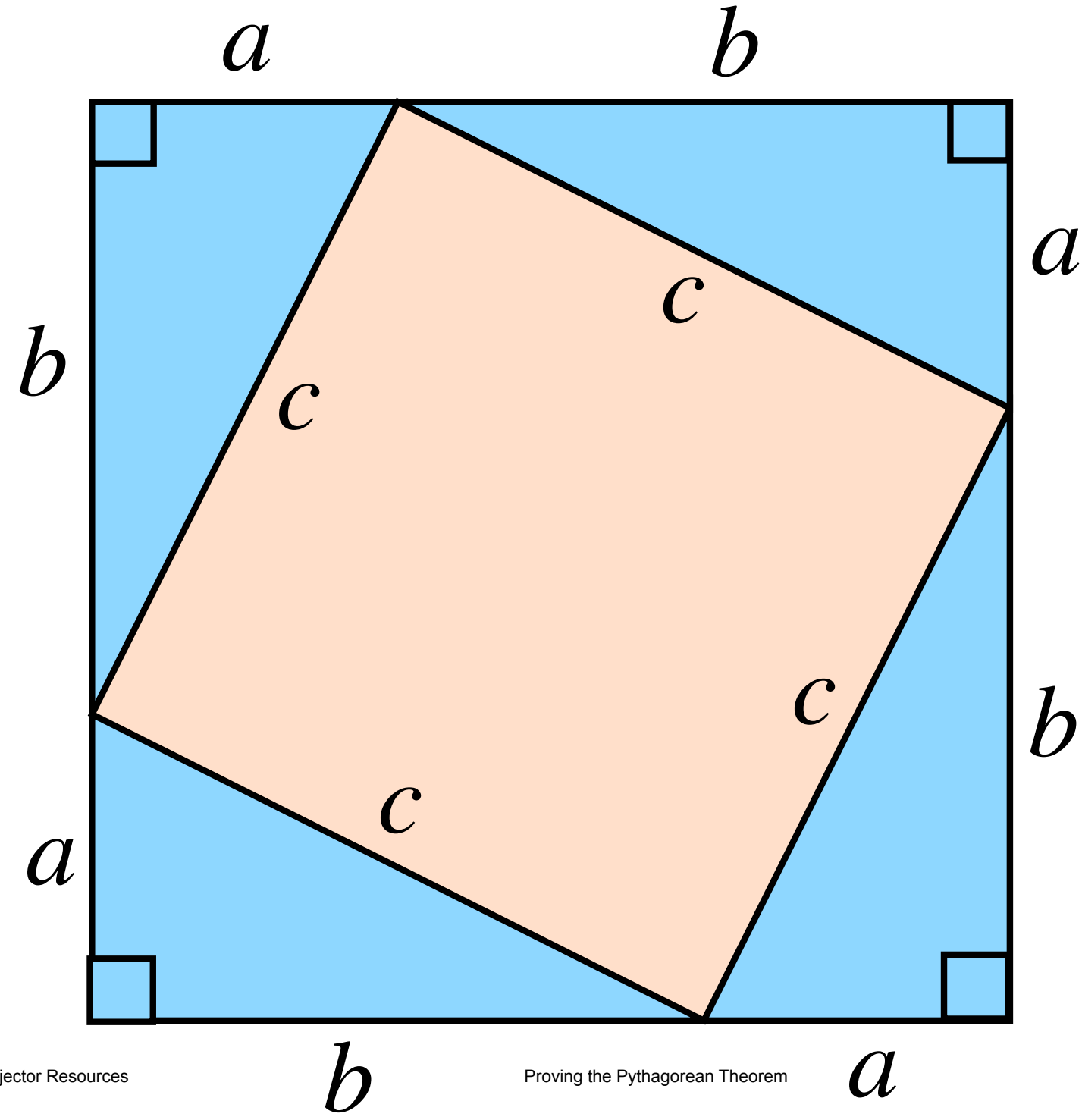

© 2015 MARS, Shell Center, University of Nottingham

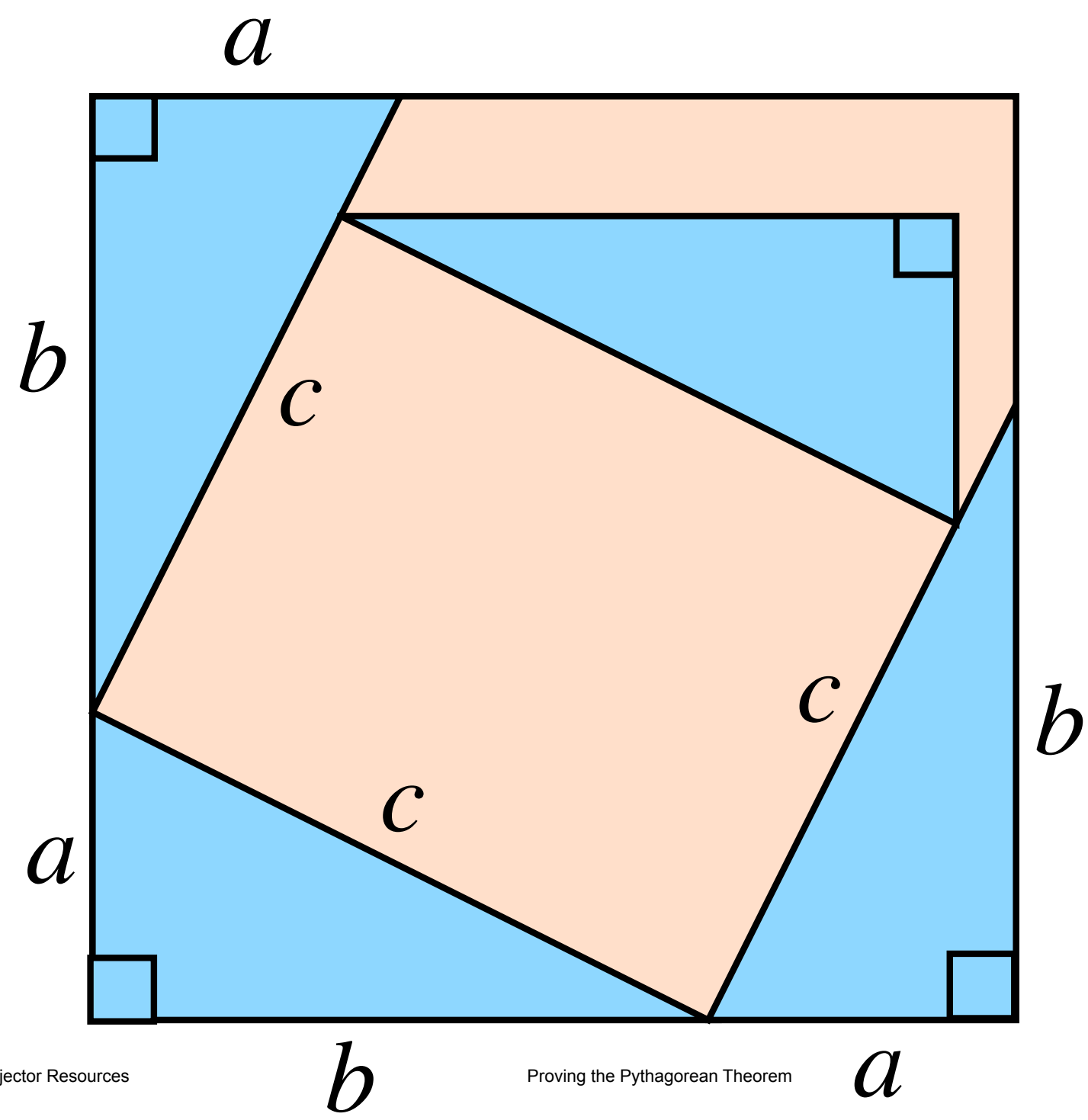

a
c
b

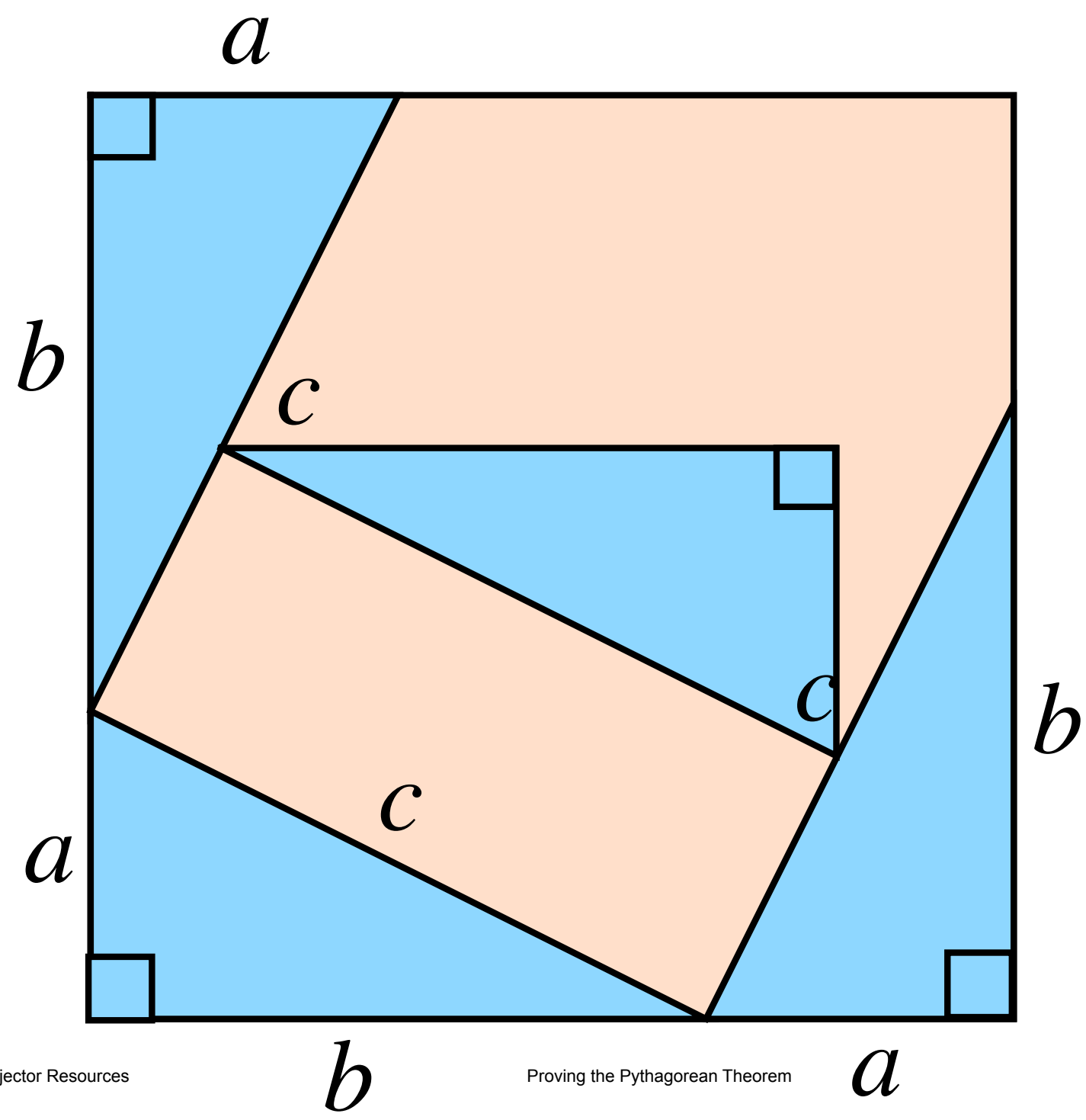

a
c
b
c
b
a

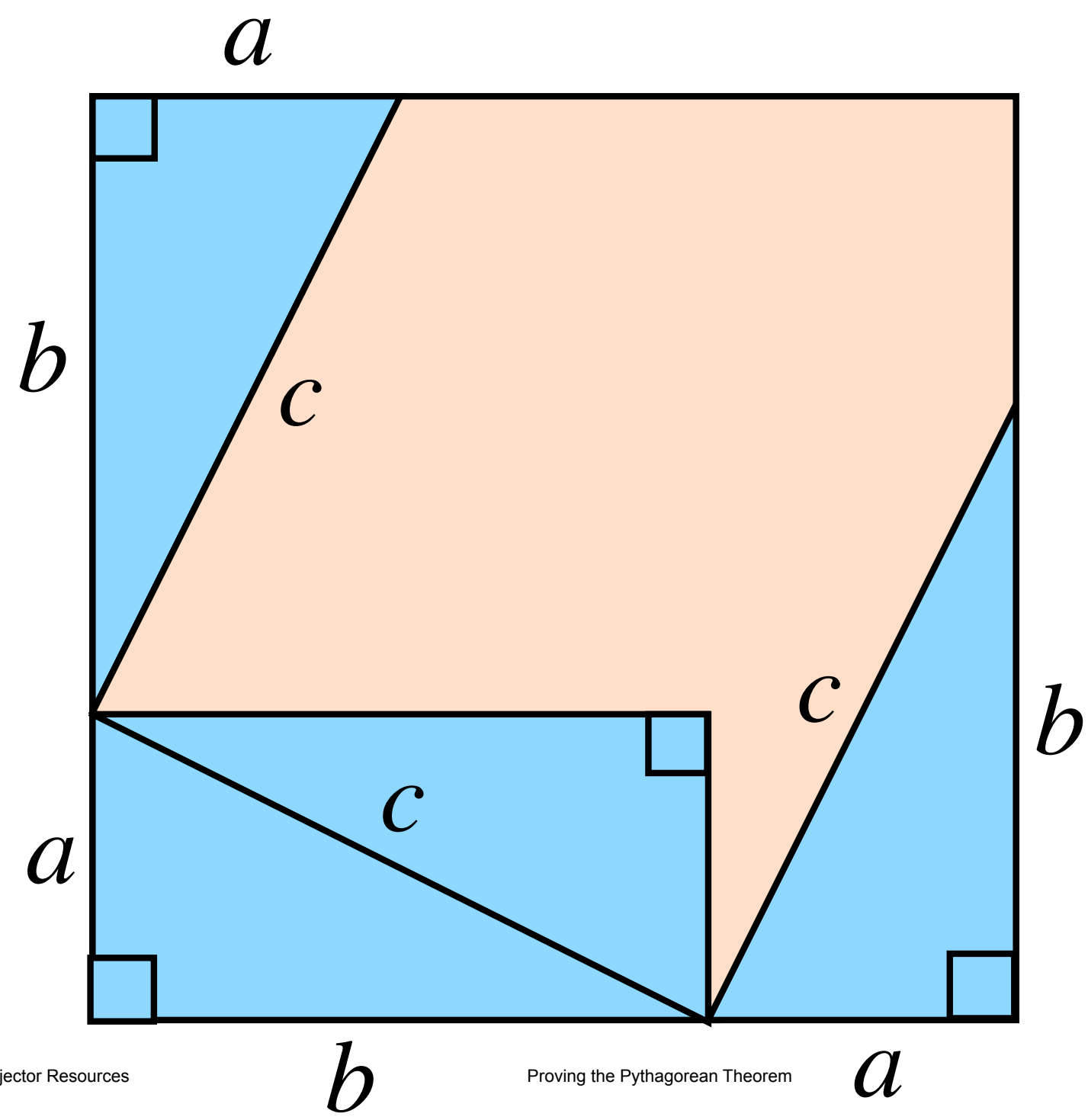

b
a
c
c
b
a
c
a
b
b
a

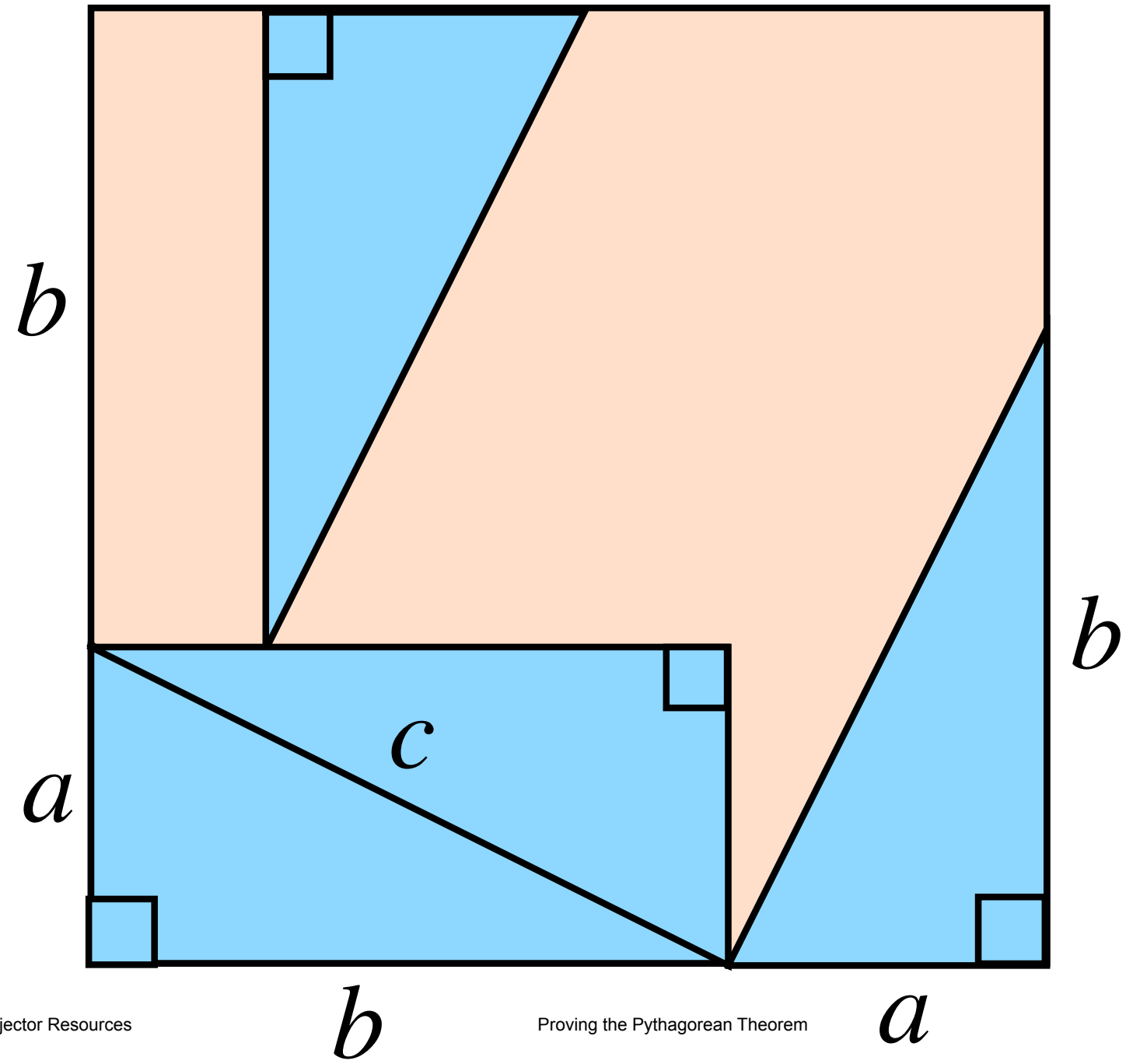

a
b
a
b
c
c
c
c
a
b
a
b

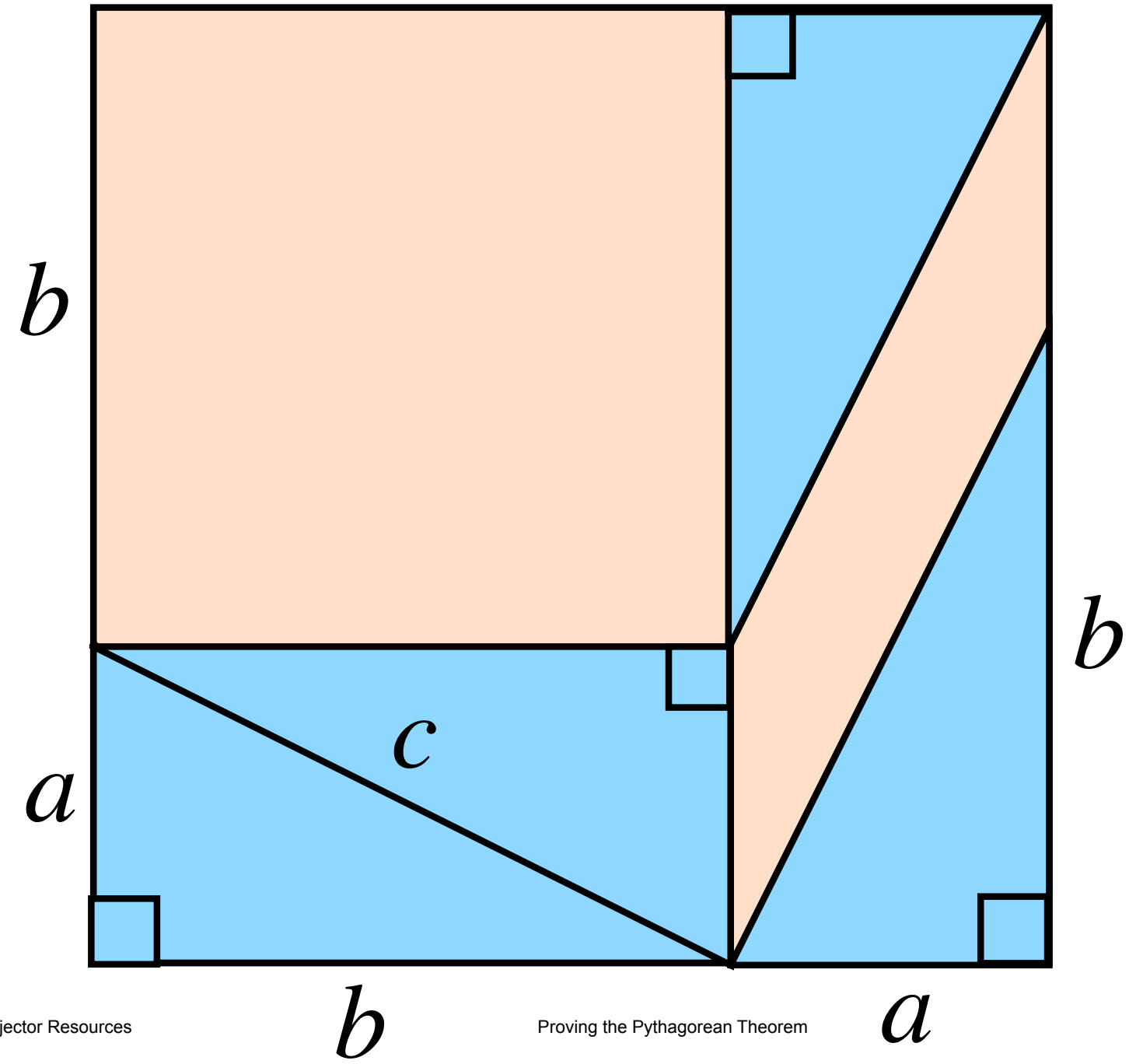

a
b
a
b
c
c
b
a
c
c
b
a

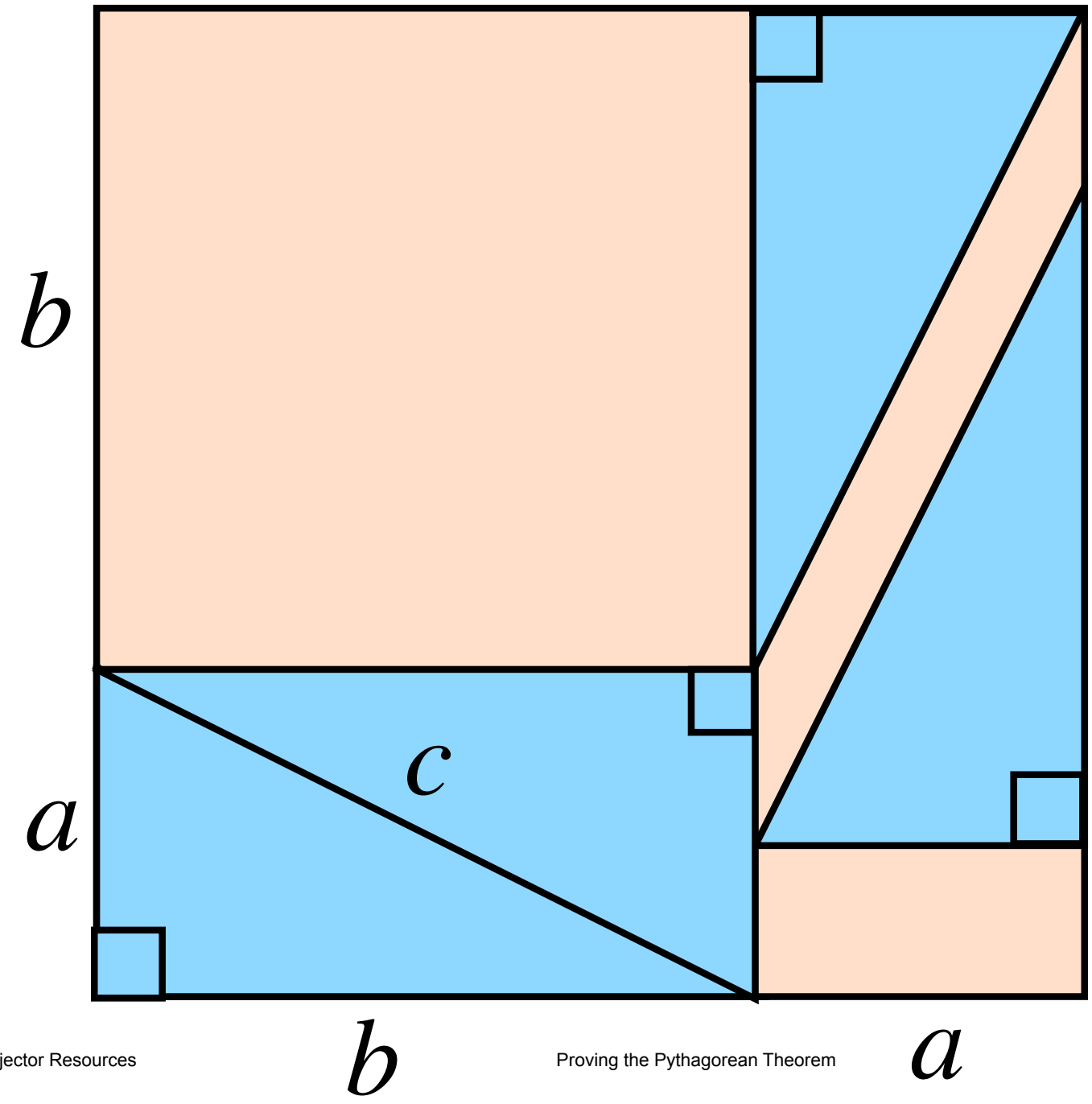

a
b
c
c
b
c
a
b
a

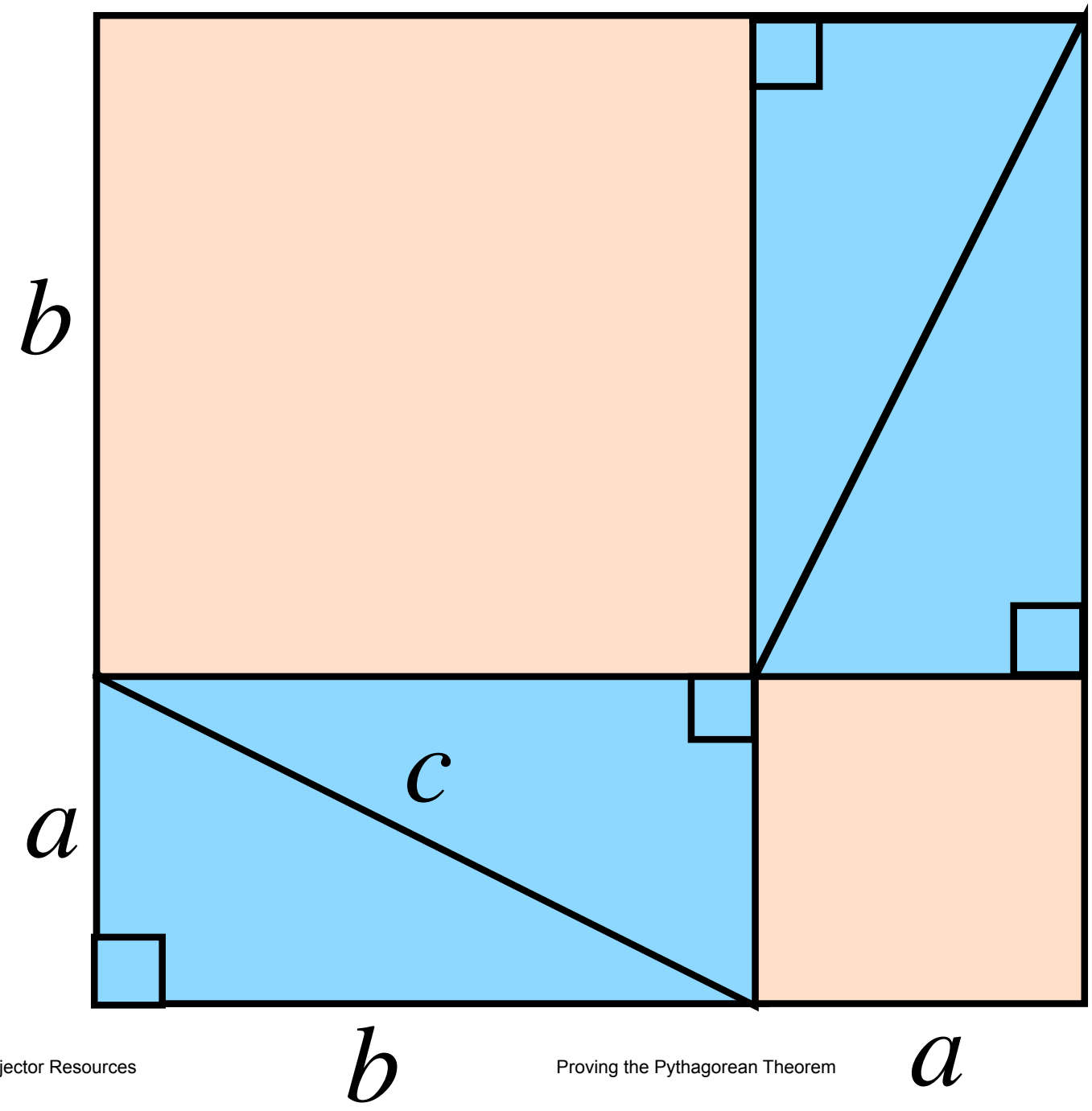

a
b
c
c
b
c
a
b
a


a
b
c
c
b
c
a
b
a


b
b
c
a
b
a


b
b
c
a
b
a


b
a
c
b
a

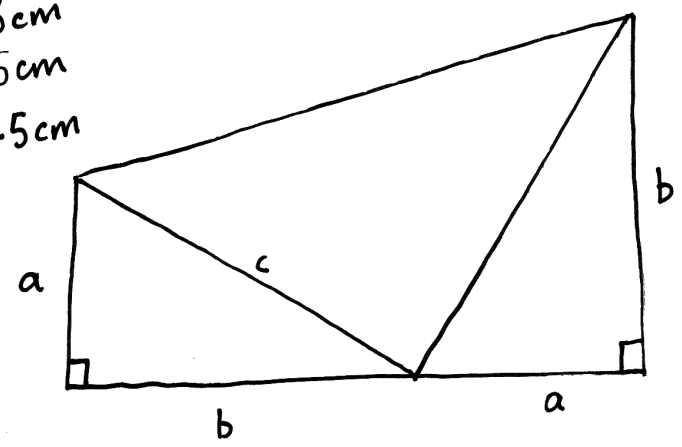

b
a
c
b
a

# Analyzing and Comparing

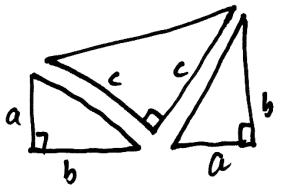
1. Describe what each student has done.
2. Will the approach lead to a proof of the theorem?
3. Explain how the work can be improved.
4. Compare the three solutions.
5. Whose solution method do you find most convincing? Why?
6. Produce a complete correct solution using your preferred method.

# Sample Methods to Discuss: Penelope

$a = 3\text{cm}$
$b = 5\text{cm}$
$c = 5.5\text{cm}$




This one trapezoid is three triangles!




Area trapezoid $= \frac{1}{2}(a+b)\times(a+b) = \frac{1}{2}(a+b)^2$

$= \frac{1}{2}(3+5)^2 = \frac{1}{2}\times 64 = 32\ \text{cm}^2$

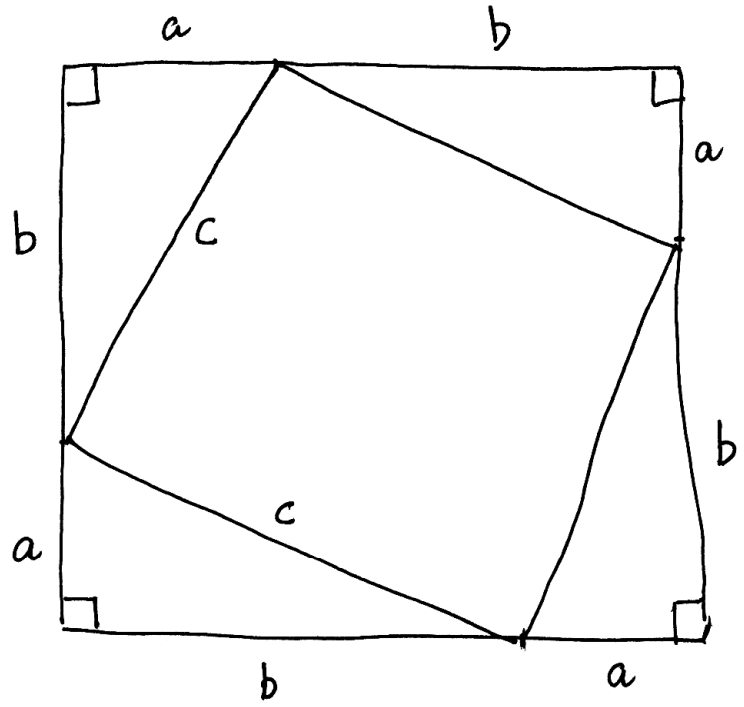
Area triangle $= \frac{1}{2}\times a\cdot b = \frac{1}{2}\times 3\times 5 = 7.5\ \text{cm}$

Two of them $= 15\ \text{cm}^2$

Other triangle $= \frac{1}{2}\times c^2 = \frac{1}{2}\times 5.5^2 = 15.125\ \text{cm}^2$

Altogether $15\text{cm}^2 + 15.125\ \text{cm}^2 \approx 32\text{cm}^2$.

# Sample Methods to Discuss: Nadia

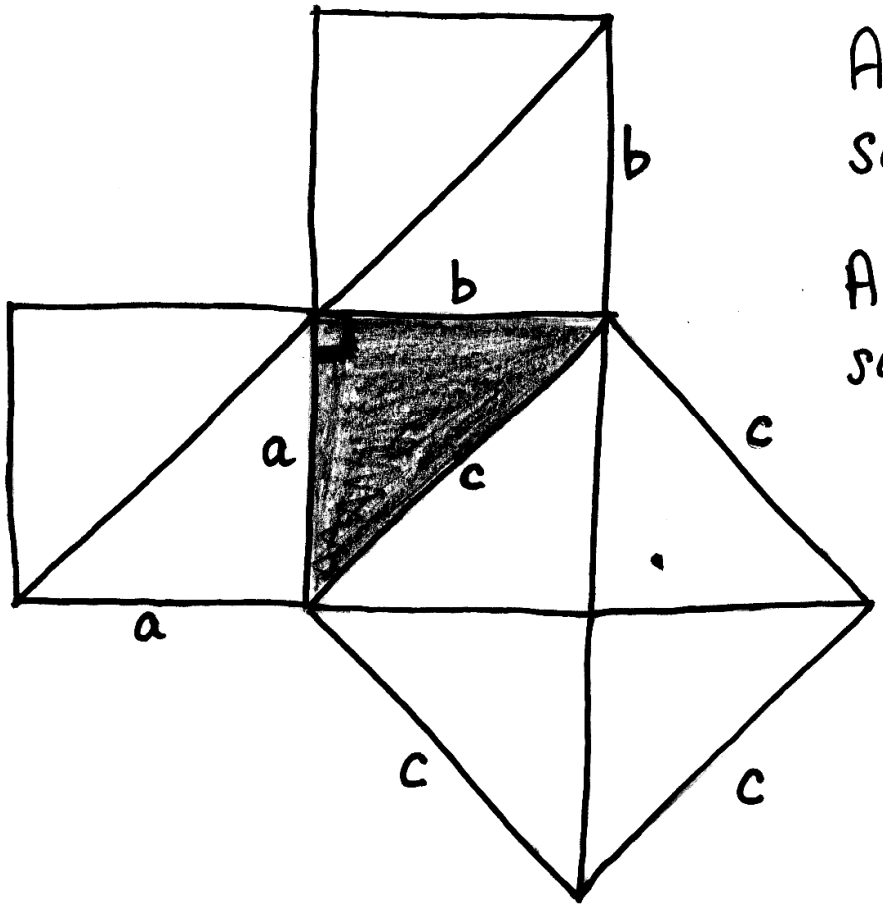



Area of middle square = $c^2$

Area of four triangles = $\frac{1}{2} \times a \times b \times 4 = 2 \times a \times b$.

Total area = $(a+b)^2 = a^2 + b^2$

# Sample Methods to Discuss: Sophie



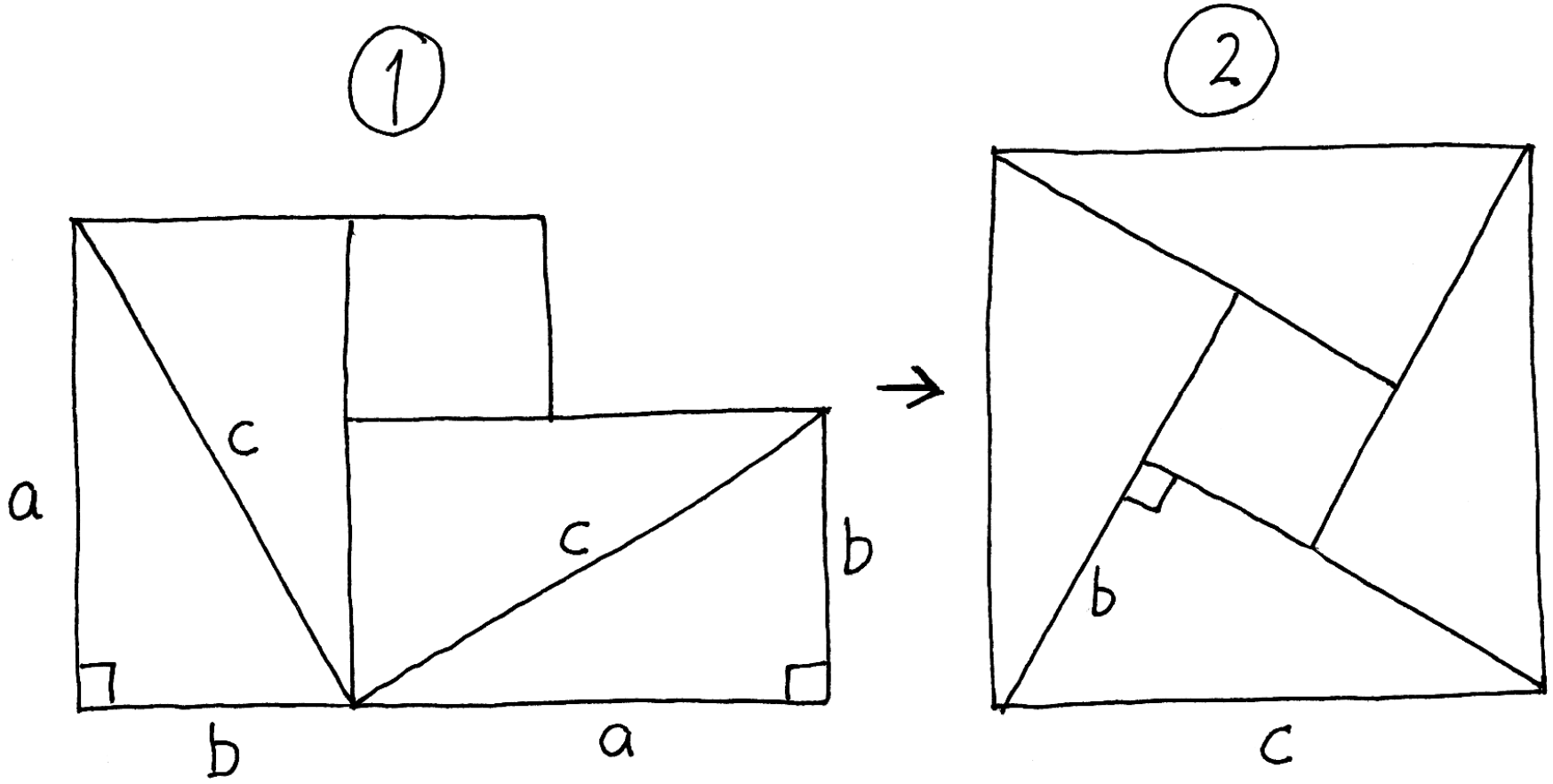

Area of small squares = $a^2 + b^2$

Area of big square = $c^2$

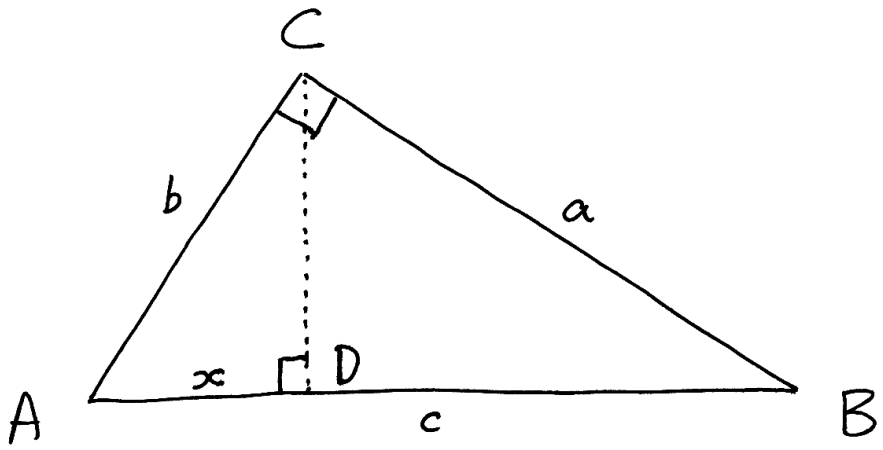
$c^2 = a^2 + b^2$

(4 triangles)

# Proving the Pythagorean Theorem (revisited)



# Proving the Pythagorean Using Similar Triangles




Triangle ABC is similar to triangles ACD and CBD

Let $AD = x$ $\quad DB = c - x$

$$\frac{x}{b} = \frac{b}{c}$$

Mathematics Assessment Project

# Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

**Malcolm Swan,**
**Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster,** and **Marie Joubert**
with
**Hugh Burkhardt, Rita Crust, Andy Noyes,** and **Daniel Pead**

We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
**David Foster, Mary Bouck,** and **Diane Schaefer**

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
**Alan Schoenfeld** at the University of California, Berkeley, and
**Hugh Burkhardt, Daniel Pead,** and **Malcolm Swan** at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
**Bill & Melinda Gates Foundation**

We are particularly grateful to
**Carina Wong, Melissa Chabran,** and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org

© 2015 MARS, Shell Center, University of Nottingham
This material may be reproduced and distributed, without modification, for non-commercial purposes,
under the Creative Commons License detailed at http://creativecommons.org/licenses/by-nc-nd/3.0/
All other rights reserved.
Please contact map.info@mathshell.org if this license does not meet your needs.