CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Translating Between Repeating Decimals and Fractions

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Translating Between Repeating Decimals and Fractions

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Translate between decimal and fraction notation, particularly when the decimals are repeating.
- Create and solve simple linear equations to find the fractional equivalent of a repeating decimal.
- Understand the effect of multiplying a decimal by a power of 10.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Common Core State Standards for Mathematical Content:*

8.NS: Know that there are numbers that are not rational, and approximate them by rational numbers.

8.EE: Analyze and solve linear equations and pairs of simultaneous linear equations.

This lesson also relates to the following *Common Core State Standards for Mathematical Practice,* with a particular emphasis on Practices 7 and 8:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

- Before the lesson, students attempt the *Repeating Decimals* task individually. You review their work and write questions for students to answer in order to improve their solutions.
- A whole-class discussion is used to introduce students to some of the patterns in repeating decimals and a method for converting such decimals to fractions.
- Students work collaboratively in small groups on a matching task that requires them to translate between fractions, decimals, and equations.
- In a final whole-class discussion, students compare and evaluate the methods.
- In a follow-up lesson, students review their initial solutions and then complete a different *Repeating Decimals* task.

MATERIALS REQUIRED

- Each student will need the task sheets *Repeating Decimals, Repeating Decimals (revisited)*, and *Fraction Conversion Challenge,* a calculator, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need Card Sets A: *Decimals*, B: *Equations*, C: *Fractions*, a large sheet of paper for making a poster, and a glue stick.

You may want to copy the Card Sets onto transparencies for use on an overhead projector.

There is a projector resource to remind students of the instructions for the group activity.

TIME NEEDED

15 minutes before the lesson, a 1-hour lesson and 20 minutes in a follow-up lesson (or for homework). Timings are only approximate. Exact timings will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Repeating Decimals* (15 minutes)

Ask the students do this task, in class or for homework, a day or more before the lesson. This will give you an opportunity to assess their work and to find out the kinds of difficulties they have with the task. You will then be able to target your help more effectively in the next lesson.

Give each student a copy of the *Repeating Decimals* task, mini-whiteboard, a pen, and an eraser.

Ask students to read through the opening text 'How to write repeating decimals' on the handout, *Repeating Decimals*. Then introduce the task briefly with a question and answer session using mini-whiteboards. This introduction should help you check that students understand the repeating decimals notation.

Repeating Decimals

How to write repeating decimals

We draw a line above decimal digits when they repeat over and over again.

For example: $0.\overline{3}$ is the same as the decimal 0.333333333333… (and so on)

$0.\overline{25}$ is the same as the decimal 0.252525252525… (and so on)

$0.12\overline{34}$ is the same as the decimal 0.1234343434… (and so on)

1. Use a calculator to change these fractions into decimals.
Try to use the notation described above.

$\frac{2}{3}$ = …………………… $\frac{3}{11}$ = …………………… $\frac{5}{6}$ = ……………………

2. Complete the following explanation to convert $0.1\overline{6}$ into a fraction:

Let $x = 0.1\overline{6}$
$100x$ = ……………………
$10x$ = ……………………
$100x - 10x$ = ……………………

So, x ……………………

3. Change each of these decimals into fractions.
Please show your method clearly next to each question.

0.54 = ………………	
$0.\overline{4}$ = ………………	
$0.\overline{54}$ = ………………	
$0.5\overline{9}$ = ………………	

On your whiteboards, write the following using repeating decimals notation:

0.3333333333…

0.1222222222…

0.1212121212…

0.1232323232…

0.1231231231…

If students are still struggling with the notation:

On your whiteboards, write out the following repeating decimals to 8 decimal places. Do not use the 'bar' notation:

$0.\overline{5}$

$0.\overline{26}$

$0.1\overline{5}$

Now explain what you are asking students to do.

Spend fifteen minutes on your own, answering these questions as carefully as you can. Show all your work so that I can understand your reasoning. There will be a lesson on this material [tomorrow] that will help you improve your work.

It is important that, as far as possible, students are allowed to answer the questions without assistance. If students are struggling to get started then ask questions that help them understand what is required, but make sure you do not do the task for them.

Students should not worry too much if they cannot understand or do everything because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their problem solving strategies.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on the next page. We suggest that you make a list of your own questions, based on your students' work, using the ideas on the following page. We recommend you either:

- write one or two questions on each student's work,

or

- give students a printed version of your list of questions highlighting the questions appropriate to each student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work.

Common issues:	**Suggested questions and prompts:**
Does not relate fraction notation to division or divides the denominator by the numerator (Q1)	• How can you write $\frac{2}{3}$ as a division? • If you divide two bars of chocolate equally between three people, how much will each person get?
Is not aware of the 'bar' notation for showing repeating decimals For example: The student copies from the calculator screen all the repeating digits (Q1).	• Look at the pattern for $\frac{2}{3}$. How can we write this using the 'bar' notation?
Does not understand the effect of multiplying decimals by powers of ten (Q2)	• What do you get when you multiply 0.4 by 10? By 100? By 1000? • What is the difference between 0.4 and 0.40? • Try multiplying other decimal numbers by powers of ten using your calculator. Try to explain what happens.
Has difficulty with algebraic notation (Q2 and Q3)	• If x = 0.1666666… • What do we get if we multiply x by 10? • What if we multiply x by 100? • What is the difference between your two answers? • Can you write this as an equation? • What is the purpose of the subtraction? • What is the result of subtracting $10x$ from $100x$? / subtracting x from $100x$? / subtracting x from $10x$? Which one of these sums will be most helpful in representing the repeating decimal as a fraction?
Does not provide sufficient explanation (Q2 and Q3)	• Now write down your method. • Does your method work for a decimal that repeats over and over again?
Completes the task successfully	• Can you write any repeating decimal as a fraction? Make up a hard example to convince me. • Are there any decimals that cannot be written as fractions? Can you think of any examples?

SUGGESTED LESSON OUTLINE

Introduction: *Fraction Conversion Challenge* (20 minutes)

Give each student a copy of the *Fraction Conversion Challenge* sheet, a mini-whiteboard, a pen, and an eraser.

I am going to give you about 5 minutes to change these fractions into decimals.

Do as many as you can.

Some you could be able to do mentally and for some you may need to do some working out on paper.

You might see some patterns that will help you!

Fraction Conversion Challenge

$\frac{1}{2}=$	$\frac{2}{2}=$							
$\frac{1}{3}=$	$\frac{2}{3}=$	$\frac{3}{3}=$						
$\frac{1}{4}=$	$\frac{2}{4}=$	$\frac{3}{4}=$	$\frac{4}{4}=$					
$\frac{1}{5}=$	$\frac{2}{5}=$	$\frac{3}{5}=$	$\frac{4}{5}=$	$\frac{5}{5}=$				
$\frac{1}{6}=$	$\frac{2}{6}=$	$\frac{3}{6}=$	$\frac{4}{6}=$	$\frac{5}{6}=$	$\frac{6}{6}=$			
$\frac{1}{7}=$	$\frac{2}{7}=$	$\frac{3}{7}=$	$\frac{4}{7}=$	$\frac{5}{7}=$	$\frac{6}{7}=$	$\frac{7}{7}=$		
$\frac{1}{8}=$	$\frac{2}{8}=$	$\frac{3}{8}=$	$\frac{4}{8}=$	$\frac{5}{8}=$	$\frac{6}{8}=$	$\frac{7}{8}=$	$\frac{8}{8}=$	
$\frac{1}{9}=$	$\frac{2}{9}=$	$\frac{3}{9}=$	$\frac{4}{9}=$	$\frac{5}{9}=$	$\frac{6}{9}=$	$\frac{7}{9}=$	$\frac{8}{9}=$	$\frac{9}{9}=$

Students will probably be able to do many of these mentally:

$$\frac{2}{2}=1 \quad \frac{3}{3}=1 \quad \frac{4}{4}=1 \quad \frac{5}{5}=1 \text{ and so on.}$$

$$\frac{1}{2}=0.5 \quad \frac{2}{4}=0.5 \quad \frac{3}{6}=0.5 \text{ and so on}$$

$$\frac{1}{5}=0.2 \quad \frac{2}{5}=0.4 \quad \frac{3}{5}=0.6 \text{ and so on}$$

After five minutes collect some student answers on the board and ask for explanations.

You may want to use Slide P-1 of the projector resource.

Then, give out calculators.

Using these calculators, try to finish the sheet.

Write down all the patterns you can see in the table.

Some students may not realize that they can easily convert fractions to decimals by division. You may need to point out that sometimes the calculator rounds decimals so some digits may be different.

As students complete the sheet encourage them to discuss their answers with a partner.

Patterns in the Table

Invite students to discuss the patterns that emerge, particularly focusing on the patterns in the repeating decimals.

Look at the pattern in the thirds. 0.3333333…

Do the threes in this pattern go on forever? How can you be sure?

What does the pattern for two thirds and three thirds look like?

Students may like to do the division by hand, $1 \div 3$, and see how the carry digits repeat. Ensure that all students are now aware of the 'bar' notation for showing repeating decimals:

Look at the pattern in the sixths. 0.16666… or 0.16666….7 (if the calculator rounds the number)

Is a sixth a repeating decimal?

How can we write this in our notation?

Check that they know how to show: $0.1\overline{6}$

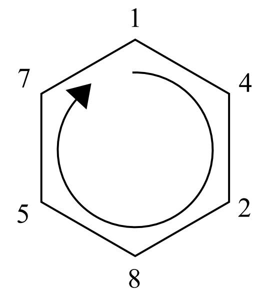
Students may also notice a pattern in the sevenths, however calculators with smaller screens may make it difficult for students to notice a pattern. Working out some by hand will resolve this.

What patterns about the sevenths?

0.142857142…; 0.285714285…; 0.428571428…; …

How can we write these in our notation?

Check that students can show these as: $0.\overline{142857}$, $0.\overline{285714}$, $0.\overline{428571}$



The patterns in the sevenths could lead **some** classes to a very interesting discussion. Digits repeat in a cycle of six numbers, each number starting at a different place and opposite digits add to nine (see diagram). The relationship between complementary fractions may be seen, for example, by adding: 0.142857… + 0.857142…= 0.999999…= 1

Now look at the ninths: 0.111111…; 0.222222…; 0.333333…; 0.444444…;

How does this pattern continue? What happens with nine ninths?

This leads to a very important idea. Many students will argue that $0.\overline{9}$ is less than one. This is also important as it shows a method for converting repeating decimals to fractions – one that is used in the collaborative group activity.

Write the number $0.\overline{9}$ on the board.

We are trying to find out what number $0.\overline{9}$ *really is.*

Suppose $x = 0.\overline{9}$

So what is $10x - x$ *?* *[* $9.\overline{9} - 0.\overline{9} = 9$ *]*

So if $10x - x = 9$ *what is the value of x?* *[* $9x = 9$ *so* $x = 1$ *]*

Students may of course not believe this. The method is useful as it may be generalized to convert any repeating decimal to a fraction.

Converting a Repeating Decimal to a Fraction

This task focuses on the purpose of subtracting algebraic expressions when converting a repeating decimal to a fraction.

Write the following expression on the board: $x = 0.1\overline{7}$

Explain to students that to figure out the fractional value of this decimal they could perform the following subtractions:

$$10x - x \quad \text{or} \quad 100x - x \quad \text{or} \quad 100x - 10x$$

Students should try to figure out which expression is the best one to use and then use it to work out the fractional value for the decimal.

Allow students a few minutes to think about this task individually and then discuss it with a partner before asking one or two students for an explanation. Try not to correct any answers, but encourage students to challenge each other's answer.

What is the purpose of the subtraction? [To eliminate the repeating part of the number]

Which of these expressions would you use?

All of the expressions will provide the correct fraction, however $100x - 10x$ is the most efficient.

$x = 0.1\overline{7}$

$10x = 1.\overline{7}$

$100x = 17.\overline{7}$

$$10x - x = 1.\overline{7} - 0.1\overline{7} = 1.6$$
$$9x = 1.6$$
$$x = \frac{1.6}{9} = \frac{16}{90} = \frac{8}{45}$$

Or

$$100x - x = 17.6$$
$$99x = 17.6$$
$$x = \frac{17.6}{99} = \frac{176}{990} = \frac{8}{45}$$

Or

$$100x - 10x = 16$$
$$90x = 16$$
$$x = \frac{16}{90} = \frac{8}{45}$$

What made you decide to use this expression?

Collaborative activity: *Matching Decimals, Equations, and Fractions* (20 minutes)

Ask students to work in small groups of two or three.

Give each group copies of the three card sets: A: *Decimals*, B: *Equations*, and C: *Fractions* along with a large sheet of paper, and a glue stick for making a poster. Giving out all the cards at once allows students to develop their own strategies. Trials of this lesson have shown that some students start the activity by using division to convert a fraction to a decimal. The blank fraction and equation cards encourage students to use algebra to convert a repeating decimal to a fraction.

You are now going to work together matching cards.

Take turns to find three cards that match.

Each time you do this explain your thinking clearly and carefully.

If you think there is no suitable card that matches, write one of your own. There are some blank cards for doing this.

When you have agreed on three cards that go together, stick them side-by-side on your large sheet of paper.

Write your reasoning on the poster. Show clearly how you know that the matching is correct.

You could display the Slide P-2 of the projector resource, *Matching Decimals and Fractions*, which summarizes these instructions.

While students work in small groups you have two tasks: to note their different approaches to the task and to support their reasoning.

Note different student approaches to the task

Listen and watch students carefully and note different student approaches to the task. In particular, notice any difficulties that students encounter, and the ways they justify and explain to each other. Do students correctly relate fraction notation to division? Do students struggle to multiply decimals by 10 or 100? Do students understand the purpose of subtraction on the *Equation* cards? Do students use guess and check? Do students write down explanations of their matches? What do they do if they get stuck?

You can then use this information to focus your questioning in the whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking. You may want to use some of the questions from the *Common issues* table. If the whole class is struggling on the same issue, write relevant questions on the board and hold a brief whole-class discussion.

Encourage students to write down their explanations on the poster.

Which is bigger 0.6 or $0.\bar{6}$*? How do you know?*

What is the difference between 0.6, 0.60 and 6?
What is the value of 0.6 × 10?
What is the value of $0.\bar{6} \times 10$*?*

What does $0.\bar{6}7$ *mean? [This number does not make sense.]*

Some students select a fraction card and look for a decimal to match:

How can you read this fraction as a division?
Can you do the calculation without using a calculator?

Show me a Decimal card that can easily be matched to a Fraction card.
[Students are likely to select non-repeating decimals.]

Some students may select an *Equation* card and look for a *Fraction* card to match, or use 'guess and check' to substitute values of *x* into the *Equation* cards.

Show me the solution to your equation.

Show me an Equation card that can easily be matched to a Fraction card.
[Students are likely to select an equation that does not contain a subtraction.]

Some students will begin by selecting a *Decimals* card and looking for an *Equation* card to match:

Suppose you let your decimal be called x.

How would you write x without the repeating notation?

What is the value of $10x$? $100x$?

How can you use arithmetic on these expressions to eliminate the repeating notation?

What is the difference between $100x$ and $10x$? Does this eliminate the repeating notation?

Encourage students to use a variety of strategies. If, for example, they have matched several fraction cards to decimal ones, ask them to match an equation card or a decimal card to a fraction card.

Some cards have no partner and students will need to construct these. Some students may also notice that there are fewer fraction cards than decimal cards. Leave them to realize, or discover by solving the equations, that $0.4\bar{9} = 0.5$ and that $0.1\bar{9} = 0.2$.

Whole-class discussion: comparing posters (20 minutes)

You may want to use transparencies of the cards to support the discussion.

The intention is that this discussion focuses on the justification of a few examples rather than checking students all have the correct matches. You may want to first, select a pair of cards that most groups matched correctly as this may encourage good explanations, then select one or two matches that most groups found difficult. Ask students to share their thinking with the whole class.

How did you decide that that equation matched this decimal?

How did you solve the equation to find the matching fraction?

Did anyone use a different method?

If you have time, some classes may want to find fraction equivalents for harder examples.

Find a fraction that represents one of these decimals:

$1.\overline{36},\ 0.0\bar{6},\ 0.\overline{123},\ \ldots$

Show me your solutions using your mini-whiteboards.
Show me your answers in their lowest terms.

Show me an example of a repeating decimal for which the method $1000x - 100x$ would be useful. Can you explain?

[

$0.12\bar{3},\ 0.00\bar{1},\ 0.01\bar{2}$, etc.

For example, if $x = 0.12\bar{3}$

$$\begin{aligned} 1000x &= 123.\bar{3} \\ 100x &= 12.\bar{3} \\ 1000x - 100x &= 123.\bar{3} - 12.\bar{3} \\ 900x &= 111 \\ x &= \frac{111}{900}. \end{aligned}$$

]

Could you convert the repeating decimal to a fraction using a different expression?
[Yes, but the method would not be as efficient.]

$$\begin{aligned}
\text{For example, if } x &= 0.12\bar{3} \\
100x &= 12.\bar{3} \\
100x - x &= 12.\bar{3} - 0.12\bar{3} = 12.21 \\
99x &= 12.21 \\
x &= \frac{12.21}{99} = \frac{1221}{9900} = \frac{111}{900}.
\end{aligned}$$

Finally, ask students:

If the area of a square is 5, what is the length of its sides? [$\sqrt{5}$.]
What is special about this number? [It does not terminate nor repeat.]

Can you give me another example of a decimal that does not terminate nor repeat?

Explain that these numbers are called irrational because they cannot be written as fractions. Encourage students to find a few examples from their prior knowledge, or to create some of their own:

For example: *0.12345678910111213141516…*

$\sqrt{2}, \sqrt{3}, \sqrt{5}$

π, 2π, 3π, …$n\pi$

Suggestion for an extension activity

Invite students to create their own set of decimals, equations and fractions cards. They may then like to try these out on some other students.

Follow-up lesson: Reviewing the assessment task (20 minutes)

Return the original assessment *Repeating Decimals* to the students together with a copy of *Repeating Decimals (revisited)*.

If you have not added questions to individual pieces of work, or highlighted questions on a printed list of questions, then write your list of questions on the board. Students should select only the questions from this list they think are appropriate to their own work.

Look at your original responses and the questions (on the board/written on your script.)

Use what you have learned to answer these questions.

Now look at the new task sheet, Repeating Decimals (revisited). Use what you have learned to answer these questions.

If you are short of time, then you could set this task as homework.

SOLUTIONS

Fraction Conversion Challenge

$\frac{1}{2} = 0.5$	$\frac{2}{2} = 1$							
$\frac{1}{3} = 0.\overline{3}$	$\frac{2}{3} = 0.\overline{6}$	$\frac{3}{3} = 1$						
$\frac{1}{4} = 0.25$	$\frac{2}{4} = 0.5$	$\frac{3}{4} = 0.75$	$\frac{4}{4} = 1$					
$\frac{1}{5} = 0.2$	$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$	$\frac{4}{5} = 0.8$	$\frac{5}{5} = 1$				
$\frac{1}{6} = 0.1\overline{6}$	$\frac{2}{6} = 0.\overline{3}$	$\frac{3}{6} = 0.5$	$\frac{4}{6} = 0.\overline{6}$	$\frac{5}{6} = 0.8\overline{3}$	$\frac{6}{6} = 1$			
$\frac{1}{7} = 0.\overline{142857}$	$\frac{2}{7} = 0.\overline{285714}$	$\frac{3}{7} = 0.\overline{428571}$	$\frac{4}{7} = 0.\overline{571428}$	$\frac{5}{7} = 0.\overline{714285}$	$\frac{6}{7} = 0.\overline{857142}$	$\frac{7}{7} = 1$		
$\frac{1}{8} = 0.125$	$\frac{2}{8} = 0.25$	$\frac{3}{8} = 0.375$	$\frac{4}{8} = 0.5$	$\frac{5}{8} = 0.625$	$\frac{6}{8} = 0.75$	$\frac{7}{8} = 0.875$	$\frac{8}{8} = 1$	
$\frac{1}{9} = 0.\overline{1}$	$\frac{2}{9} = 0.\overline{2}$	$\frac{3}{9} = 0.\overline{3}$	$\frac{4}{9} = 0.\overline{4}$	$\frac{5}{9} = 0.\overline{5}$	$\frac{6}{9} = 0.\overline{6}$	$\frac{7}{9} = 0.\overline{7}$	$\frac{8}{9} = 0.\overline{8}$	$\frac{9}{9} = 1$

Repeating Decimals

1. $\frac{2}{3} = 0.\overline{6} \quad \frac{3}{11} = 0.\overline{27} \quad \frac{5}{6} = 0.8\overline{3}$

2.

$$
\begin{aligned}
x &= 0.1\overline{6} \\
\Rightarrow 100x &= 16.\overline{6} \\
\Rightarrow 10x &= 1.\overline{6} \\
\Rightarrow 90x &= 15 \\
\Rightarrow x &= \frac{15}{90} = \frac{1}{6}
\end{aligned}
$$

3. $0.54 = \frac{27}{50} \quad 0.\overline{4} = \frac{4}{9} \quad 0.\overline{54} = \frac{6}{11} \quad 0.5\overline{9} = \frac{3}{5}$

Repeating Decimals (revisited)

1. $\frac{4}{3} = 1.\overline{3} \quad \frac{4}{11} = 0.\overline{36} \quad \frac{1}{6} = 0.1\overline{6}$

2.

$$
\begin{aligned}
x &= 0.8\overline{3} \\
\Rightarrow 100x &= 83.\overline{3} \\
\Rightarrow 10x &= 8.\overline{3} \\
\Rightarrow 90x &= 75 \\
\Rightarrow x &= \frac{75}{90} = \frac{5}{6}
\end{aligned}
$$

3. $0.\overline{5} = \frac{5}{9} \quad 0.\overline{45} = \frac{5}{11} \quad 0.4\overline{9} = \frac{1}{2}$

Matching Decimals, Equations, Fractions

<table>
<tr><th>Decimals</th><th>Equations</th><th>Fractions</th></tr>
<tr><td>1.2</td><td>$10x = 12$</td><td rowspan="2">$\frac{6}{5}$</td></tr>
<tr><td>$1.1\overline{9}$</td><td>$100x - 10x = 108$</td></tr>
<tr><td>1.5</td><td>$10x = 15$</td><td rowspan="2">$\frac{3}{2}$</td></tr>
<tr><td>$1.4\overline{9}$</td><td>$100x - 10x = 135$</td></tr>
<tr><td>$0.1\overline{2}$</td><td>$100x - 10x = 11$</td><td>$\frac{11}{90}$</td></tr>
<tr><td>$0.1\overline{5}$</td><td>New card:
$100x - 10x = 14$</td><td>New card: $\frac{7}{45}$</td></tr>
<tr><td>0.12</td><td>$100x = 12$</td><td>$\frac{3}{25}$</td></tr>
<tr><td>$1.\overline{5}$</td><td>$10x - x = 14$</td><td>$\frac{14}{9}$</td></tr>
<tr><td>$1.\overline{2}$</td><td>New card:
$10x - x = 11$</td><td>New card: $\frac{11}{9}$</td></tr>
<tr><td>$0.\overline{15}$</td><td>$100x - x = 15$</td><td>$\frac{5}{33}$</td></tr>
<tr><td>$0.\overline{12}$</td><td>New card:
$100x - x = 12$</td><td>New card: $\frac{4}{33}$</td></tr>
<tr><td>0.15</td><td>$100x = 15$</td><td>$\frac{3}{20}$</td></tr>
</table>

Repeating Decimals

How to write repeating decimals

We draw a line above decimal digits when they repeat over and over again.

For example: $0.\overline{3}$ is the same as the decimal 0.333333333333… (and so on)

$0.\overline{25}$ is the same as the decimal 0.252525252525… (and so on)

$0.12\overline{34}$ is the same as the decimal 0.1234343434… (and so on)

1. Use a calculator to change these fractions into decimals.
   Try to use the notation described above.

   $\frac{2}{3}$ = …………………… $\frac{3}{11}$ =…………………… $\frac{5}{6}$ =……………………

2. Complete the following explanation to convert $0.1\overline{6}$ into a fraction:

   Let $x = 0.1\overline{6}$

   $100x$ = ...

   $10x$ = ...

   $100x - 10x$ = ...

   So, x ....

3. Change each of these decimals into fractions.
   Please show your method clearly next to each question.

0.54 = ……………….	
$0.\overline{4}$ = ……………….	
$0.\overline{54}$ = ……………….	
$0.5\overline{9}$ = ……………….	

Fraction Conversion Challenge

$\frac{1}{2} =$	$\frac{2}{2} =$							
$\frac{1}{3} =$	$\frac{2}{3} =$	$\frac{3}{3} =$						
$\frac{1}{4} =$	$\frac{2}{4} =$	$\frac{3}{4} =$	$\frac{4}{4} =$					
$\frac{1}{5} =$	$\frac{2}{5} =$	$\frac{3}{5} =$	$\frac{4}{5} =$	$\frac{5}{5} =$				
$\frac{1}{6} =$	$\frac{2}{6} =$	$\frac{3}{6} =$	$\frac{4}{6} =$	$\frac{5}{6} =$	$\frac{6}{6} =$			
$\frac{1}{7} =$	$\frac{2}{7} =$	$\frac{3}{7} =$	$\frac{4}{7} =$	$\frac{5}{7} =$	$\frac{6}{7} =$	$\frac{7}{7} =$		
$\frac{1}{8} =$	$\frac{2}{8} =$	$\frac{3}{8} =$	$\frac{4}{8} =$	$\frac{5}{8} =$	$\frac{6}{8} =$	$\frac{7}{8} =$	$\frac{8}{8} =$	
$\frac{1}{9} =$	$\frac{2}{9} =$	$\frac{3}{9} =$	$\frac{4}{9} =$	$\frac{5}{9} =$	$\frac{6}{9} =$	$\frac{7}{9} =$	$\frac{8}{9} =$	$\frac{9}{9} =$

Card Set A: Decimals

1.2	$1.\overline{5}$
1.5	$1.\overline{2}$
$0.1\overline{2}$	$0.\overline{15}$
$0.1\overline{5}$	$0.\overline{12}$
0.12	$1.1\overline{9}$
$1.4\overline{9}$	0.15

Card Set B: Equations

$100x = 12$	$10x - x = 14$
$100x - x = 15$	$10x = 12$
E	$100x = 15$
$100x - 10x = 11$	$100x - 10x = 108$
E	$10x = 15$
$100x - 10x = 135$	E

Card Set C: Fractions

$\frac{3}{25}$	$\frac{6}{5}$
$\frac{3}{20}$	$\frac{14}{9}$
F	$\frac{11}{90}$
$\frac{5}{33}$	F
F	$\frac{3}{2}$

Repeating Decimals (revisited)

How to write repeating decimals

We draw a line above decimal digits when they repeat over and over again.

For example: $0.\overline{3}$ is the same as the decimal 0.333333333333… (and so on)

$0.\overline{25}$ is the same as the decimal 0.252525252525… (and so on)

$0.12\overline{34}$ is the same as the decimal 0.1234343434… (and so on)

1. Use a calculator to change these fractions into decimals.
Try to use the notation described above.

$\frac{4}{3}$ = …………………… $\frac{4}{11}$ =…………………… $\frac{1}{6}$ =……………………

2. Complete the following explanation to convert $0.8\overline{3}$ into a fraction:

Let $x = 0.8\overline{3}$

$100x$ = ...

$10x$ = ...

$100x - 10x$ = ...

So, x ....

3. Change each of these repeating decimals into fractions.
Please show your method clearly next to each question.

$0.\overline{5}$ = ……………….	
$0.\overline{45}$ = ……………….	
$0.4\overline{9}$ = ……………….	

$\frac{1}{2} =$	$\frac{2}{2} =$							
$\frac{1}{3} =$	$\frac{2}{3} =$	$\frac{3}{3} =$						
$\frac{1}{4} =$	$\frac{2}{4} =$	$\frac{3}{4} =$	$\frac{4}{4} =$					
$\frac{1}{5} =$	$\frac{2}{5} =$	$\frac{3}{5} =$	$\frac{4}{5} =$	$\frac{5}{5} =$				
$\frac{1}{6} =$	$\frac{2}{6} =$	$\frac{3}{6} =$	$\frac{4}{6} =$	$\frac{5}{6} =$	$\frac{6}{6} =$			
$\frac{1}{7} =$	$\frac{2}{7} =$	$\frac{3}{7} =$	$\frac{4}{7} =$	$\frac{5}{7} =$	$\frac{6}{7} =$	$\frac{7}{7} =$		
$\frac{1}{8} =$	$\frac{2}{8} =$	$\frac{3}{8} =$	$\frac{4}{8} =$	$\frac{5}{8} =$	$\frac{6}{8} =$	$\frac{7}{8} =$	$\frac{8}{8} =$	
$\frac{1}{9} =$	$\frac{2}{9} =$	$\frac{3}{9} =$	$\frac{4}{9} =$	$\frac{5}{9} =$	$\frac{6}{9} =$	$\frac{7}{9} =$	$\frac{8}{9} =$	$\frac{9}{9} =$

Matching Decimals and Fractions

1. Take turns to find matching cards.
2. Explain your thinking clearly to your group.
3. If you find a card that has no match, create one on a blank card.
4. When you have three cards that match, stick them side-by-side on your poster.
5. Record your reasoning on the poster.

You all need to be able to agree on and explain the matching of every card.

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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