

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Estimating Length Using Scientific Notation

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

For more details, visit: <http://map.mathshell.org>
© 2015 MARS, Shell Center, University of Nottingham
May be reproduced, unmodified, for non-commercial purposes under the Creative Commons license
detailed at <http://creativecommons.org/licenses/by-nc-nd/3.0/> - all other rights reserved

Estimating Length Using Scientific Notation

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Estimate lengths of everyday objects.
- Convert between decimal and scientific notation.
- Make comparisons of the size of numbers expressed in both decimal and scientific notation.

STANDARDS ADDRESSED

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8-EE: Work with radicals and integer exponents.

This lesson also relates to **all** the *Standards for Mathematical Practice*, with a particular emphasis on:

2. Reason abstractly and quantitatively.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their responses and create questions for students to consider in order to improve their work.
- After a whole-class introduction, students work on a collaborative task, matching measurements expressed in decimal and scientific notation. These are then matched to everyday objects.
- Students then make comparisons between the objects by investigating the size of one object in relation to another. To end the lesson there is a whole-class discussion.
- In a follow-up lesson, students work alone again on a similar task to the introductory task.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks *Size It Up* and *Resize It Up!*, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need *Card Set A: Measurements*, *Card Set B: Objects*, *Card Set C: Comparisons* (all cards cut out before the lesson), a large sheet of paper, and a glue stick.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson for the assessment task, an 80-minute lesson (or split into two shorter lessons), and 15 minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Size It Up* (15 minutes)

Ask the students to complete this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the task *Size It Up*.

Read through the questions and try to answer them as carefully as you can.

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry if they cannot understand or do everything because in the next lesson they will work on a similar task, which should help them. Explain that by the end of the next lesson, students should be able to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task, and note down what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counter-productive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on the next page. We suggest that you make a list of your own questions, based on your students' work, using the ideas in the table. We recommend you write one or two questions on each student's work. If you do not have time to do this, you could:

- Select a few questions that will be of help to the majority of students, and write these questions on the board when you return the work to the students; or
- Give each student a printed version of your list of questions and highlight the questions for each individual student.

Size It Up

1. The table contains seven measurements written in decimal and scientific notation.

(a) Complete the table so that each measurement is written in *both decimal and scientific notation*.

(b) In the last column, rank the measurements in order of size.
(1 = smallest, 2 = next smallest, and so on up to 7 = largest)

Decimal Notation	Scientific Notation	Rank 1 = smallest 7 = largest
	1×10^{-2} m	
0.004 m	=	
200 m	=	
	8×10^5 m	
40,000,000 m	=	
40 m	=	
	8×10^{-4} m	

2 (a) Complete the following statement using two numbers in *decimal notation* from the table.

..... $\times 4000 =$

2 (b) Complete the following statement using two numbers in *scientific notation* from the table.

..... $\times 50,000 =$

Common issues:**Suggested questions and prompts:**

<p>Converts measurements from scientific to decimal notation incorrectly</p> <p>For example: The student writes 1×10^{-2} m as 0.001 m.</p>	<ul style="list-style-type: none"> • What does $\times 10^{-1}$ mean? What does $\times 10^{-2}$ mean? • How can you check your answer is correct?
<p>Does not remember the requirements for scientific notation</p> <p>For example: The student writes 40,000,000 m as 40×10^6 m.</p>	<ul style="list-style-type: none"> • Have any of these numbers been written in scientific notation? 20×10^{-2}, 0.6×10^4, 5×10^3 • How do you know?
<p>Does not fully understand the effects of negative exponents</p> <p>For example: The student assumes that 8×10^{-4} is bigger than 1×10^{-2}.</p>	<ul style="list-style-type: none"> • Is 10^{-4} bigger than 10^{-2}? How do you know? • In scientific notation, which has a bigger effect on the size of a number, the coefficient or the exponent? Explain. • Is 9×10^1 bigger than 1×10^2? What is the difference between the two numbers? • Is 9×10^{-2} bigger than 1×10^{-1}? What is the difference between the two numbers?
<p>Fails to complete the size rankings for the measurements</p>	<ul style="list-style-type: none"> • Which measurement is the smallest? Which measurements is the biggest? How do you know?
<p>Makes incorrect assumptions when comparing measurements</p> <p>For example: The student assumes that question 2 cannot be completed as the coefficients in the scientific notation are limited to 1, 2, 4 and 8 and so you can't multiply any of these by either 50 or 4000 to get another listed coefficient.</p>	<ul style="list-style-type: none"> • Can you use the decimal notation to help you to complete the second question? • Numbers in scientific notation include an integer power of ten. What effect does this have?
<p>Completes the task</p> <p>The student needs an extension task.</p>	<ul style="list-style-type: none"> • Choose two measurements that you have not yet used in Q2. Can you work out how much bigger one measurement is than the other? • How much bigger is the measurement you have ranked as 2 than rank 1? Can you compare your other rankings in this way?

SUGGESTED LESSON OUTLINE

Whole-class introduction (20 minutes)

Give each student a mini-whiteboard, pen and eraser.

Display slide P-1 of the projector resource:

A slide titled "What's the Number?" with three boxes containing mathematical expressions:

$$30,000 \times 10^{-1}$$
$$3 \times 10^3$$
$$0.3 \times 10^4$$

*These three expressions all represent the same number.
Write the number on your whiteboard [3000.]*

*Which of the expressions did you use to work out the number?
Why did you choose this expression?*

Did you check that the other expressions gave the same number?

It is likely that students will use the second representation to figure out the number. Ask students to justify why all three expressions are equal.

Display slide P-2 of the projector resource, which contains the same numbers as slide 1.

Which of these three expressions is written in scientific notation?

Some students may assume that all three expressions are in scientific notation. Spend some time discussing student responses and address any misconceptions that arise. Students should have a clear understanding that 3×10^3 is the only expression written using scientific notation and be able to justify why the other two representations are not scientific notation.

A number is written in scientific notation when it is of the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

(Note that, when the decimal number is negative, $-10 < a \leq -1$, but in this lesson we focus on physical lengths, which are greater than zero.)

Display slide P-3 of the projector resource:

A slide titled "Which is Greater?" with two boxes containing mathematical expressions:

$$8 \times 10^{-3}$$
$$4 \times 10^{-1}$$

Write on your whiteboard the number you think is the greater of the two.

If students' opinions are divided, ask students to pair up with another student with the same choice.

In your pair, work out how much greater your chosen number is than the other number.

Once students have had a chance to complete this calculation, choose a few pairs of students to feedback their choice and any calculations they have carried out. A common misconception is to ignore the exponent and assume that 8×10^{-3} is twice as big as 4×10^{-1} due to the fact that $4 \times 2 = 8$. If all students have correctly identified the greater of the two numbers, it may be appropriate to explore the different methods used to compare the two numbers and address any differences in the comparison of sizes.

Collaborative activity: Card Set A: Measurements (15 minutes)

Organize the class into groups of two or three students.

Give each group *Card Set A: Measurements*.

Explain how students are to work collaboratively. Slide P-4 of the projector resource summarizes these instructions.

Take turns to match a card in decimal notation with a card in scientific notation.

Each time you match a pair of cards, explain your thinking clearly and carefully. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.

Partners should either agree with the explanation, or challenge it if it is not clear, or not complete.

It is important that everyone in the group understands the matching of each card.

You should find that two cards do not have a match. Write the alternative notation for these measurements on the blank cards to produce a pair.

If students are struggling to get started, it may be appropriate to demonstrate a correct match with the whole class. It may also be helpful for students to divide the cards into separate piles dependent on notation, before looking for matches.

As students work, you have two tasks: to note student approaches to the task and to support student reasoning.

Note student approaches to the task

Listen and watch students carefully. In particular, notice how students make a start on the task, where they get stuck, and how they overcome any difficulties. Do students prefer to convert from scientific to decimal notation or the other way around? How are they working out the value of the exponent? You can use this information to focus a whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards a particular matching. Instead, ask questions to help students to reason together. You may want to use some of the questions in the *Common issues* table. If you find one student within the group has matched a particular pair of cards, challenge another student in the group to explain the match.

Chen matched these two cards. Christie, why does Chen think these two cards match?

When completing the blank measurement cards, some students may assume that for negative exponents, the magnitude of the exponent corresponds to the number of zeros after the decimal point (for example, assuming that $1.2 \times 10^{-2} = 0.0012$ rather than 0.012.) Encourage students to explain their reasoning and use this process as a method for checking their work.

Extending the lesson over two days

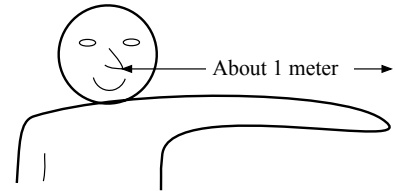
If you are taking two days to complete the unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of their previous work before moving on to the next collaborative activity.

Collaborative activity: *Card Set B: Objects* (15 minutes)

When students have finished matching the measurement cards, give each group *Card Set B: Objects*.

Students may already have noticed that each measurement card has an ‘m’ at the end. Ask students to suggest what this represents and clarify that all of the measurements are in meters.

For the next part of the lesson students will be matching up an everyday object from *Card Set B* with the appropriate measurement cards from *Card Set A*. It might be helpful to illustrate the rough size of a meter. Explain to students that the distance from your nose to a fingertip is about one meter.



Ask students to match the objects in *Card Set B* with the corresponding paired measurements from *Card Set A*.

If students are struggling, suggest putting the object cards in order of size first and then matching them up to the measurements cards based on order of size.

As students complete all matches, ask them to check their work against that of a neighboring group.

If there are differences between your matching, ask for an explanation. If you still don't agree, explain your own thinking.

You must not move the cards for another group. Instead, explain why you disagree with a particular match and explain which cards you think need to be changed.

Slide P-5 of the projector resource summarizes these instructions.

Collaborative activity: *Card Set C: Comparisons* (20 minutes)

When students are satisfied with their matches, give each group *Card Set C: Comparisons*, a glue stick, and a large sheet of paper for making a poster:

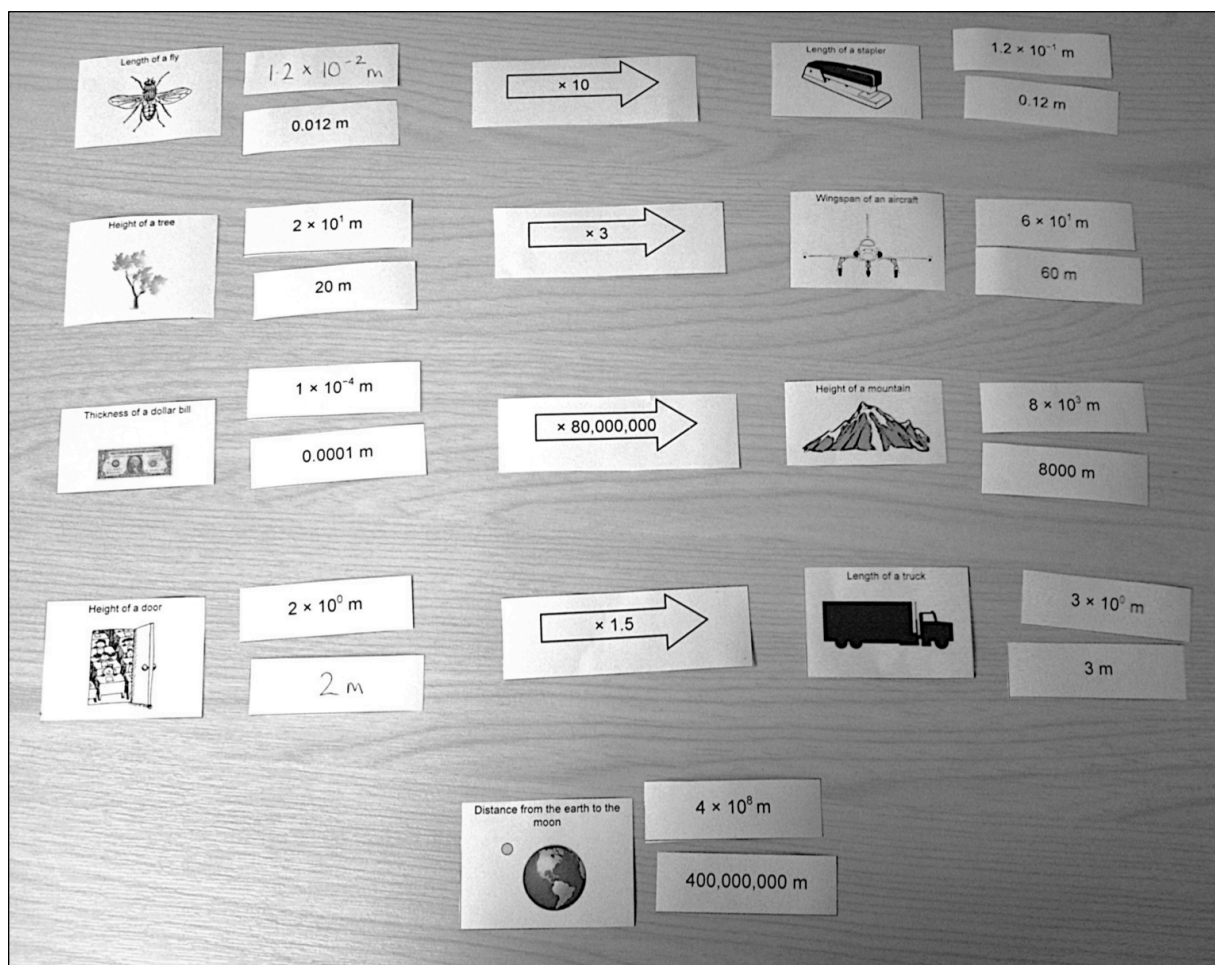
You are now to place an arrow card between two sets of measurements/objects to show how much bigger one object is than another.

For example, you may choose to start with the arrow that has “ $\times 3$ ” on it. Are there two objects that have measurements such that the measurement for one of the objects multiplied by 3 gives the measurement for the other object?

Alternatively, you may want to start with two measurements/objects and see if there is an arrow that describes how much bigger one object is compared to the other. There are some blank arrow cards for you to use if needed.

Once you have linked a pair of measurements/objects, stick the cards down on your poster and see if you can compare another pair of objects. Do this for as many pairs as possible.

Their finished poster may look something like this:



Students will be able to make a maximum of four paired comparisons with one set of object/measurement cards left over. These can be used in the extension activity if time allows. They will not need all of the arrow cards.

As students work on the task, encourage them to check their work. They may find it helpful to think about the matched objects to check whether or not their comparison is realistic, before carrying out any calculations:

Roughly how many times bigger is the height of a tree than the height of a door? Is this a realistic comparison? Can you now check it mathematically?

Students should be encouraged to justify the comparisons made and describe their methods:

Can you explain why you placed this arrow here?

In this task, do you find it easier to use the decimal notation card or the scientific notation card? Why?

When students have completed as many comparisons as possible and stuck them down, ask them to stick down any remaining matched up object/measurement cards on their poster as well.

Extension activity

If a group successfully completes four comparisons, ask them to compare their remaining object with an object of their own choice. They need to decide on an object and its measurement and state how many times bigger it is. They can either select one of the unused arrow cards or write their own.

Whole-class discussion (10 minutes)

Organize a discussion about what has been learned and encourage students to explore the different methods used. It is likely that each poster will be slightly different, depending on the comparisons made.

Michael, explain how you matched one arrow card with two measurement/object cards.

Did anyone else complete the same comparison?

In addition to asking for a variety of methods, pursue the theme of listening and comprehending others' methods by asking students to rephrase others' reasoning.

Can someone else put that into your own words?

Does anyone disagree with this comparison?

There will not be time to discuss all the comparisons, but a good coverage of measurements should be encouraged.

Follow-up lesson: Reviewing the assessment task (15 minutes)

Give each student a copy of the review task, *Resize It Up!*, and their original scripts from the assessment task, *Size It Up*. If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Read through your scripts from Size It Up, and the questions [on the board/written on your script.]

Use what you have learned to answer these questions.

Now look at the new task sheet, Resize It Up! Can you use what you have learned to answer these questions?

SOLUTIONS

Assessment Task: *Size It Up*

1.

Decimal Notation	Scientific Notation	Rank order
0.01 m	$1 \times 10^{-2} m$	3
$0.004 m$	$4 \times 10^{-3} m$	2
$200 m$	$2 \times 10^2 m$	5
800,000 m	$8 \times 10^5 m$	6
$40,000,000 m$	$4 \times 10^7 m$	7
$40 m$	$4 \times 10^1 m$	4
0.0008 m	$8 \times 10^{-4} m$	1

2 (a) There are two alternatives:

$$0.01 m \quad \times 4000 \quad = \quad 40 m$$

$$200 m \quad \times 4000 \quad = \quad 800,000 m$$

2 (b) There are two alternatives:

$$8 \times 10^{-4} m \quad \times 50,000 \quad = \quad 4 \times 10^1 m$$

$$4 \times 10^{-3} m \quad \times 50,000 \quad = \quad 2 \times 10^2 m$$

Assessment Task: Resize It Up!

1.

Decimal Notation	Scientific Notation	Rank order
6 m	$6 \times 10^0 m$	4
120,000 m	$1.2 \times 10^5 m$	7
0.0012 m	$1.2 \times 10^{-3} m$	2
300 m	$3 \times 10^2 m$	5
0.6 m	$6 \times 10^{-1} m$	3
0.0009 m	$9 \times 10^{-4} m$	1
6,000 m	$6 \times 10^3 m$	6

2 (a) There are two alternatives:

$$300 \text{ m} \quad \times 20 \quad = \quad 6,000 \text{ m}$$

$$6,000 \text{ m} \quad \times 20 \quad = \quad 120,000 \text{ m}$$

2 (b) There are two alternatives:

$$6 \times 10^{-1} \text{ m} \quad \times 500 \quad = \quad 3 \times 10^2 \text{ m}$$

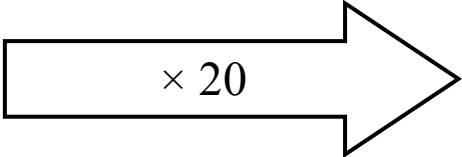
$$1.2 \times 10^{-3} \text{ m} \quad \times 500 \quad = \quad 6 \times 10^{-1} \text{ m}$$

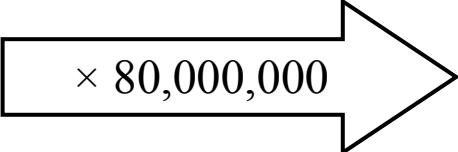
Collaborative activity

The measurements in **bold** correspond to blank cards.

Scientific Notation	Decimal Notation	Object
$1 \times 10^{-4} \text{ m}$	0.0001 m	Thickness of a dollar bill
$1.2 \times 10^{-2} \text{ m}$	0.012 m	Length of a fly
$1.2 \times 10^{-1} \text{ m}$	0.12 m	Length of a stapler
$2 \times 10^0 \text{ m}$	2 m	Height of a door
$3 \times 10^0 \text{ m}$	3 m	Length of a truck
$2 \times 10^1 \text{ m}$	20 m	Height of a tree
$6 \times 10^1 \text{ m}$	60 m	Wingspan of an aircraft
$8 \times 10^3 \text{ m}$	8000 m	Height of a mountain
$4 \times 10^8 \text{ m}$	400,000,000 m	Distance from the earth to the moon

There are numerous comparisons that can be made. For example:

Length of a truck $3 \times 10^0 \text{ m} = 3 \text{ m}$  Wingspan of an aircraft $6 \times 10^1 \text{ m} = 60 \text{ m}$

Thickness of a dollar bill $1 \times 10^{-4} \text{ m} = 0.0001 \text{ m}$  Height of a mountain $8 \times 10^3 \text{ m} = 8000 \text{ m}$

Some of the arrows will be redundant depending on the comparisons made.

Size It Up

1. The table contains seven measurements written in decimal and scientific notation.
- (a) Complete the table so that each measurement is written in *both* decimal *and* scientific notation.
- (b) In the last column, rank the measurements in order of size.
(1 = smallest, 2 = next smallest, and so on up to 7 = largest)

Decimal Notation	Scientific Notation	Rank 1 = smallest 7 = largest
	$1 \times 10^{-2} \text{ m}$	
0.004 m	=	
200 m	=	
	$8 \times 10^5 \text{ m}$	
40,000,000 m	=	
40 m	=	
	$8 \times 10^{-4} \text{ m}$	

- 2 (a) Complete the following statement using two numbers in *decimal notation* from the table.

$$\text{.....} \times 4000 = \text{.....}$$

- 2 (b) Complete the following statement using two numbers in *scientific notation* from the table.

$$\text{.....} \times 50,000 = \text{.....}$$

Card Set A: Measurements

Decimal notation

20 m	60 m	0.012 m
0.12 m	3 m	8000 m
400,000,000 m	0.0001 m	

Scientific notation

2×10^0 m	1×10^{-4} m	8×10^3 m
3×10^0 m	2×10^1 m	4×10^8 m
6×10^1 m	1.2×10^{-1} m	

Card Set B: Objects

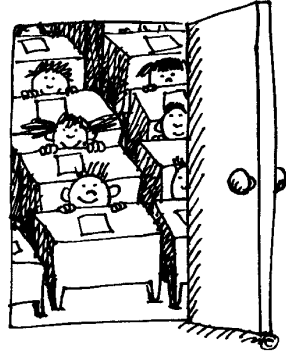
Height of a tree



Distance from the earth to the moon



Height of a door



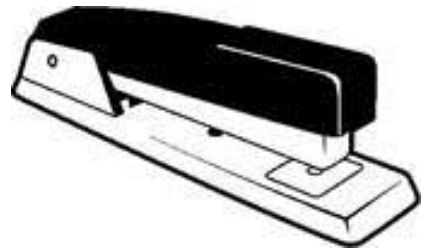
Height of a mountain



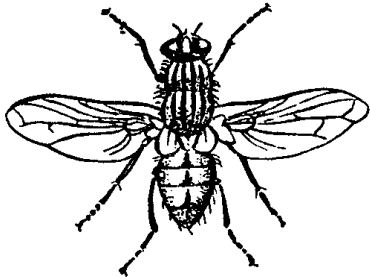
Thickness of a dollar bill



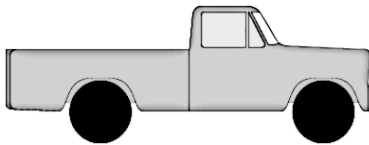
Length of a stapler



Length of a fly



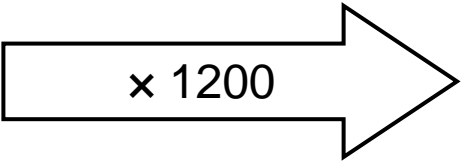
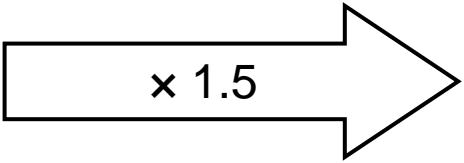
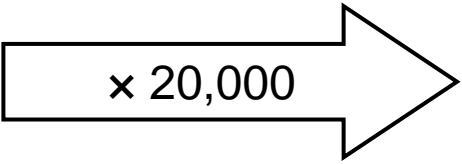
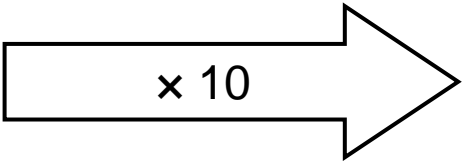
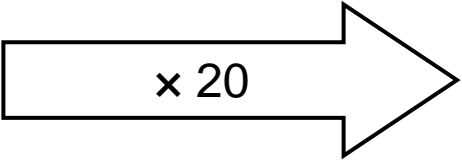
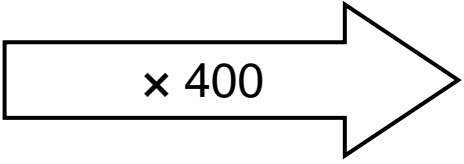
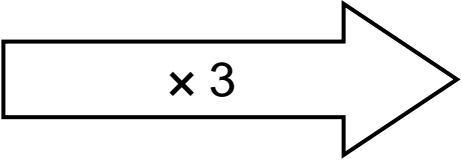
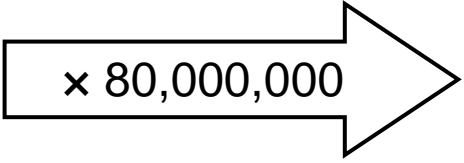
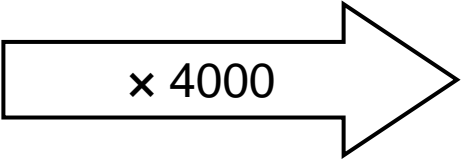
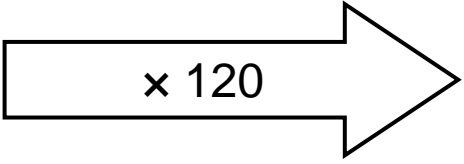
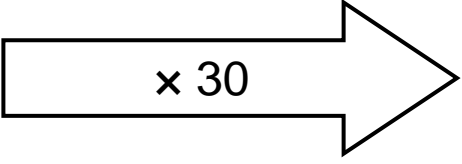

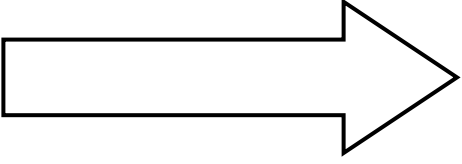
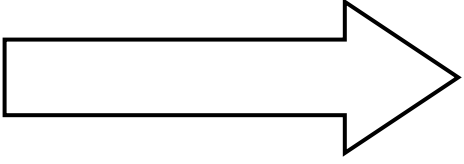
Length of a truck



Wingspan of an aircraft



Card Set C: Comparisons

Resize It Up!

1. The table contains seven measurements written in decimal and scientific notation.

- (a) Complete the table so that each measurement is written in *both* decimal *and* scientific notation.
- (b) In the last column, rank the measurements in order of size.
(1 = smallest, 2 = next smallest, and so on up to 7 = largest)

Decimal Notation	Scientific Notation	Rank 1 = smallest 7 = largest
	= $6 \times 10^0 \text{ m}$	
120,000 m	=	
0.0012 m	=	
	= $3 \times 10^2 \text{ m}$	
0.6 m	=	
	= $9 \times 10^{-4} \text{ m}$	
6000 m	=	

2 (a) Complete the following statement using two numbers in *decimal notation* from the table.

$$\text{.....} \times 20 = \text{.....}$$

2 (b) Complete the following statement using two numbers in *scientific notation* from the table.

$$\text{.....} \times 500 = \text{.....}$$

What's the Number?

$$30,000 \times 10^{-1}$$

$$3 \times 10^3$$

$$0.3 \times 10^4$$

Scientific Notation or Not?

$$30,000 \times 10^{-1}$$

$$3 \times 10^3$$

$$0.3 \times 10^4$$

Which is Greater?

$$8 \times 10^{-3}$$

$$4 \times 10^{-1}$$

Matching Card Set A

1. Take turns to match a card in scientific notation with a card in decimal notation.
2. Each time you match a pair of cards explain your thinking clearly and carefully. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.
3. Partners should either agree with the explanation, or challenge it if it is not clear or not complete.
4. It is important that everyone in the group understands the matching of each card.
5. You should find that two cards do not have a match. Write the alternative notation for these measurements on the blank cards to produce a pair.

Matching *Card Set B*

1. Match the objects in *Card Set B* with the corresponding paired measurements from *Card Set A*. You may want to put the objects in size order first to help you.
2. It is important that everyone in the group agrees on each match.
3. When you have finished matching the cards, check your work against that of a neighboring group.
4. If there are differences between your matching, ask for an explanation. If you still don't agree, explain your own thinking.
5. You must not move the cards for another group. Instead, explain why you disagree with a particular match and explain which cards you think need to be changed.

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

Malcolm Swan,
Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and Marie Joubert
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and Diane Schaefer

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and Malcolm Swan at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee

The full collection of Mathematics Assessment Project materials is available from

<http://map.mathshell.org>