CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Analyzing Games of Chance

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Analyzing Games of Chance

MATHEMATICAL GOALS

This unit is designed to help students to:

- Confront and overcome common probability misconceptions.
- Count equally likely outcomes using diagrams.
- Discuss relationships between theoretical probabilities, observed outcomes, and sample sizes.
- Calculate probabilities of independent events.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.SP: Investigate chance processes and develop, use, and evaluate probability models.

This lesson also relates to **all** the *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2, 3, 4, 5 and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on a task designed to reveal their current level of understanding. You review their scripts and write questions to help them to improve their work.
- In the lesson, students are asked to work collaboratively on some simple games. They make predictions of the outcomes and then conduct the experiments and gather data.
- In a follow-up lesson, students use their learning and your questions to review their initial answers and to complete a follow-up task.

MATERIALS REQUIRED

- Each individual student will need a mini-whiteboard, a pen, an eraser, a copy of the assessment task *Spinner Bingo,* and the assessment task *Number Game*.
- Each small group of students will need two dice, 11 counters or buttons, a copy of the task *The Horse Race,* a copy of the *Rules of Horse Race,* and the *Race Results Sheet*. The race board should be enlarged if possible. Each small group will also need copies of the *Sample Student Reasoning* sheets and the *Other Horse Races* sheet.
- If a computer is available, the software may be used as well to make it possible to play many more games. The software and projector slides will also support whole-class discussions.

TIME NEEDED

20 minutes before the lesson, a 120-minute lesson (or two shorter lessons), and 10 minutes in a follow-up lesson. Exact timings will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Spinner Bingo* (20 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Make sure that students understand the context of the task:

Who has played Bingo?
What is the aim of the game?

In this Bingo game the numbers called are obtained by adding the scores on two spinners. Each spinner gives a number (say red spinner gives 2 and blue spinner gives 3, then the number called is the sum of the two numbers, 5.)

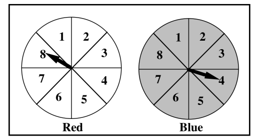
Now explain what students need to do:

Read through the questions and try to answer them as carefully as you can. Show all your work so that I can understand your reasoning.

Do not be too concerned if you cannot finish everything. [Tomorrow] we will have a lesson on these ideas, which should help you to make further progress.

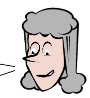
Spinner Bingo

Sally has made a Spinner Bingo game for her class.



Write down 9 different numbers on your card.

I will spin both spinners and add up the two numbers I get.
If you have that total on your Bingo card, you cross it off.
The first person to cross off all the numbers wins the prize.



Here are three Bingo cards the players made:

Card A

4	13	5
12	9	6
8	11	15

Card B

14	6	17
7	10	4
1	15	12

Card C

5	15	4
14	3	16
2	13	10

1. Which card has the best possible chance of winning? Give reasons for your answer.

....

....

....

2. Fill in a card so that it has the best chance of winning. Explain how you chose the numbers for your card.

....

....

....

....

....

3. Would you be guaranteed to win playing with the card containing your chosen numbers? Explain why / why not.

....

....

Assessing students' responses

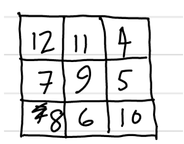
Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and any difficulties they encounter.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We recommend that you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students in the follow-up lesson.

Common issues	Suggested questions and prompts
Does not show any evidence of finding possible scores or probabilities For example: The student writes: "Card B contains a 1 so that is impossible. Card C has less chance because all the numbers are close together so Card A is the best." (Q1).	• What range of scores can you get from the pair of spinners? • Which numbers on the cards are impossible to get? • Why do you think that numbers close together have 'less chance'? • What does this tell you about the most likely and least likely totals?
Lists possible totals in a systematic way but there remain omissions For example: The student considers 2 + 1 = 3 but not 1 + 2 = 3 in the list.	• What other scores on the red and blue spinners would give a total score of 3? • How does this answer affect the total number of ways you have calculated? • How might we attach a probability to each of the possible total numbers?
Considers the likelihood of some totals appearing, but this is incomplete and unsystematic For example: The student designs the following Bingo card and says that "the numbers I chose have lots of adding factors, like 8 = 1 + 7; 8 = 4 + 4 and 8 = 3 + 5." (Q2). 	• Can you explain what you mean by "they have lots of 'adding factors'"? • How could you record all the possible totals and the ways they can be obtained? • Can you find a systematic way of making sure you have them all? • How might this information help in producing a card that has the best chance of winning?
Creates a valid Bingo card but insufficient reasons are given for the numbers chosen (Q2)	• Can you explain why you have chosen these totals? • Are there other numbers you could have used that would give you an equal chance of winning? How do you know?
Fails to distinguish between theoretical and experimental probabilities For example: The student explains that they will always win with their Bingo card as they have only included the most likely totals. (Q3).	• If a total has a high probability, is it certain to be spun in a game?
Completes the task	• Now suppose the numbers on the spinners are multiplied together. What is the best winning card this time?

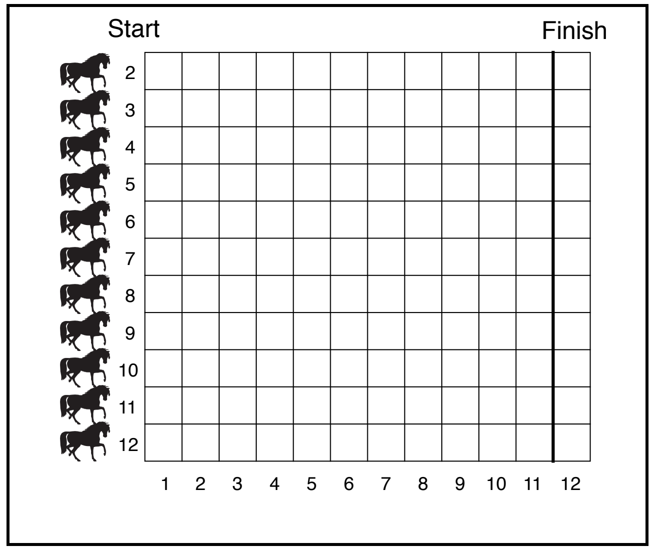
SUGGESTED LESSON OUTLINE

Whole-class introduction: *The Horse Race* (10 minutes)

This activity may be presented using the software provided. Alternatively, you can use a drawing of the board and real dice. The board can be displayed using Slide P-1 of the projector resource.

Give each student a mini-whiteboard, pen, and eraser.

> *Eleven horses enter a race. The first one to pass the finish line wins.*
>
> *Place counters on the starting squares labeled 2 to 12.*
>
> *Roll the two dice and add the scores. The horse with that number moves one square forward. Keep rolling the dice. The horse that is first past the finishing line wins.*
>
> *Stop rolling when the first horse wins and record the position of all the horses on your recording sheet.*



It is helpful to demonstrate the beginning of the game with the whole-class. The computer software will help you to do this. After a few rolls of the dice, stop and ask students to predict what will happen. Ask students to write answers on their mini-whiteboards and show them to you.

> *Which horse do you think will win the race? Can you explain your reasoning?*

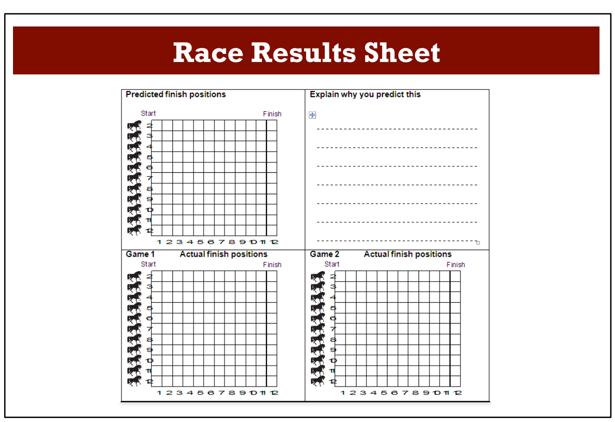
It is likely at this stage that students will base their predictions on intuition or the experimental outcomes rather than considering the theoretical probabilities of different sums of the two dice scores. They will have the opportunity to explore this further as the lesson progresses.

Small-group work: *Playing the Game* (20 minutes)

Ask students to work in groups of three.

Give out an enlarged copy of *The Horse Race* board, the *Rules of Horse Race* and the *Race Results Sheet*. Give each group 11 counters or buttons and two dice.

Display Slide P-2 of the projector resource:



The procedure for the race is as follows:

Predict the outcome of the race before starting it. Show your prediction on the Race Results Sheet, by drawing the positions of all the horses as the first one crosses the finishing line using the first diagram.

Play the game using the board and counters. Record the finishing positions of each horse when one reaches the finish. Now run the race for a second time.

Reflect on what happened and try to explain this at the bottom of the sheet.

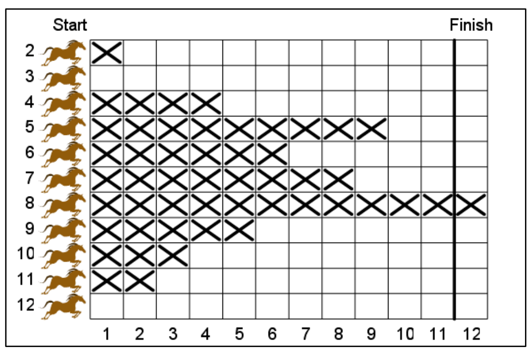
These instructions are summarized on Slide P-3 of the projector resource.

While students play the game, encourage them to write any patterns they find on the *Rules of Horse Race* sheet.

Whole-class discussion: collecting the empirical results (20 minutes)

After a set of races, collect together the results from the whole-class and hold a discussion about the results.

Maxine, can you tell me which of your horses won the first race? [E.g. Horse 8.]
How many squares had the other horses moved forward when this happened? [E.g. See diagram below.]
We'll write this in the column headed Group 1.



On the board, build up a collection of results from around the class. Slide P-4 of the projector resource may help you do this. Collect one set of results from each group. For example (these are genuine results):

Horse number	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
2	1	4	2	0	0	2
3	0	5	4	5	2	7
4	4	2	6	1	1	9
5	9	9	11	8	5	12
6	6	9	8	4	7	9
7	8	8	12	12	12	11
8	12	12	11	8	4	9
9	5	5	8	8	3	11
10	3	6	7	6	2	5
11	2	2	2	4	5	7
12	0	5	0	1	0	1

Give the students a few minutes to look at the data, perhaps asking them what they notice.

Discuss the following questions in an open way, asking for ideas from many students without comment. They should use the table of results collected from the class to answer these questions:

If we were to play the game yet again, which horse is most likely to win? [E.g. Horse 7 based on above table.] Why?

How likely is it, giving a number from 0 to 1 (where 0=impossible, 1=certain), that the horse you have chosen will win the race? [E.g. Horse 7 won 3 out of 6 races so the probability of it winning the next race based on this experimental data is 0.5.]

Does anyone disagree? Why?

Which horse is most likely to come second? [E.g. Horse 8.] Why?

Which horses are least likely to win? [E.g. Horses 12 and 2.] Why?

Will the outcome vary very much from race to race? Why?

Do you think the finishing order is predictable?

How does the number of races influence how predictable the finishing order is?

Students may suggest finding the mean of the finishing positions to help them predict the next race.

It does not matter if these questions are unresolved at this stage; they will be further explored later in the lesson.

Extending the lesson over two days

If your lesson is about an hour long, you may wish to break it here and leave the rest of the plan below for the following lesson.

Small-group work: *Looking at Reasoning* (30 minutes)

Until now, the focus has been to look at the situation intuitively and empirically. Now students are going to look at the situation theoretically.

Give out copies of the *Sample Student Reasoning* to each group of students. Invite them to interpret and critique the work and comment on the conclusions reached by each student.

How has this student tackled the problem of explaining the results?

What has each student done well?

What errors has each student made? Can you correct these errors?

How could each student's work be improved and extended?

These instructions are summarized on Slide P-5 of the projector resource.

As students work, go round asking them to describe and explain the approaches that have been adopted in this work. Encourage students to be critical and to look for mistakes and to extend the work.

Whole-class discussion (20 minutes)

Discuss the work with students, as a whole-class. Try to get them to suggest criticisms and comparisons.

What has each student done incorrectly?

Which piece of work is better? Why is this?

What has Jessica done that Tom has not done?

The student work may be projected using Slides P-6 and P-7 of the projector resource.

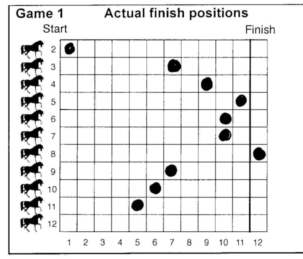
The following notes on Tom's and Jessica's reasoning may help you discuss this work with students.

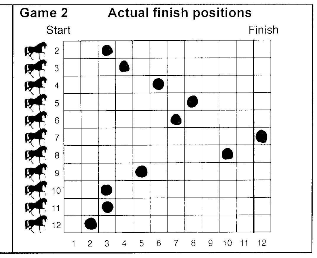
Tom has been systematic in his approach and has attempted to list all the possible combinations of the two dice.

He has, however, not realized that there are two ways of obtaining some of the results, and has not distinguished between 4 + 1 and 1 + 4, for example. This is a very common error in this type of work.

This means that his total number of ways is incorrect.

His conclusion that 6, 7 and 8 are the most likely horses to win is correct in a general sense, but his argument is incorrect.





I think the horses in the middle are most likely to win.

The dice rolls are:

1+1 = 2 2+2 = 4 3+3 = 6 4+4 = 8 5+5 = 10 6+6 = 12
1+2 = 3 2+3 = 5 3+4 = 7 4+5 = 9 5+6 = 11
1+3 = 4 2+4 = 6 3+5 = 8 4+6 = 10
1+4 = 5 2+5 = 7 3+6 = 9
1+5 = 6 2+6 = 8
1+6 = 7

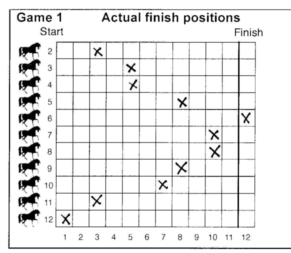
Totals	2	3	4	5	6	7	8	9	10	11	12	
ways	1	1	2	2	3	3	3	2	2	1	1	= 21

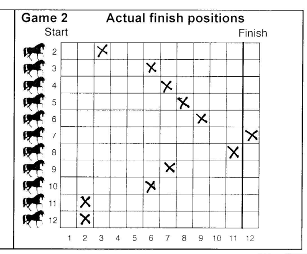
So horses 6, 7, 8 are the most likely to win and that is what happened.

Jessica has correctly and systematically listed all the possible outcomes in a table. Her conclusion that 7 will win one sixth of the time, however, is incorrect. She has shown that the probability of horse 7 moving forward one square with one roll of the two dice is one sixth. She has ***not*** shown that the probability of 7 winning the race is one sixth.

Calculating the probability of horse 7 winning the race based on theoretical probability is beyond the scope of this lesson. However, students should begin to get a feel (based on their empirical work) that the longer the racecourse, the greater the probability of horse 7 winning the race. If the course was only one square long, the probability of 7 winning would be one sixth. If it was many thousands of squares long, the probability would approach 1 (certainty.)

Students should be encouraged to use Jessica's table to find the probabilities of other horses moving one square forward.





These are the outcomes:
(there are 36)

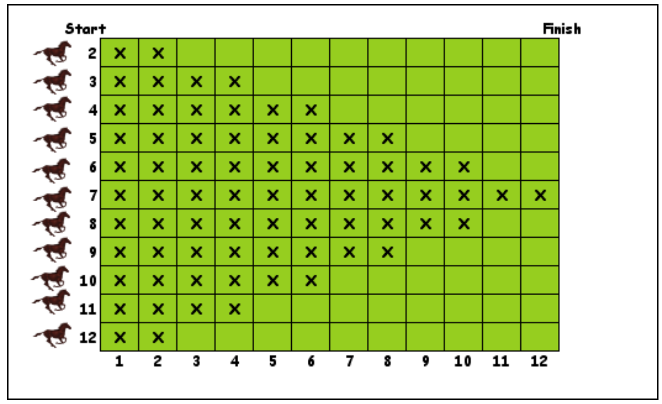
+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

7 → $\frac{6}{36} = \frac{1}{6}$ so 7 will win one sixth of the time.

Jessica's table also shows that in 36 throws of the dice one would expect horse 7 to be halfway towards its goal of winning the race.

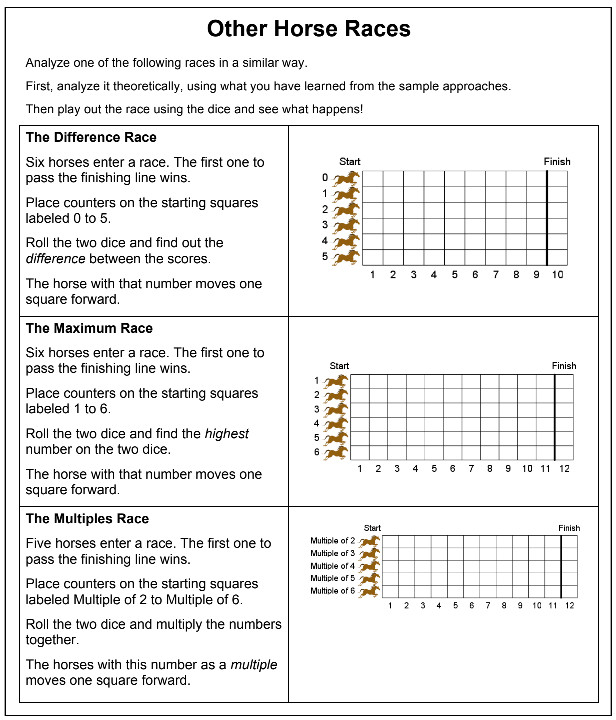
Thus, it shows that after 72 throws the statistical expectation of the positions of the horses would be as shown.

Thus, one would expect horses 6 and 8 to be 2 squares behind horse 7, and so on. This is the theoretical distribution.



Small-group work: applying what has been learned (20 minutes)

Invite students to work in groups analyzing and exploring other types of race using what they have learned. Three possibilities are shown on the sheet *Other Horse Races*:



Other Horse Races

Analyze one of the following races in a similar way.

First, analyze it theoretically, using what you have learned from the sample approaches.

Then play out the race using the dice and see what happens!

The Difference Race

Six horses enter a race. The first one to pass the finishing line wins.

Place counters on the starting squares labeled 0 to 5.

Roll the two dice and find out the *difference* between the scores.

The horse with that number moves one square forward.

The Maximum Race

Six horses enter a race. The first one to pass the finishing line wins.

Place counters on the starting squares labeled 1 to 6.

Roll the two dice and find the *highest* number on the two dice.

The horse with that number moves one square forward.

The Multiples Race

Five horses enter a race. The first one to pass the finishing line wins.

Place counters on the starting squares labeled Multiple of 2 to Multiple of 6.

Roll the two dice and multiply the numbers together.

The horses with this number as a *multiple* moves one square forward.

We suggest that students first analyze one situation and make predictions based on their analysis. Then, using the computer software, the race is held on the board and the outcomes are discussed, as before. As students do this, encourage them to draw tables to show all the possible outcomes, as Jessica did and then to calculate probabilities of each horse moving forward.

Follow-up lesson: individual review (10 minutes)

Remind students of their work on *The Horse Race* and the different methods they have seen and used.

Hand out students' attempts at the initial assessment task, *Spinner Bingo*, together with your questions. If you did not add questions to individual pieces of work, write your list of questions on the board. Students can then select questions appropriate to their own work. Ask students to read through your questions and correct their mistakes.

You may then like to ask students to tackle a second task *Number Game*, to see if they can apply their knowledge to a new situation.

SOLUTIONS

Assessment task: *Spinner Bingo*

1. Card B cannot win because it contains the numbers 1 and 17 and these totals cannot be obtained. Card C contains extreme/unlikely numbers (such as 2, 16) because there are few combinations of numbers that make these. Card A is most likely to win, because all the numbers are obtainable and this card contains more 'middle' numbers in the range 5 to 13.

2. The following sample space diagram shows all the possible outcomes for the two spinners:

Red / Blue	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

This shows that the most likely totals are (with probabilities):

9 (8/64)

8 and 10 (7/64)

7 and 11 (6/64)

6 and 12 (5/64)

5 and 13 (4/64)

These are, therefore, the numbers that should appear on the card.

Other probabilities are:

4 and 14 (3/64)

3 and 15 (2/64)

2 and 16 (1/64)

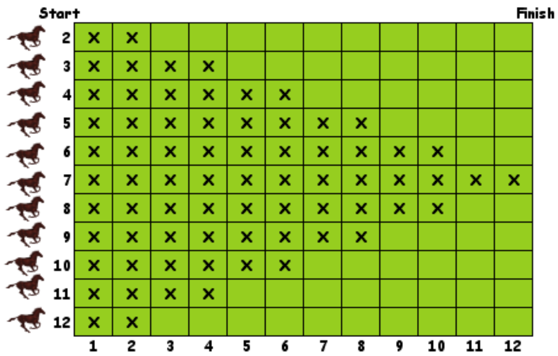
3. Students should distinguish between theoretical and experimental probability. Choosing totals that are most likely to be obtained from the spinners does not guarantee that these totals will be generated.

Lesson task: *Horse Races*

The Addition Race

In the addition race, students should notice that the horses in the center of the field tend to win more often than those at the outside. The sample space diagram shows the equally likely outcomes:

Addition race		First dice					
		1	*2*	*3*	*4*	*5*	*6*
Second dice	*1*	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

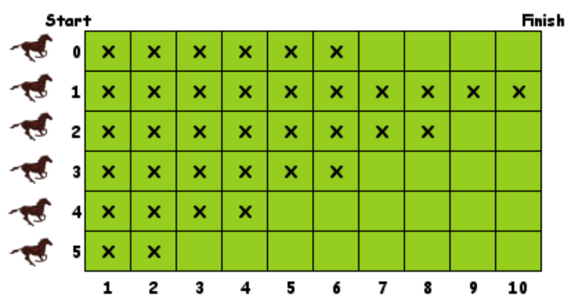


The diagram shows, for example, that we would expect horse 7 to move six times more often than horse 2. Students can calculate the probabilities that particular horses will move from this diagram. The table gives a prediction of the positions of the horses when horse 7 finishes.

The Difference Race

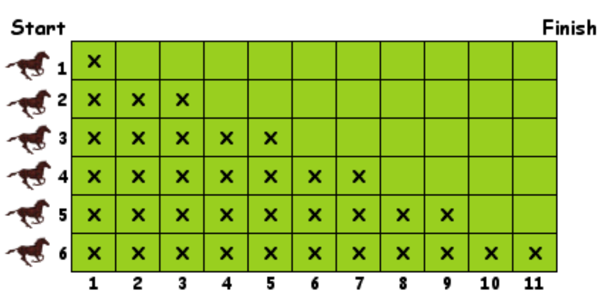
This gives an asymmetric distribution, with horse 1 the most likely to win.

Subtraction race		First dice					
		1	*2*	*3*	*4*	*5*	*6*
Second dice	*1*	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0



The Maximum Race

They should find that the results from the maximum dice race should also give an asymmetric distribution, with horse 6 the most likely to win.



Max race		First dice					
		1	*2*	*3*	*4*	*5*	*6*
Second dice	*1*	1	2	3	4	5	6
	2	2	2	3	4	5	6
	3	3	3	3	4	5	6
	4	4	4	4	4	5	6
	5	5	5	5	5	5	6
	6	6	6	6	6	6	6

The Multiples Race

This is slightly more complicated, as a particular throw of the dice may result in two or more horses moving at one time. Thus a throw of $3 \times 4 = 12$ will result in horses 2, 3, 4 and 6 all moving. Ask students to tell you which of horses 6 and 2 will be the fastest and why.

The easiest place to begin is by simply writing out a list of possible products. Students can then count the multiples systematically.

Multiples race		**First dice**					
		1	***2***	***3***	***4***	***5***	***6***
Second dice	***1***	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

There are:

27 multiples of 2 and so prob (mult of 2) = 27/36.
20 multiples of 3 and so prob (mult of 3) = 20/36.
15 multiples of 4 and so prob (mult of 4) = 15/36.
11 multiples of 5 and so prob (mult of 5) = 11/36.
15 multiples of 6 and so prob (mult of 6) = 15/36.

Note how the probabilities do not add up to 1, because they are not mutually exclusive. This enables us to predict the outcome; horse 2 should win, followed by horse 3, then 4 and 6 (together) and 5 bringing up the rear.

In each of the above situations students may question why their own results do not correspond to the theory. This is a good time to consider the issue of sample size. Small samples may not correspond to these results, but aggregated over the class, the results should correspond more closely.

Assessment task: *Number Game*

In this task there are fifteen different pairs of numbers that may be selected. Unlike the horse race, this time the order of each pair of numbers does not matter.

(1,2); (1,3); (1,4); (1,5); (1,6)

(2,3); (2,4); (2,5); (2,6)

(3,4); (3,5); (3,6)

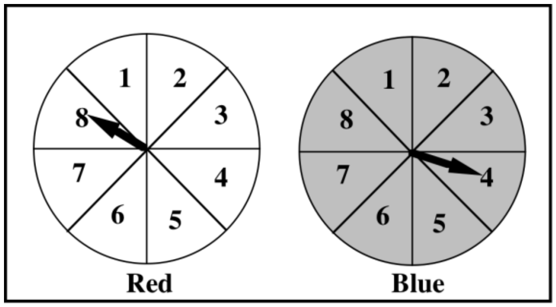
(4,5); (4,6)

(5,6)

The probability of getting the pair correct is 1/15 so for every 15 students we would expect one winner; in a class of 30 we would expect there to be two students winning a prize.

Spinner Bingo

Sally has made a Spinner Bingo game for her class.



Write down 9 different numbers on your card.

I will spin both spinners and add up the two numbers I get.
If you have that total on your Bingo card, you cross it off.
The first person to cross off all the numbers wins the prize.

Here are three Bingo cards the players made:

Card A

4	13	5
12	9	6
8	11	15

Card B

14	6	17
7	10	4
1	15	12

Card C

5	15	4
14	3	16
2	13	10

1. Which card has the best possible chance of winning? Give reasons for your answer.

....

....

....

2. Fill in a card so that it has the best chance of winning. Explain how you chose the numbers for your card.

....

....

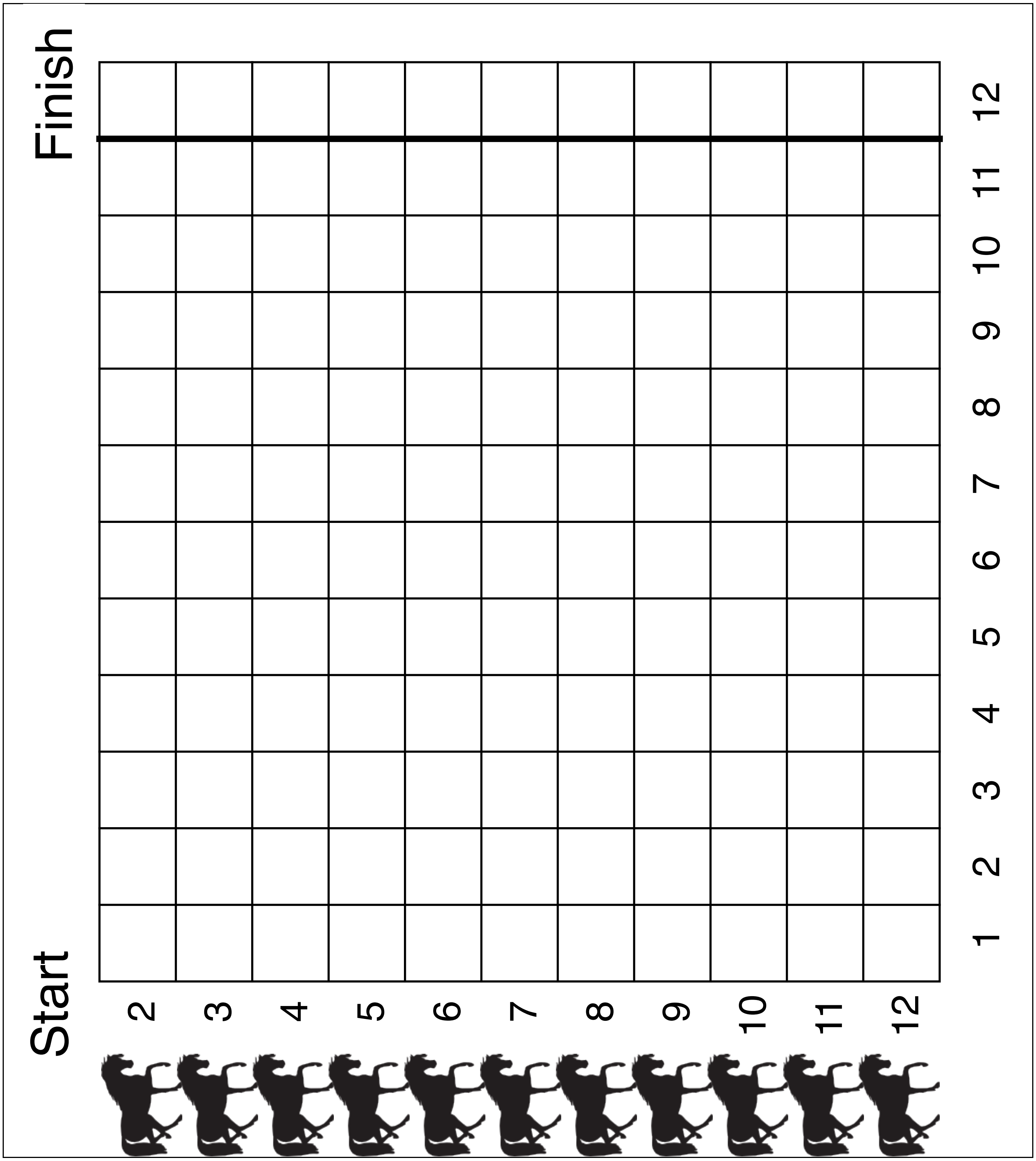
....

....

....

3. Would you be guaranteed to win playing with the card containing your chosen numbers? Explain why / why not.

The Horse Race



Rules of Horse Race

Aim of the Game

Eleven horses enter a race. The first one to pass the finish line wins.

Rules

Place counters on the starting squares labeled 2 to 12.

Share out the horses so that each person in your group has three or four horses.

How to play

Roll the two dice and add the scores.

The horse with that number moves one square forward.

Keep rolling the dice.

The horse that is first past the finishing line wins.

1. Before you start the race, write down the order you predict the horses will finish in on the Race Results sheet.
   Why do you predict this?
   Write down your reason next to your prediction in the space provided.

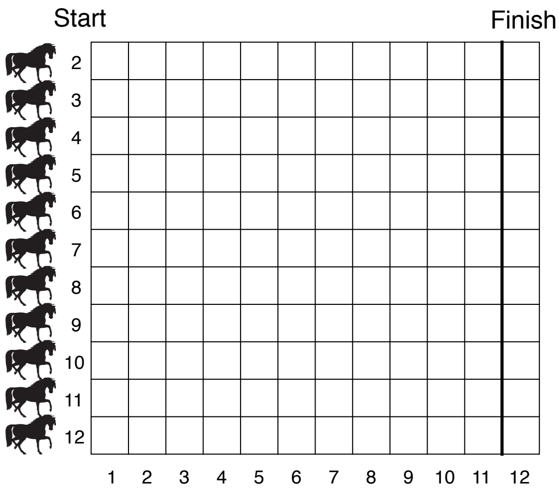
2. Play the race twice.
   Record the final positions of the horses each time.

3. Try to explain any patterns you find in your data.

   - Does the outcome vary very much from race to race?
   - Which horses are most likely to win? Why?
   - Which horses are least likely to win? Why?
   - Could the finishing order have been predicted?
   - Could the winning distance have been predicted?

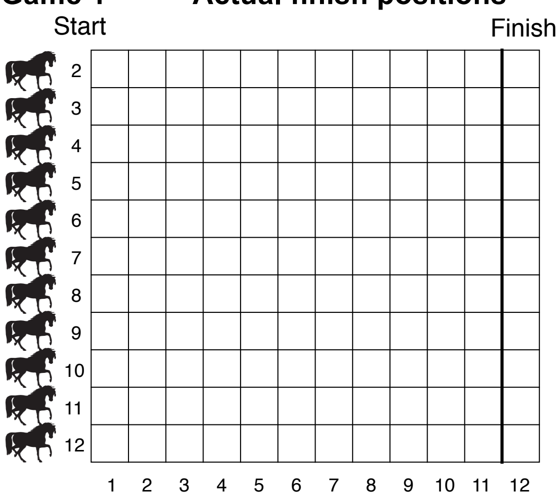
Race Results Sheet

Predicted finish positions

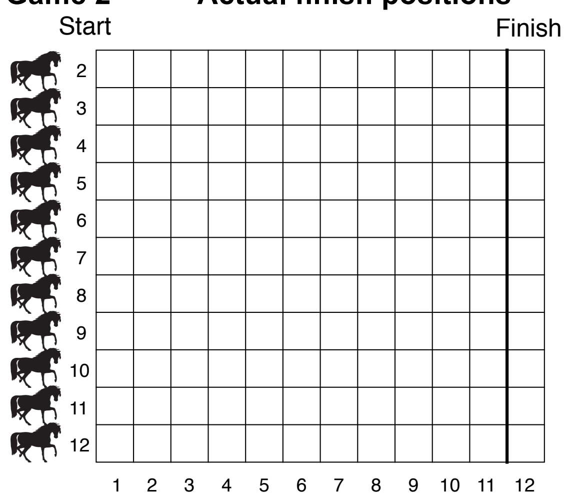


Explain why you predict this

Game 1 **Actual finish positions**

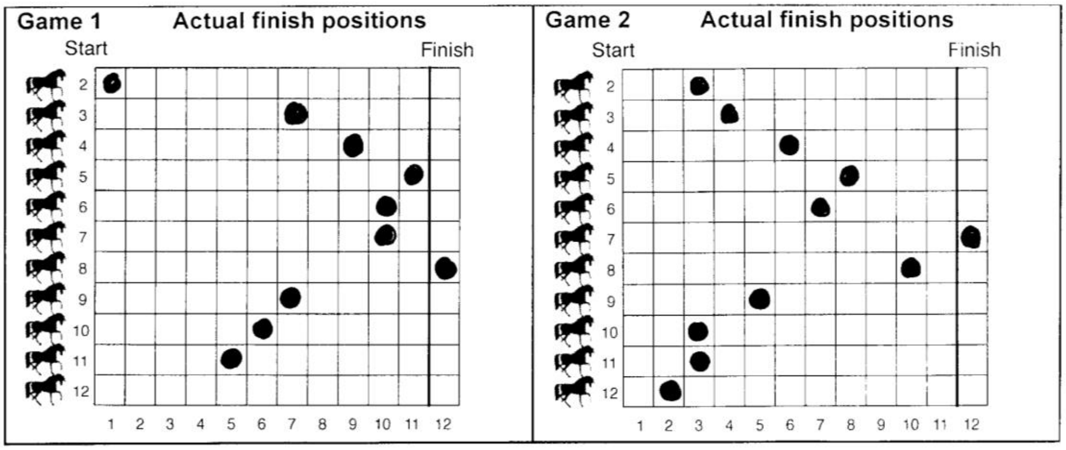


Game 2 **Actual finish positions**



Explain your results. Are they different from what you expected? Why is this?

Sample Student Reasoning: Tom



I think the horses in the middle are most likely to win.

The dice rolls are:

1+1 = 2   2+2 = 4   3+3 = 6   4+4 = 8   5+5 = 10   6+6 = 12
1+2 = 3   2+3 = 5   3+4 = 7   4+5 = 9   5+6 = 11
1+3 = 4   2+4 = 6   3+5 = 8   4+6 = 10
1+4 = 5   2+5 = 7   3+6 = 9
1+5 = 6   2+6 = 8
1+6 = 7

Totals	2	3	4	5	6	7	8	9	10	11	12	
ways	1	1	2	2	3	3	3	2	2	1	1	= 21

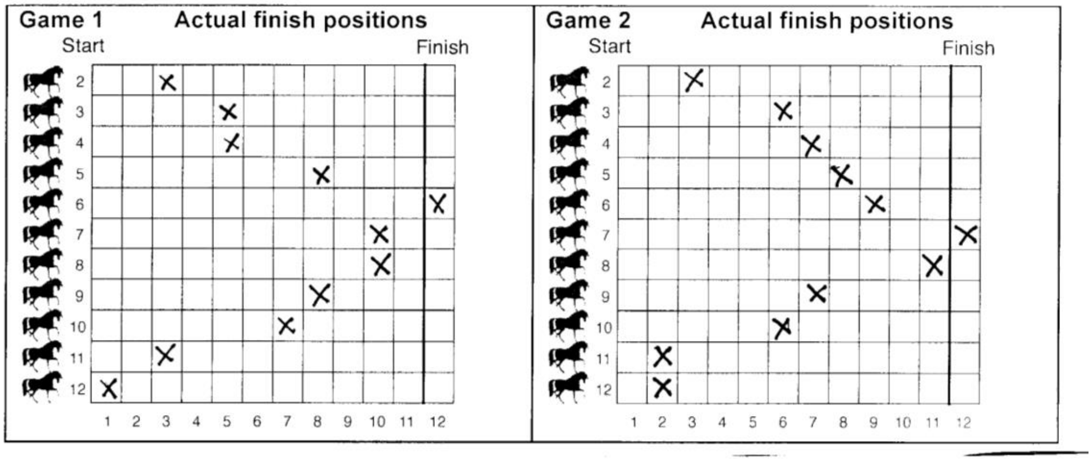
So horses 6, 7, 8 are the most likely to win and that is what happened.

1. What has Tom done well?

2. What errors has Tom made? Can you correct them?

3. Try to improve and extend Tom's work.

Sample Student Reasoning: Jessica



These are the outcomes:
(there are 36)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$7 \rightarrow \frac{6}{36} = \frac{1}{6}$ so 7 will win one sixth of the time.

1. What has Jessica done well?

2. What errors has Jessica made?

3. Try to improve and extend Jessica's work.

Other Horse Races

Analyze one of the following races in a similar way.

First, analyze it theoretically, using what you have learned from the sample approaches.

Then play out the race using the dice and see what happens!

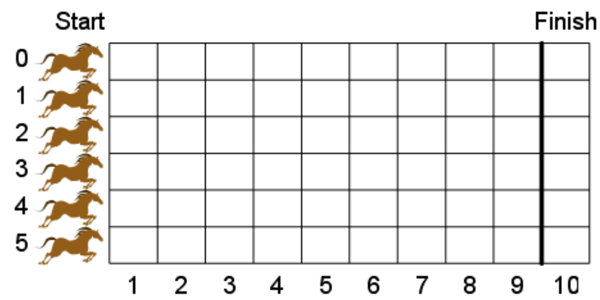
The Difference Race

Six horses enter a race. The first one to pass the finishing line wins.

Place counters on the starting squares labeled 0 to 5.

Roll the two dice and find out the *difference* between the scores.

The horse with that number moves one square forward.



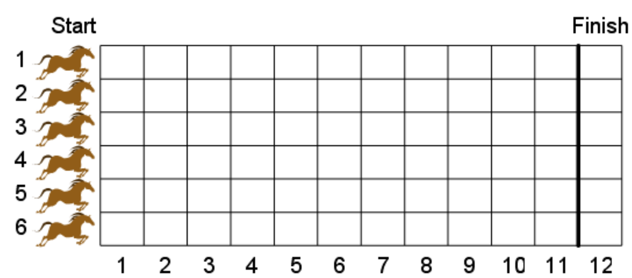
The Maximum Race

Six horses enter a race. The first one to pass the finishing line wins.

Place counters on the starting squares labeled 1 to 6.

Roll the two dice and find the *highest* number on the two dice.

The horse with that number moves one square forward.



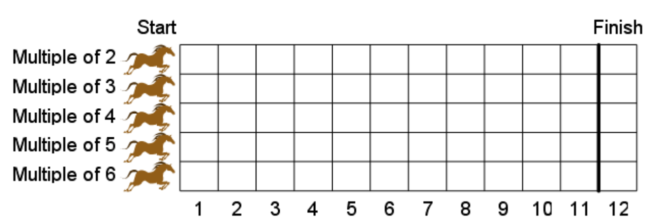
The Multiples Race

Five horses enter a race. The first one to pass the finishing line wins.

Place counters on the starting squares labeled Multiple of 2 to Multiple of 6.

Roll the two dice and multiply the numbers together.

The horses with this number as a *multiple* moves one square forward.



Number Game

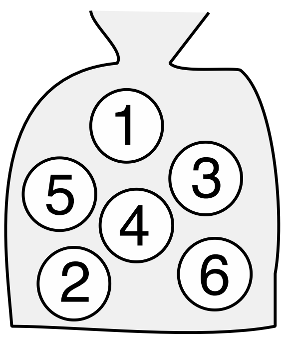
Karl's Math class is playing a number game.

Each student is given a number card containing the numbers 1 to 6:

1	2	3
4	5	6

The rules of the game are that each student must put a cross through two numbers on the card and hand it in to the teacher.

The teacher has a bag containing six balls numbered 1 to 6:

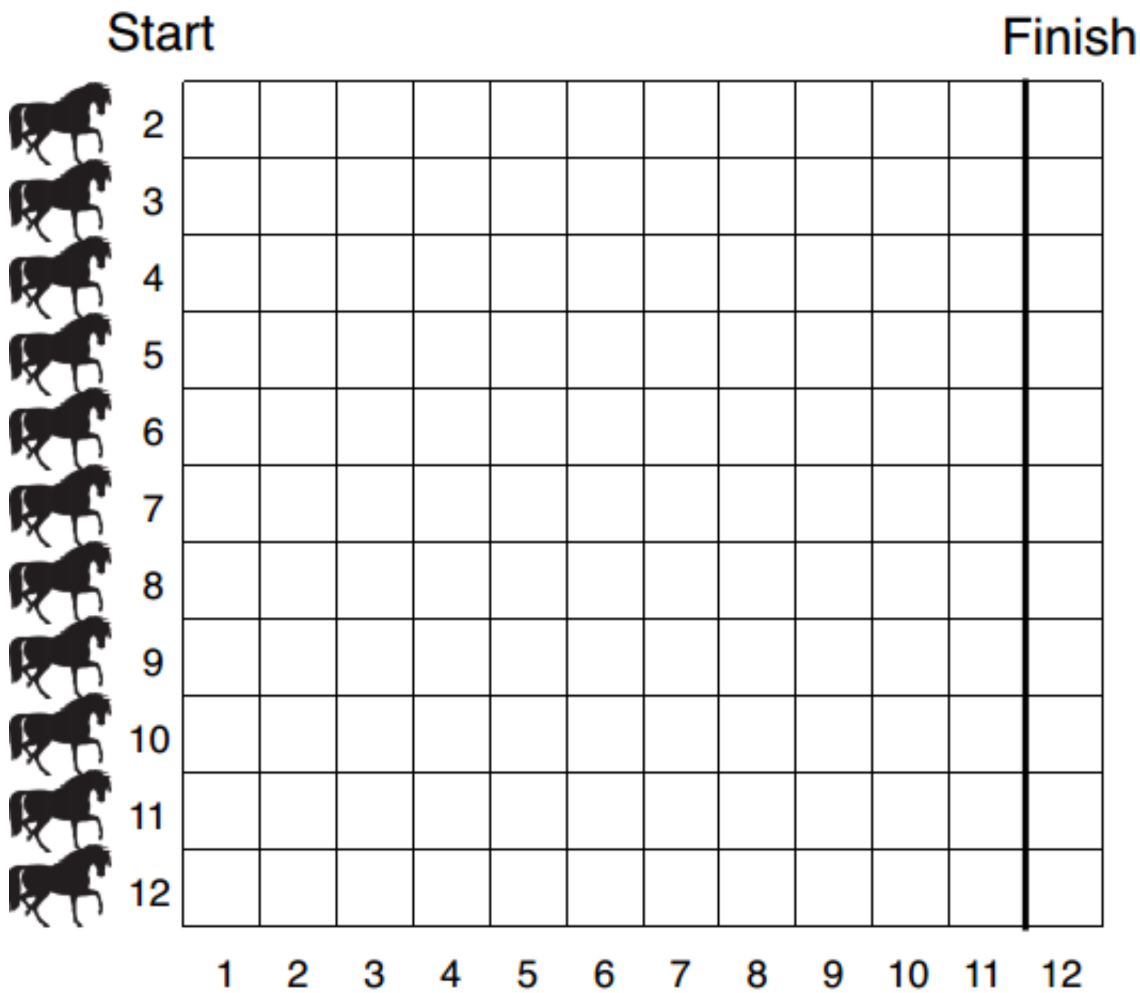


When all the number cards have been handed in the teacher draws out two balls from the bag.

Every student who has chosen the same two numbers shown on the balls wins a prize.

If there are 30 students in Karl's class, how many students are likely to win a prize? Describe your reasoning.

The Horse Race



Race Results Sheet

Predicted finish positions

Start | Finish

2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												
	1	2	3	4	5	6	7	8	9	10	11	12

Explain why you predict this

Game 1 **Actual finish positions**

Start | Finish

2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												
	1	2	3	4	5	6	7	8	9	10	11	12

Game 2 **Actual finish positions**

Start | Finish

2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												
	1	2	3	4	5	6	7	8	9	10	11	12

Playing the Game

- Predict the outcome of the race before starting it.
- Show your prediction on the *Race Results Sheet*, by drawing the positions of all the horses as the first one crosses the finishing line, using the first diagram.
- Play the game using the board and counters. Record the finishing positions of each horse when one reaches the finish. Now run the race for a second time.
- Reflect on what happened and try to explain this at the bottom of the sheet.

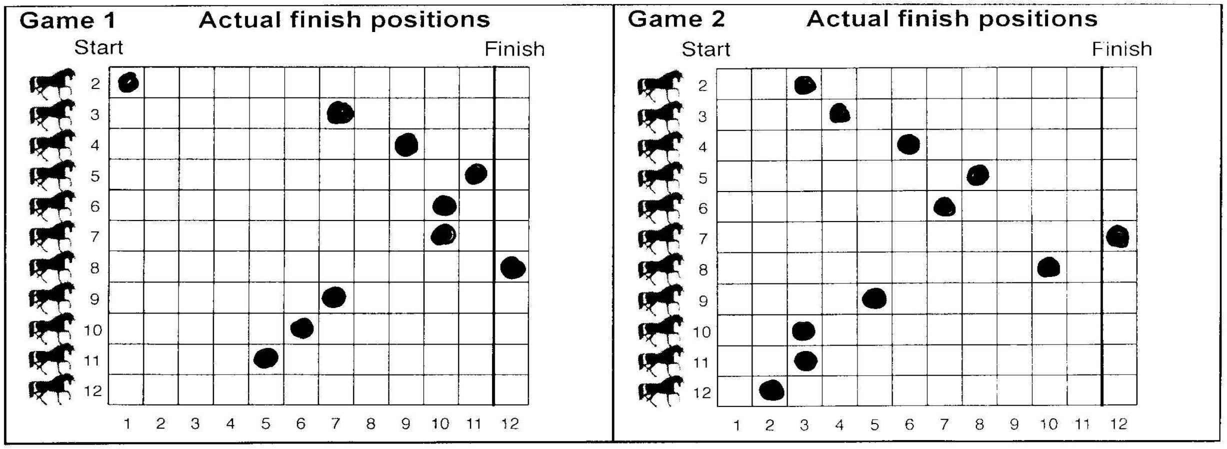
Finishing Places of Horses

Horse number	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							

Looking at Reasoning

- How has this student tackled the problem of explaining the results?
- What has each student done well?
- What errors has each student made?
  Can you correct these errors?
- How could each student's work be improved and extended?

Tom's Reasoning



I think the horses in the middle are most likely to win.

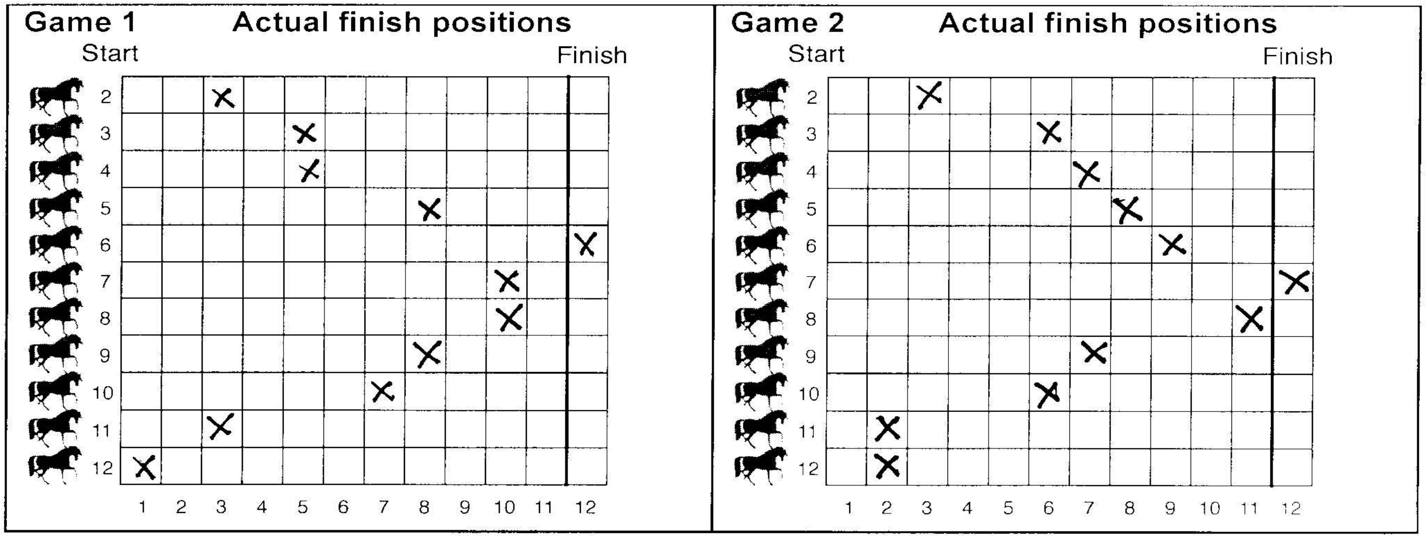
The dice rolls are:

1+1 = 2	2+2 = 4	3+3 = 6	4+4 = 8	5+5 = 10	6+6 = 12
1+2 = 3	2+3 = 5	3+4 = 7	4+5 = 9	5+6 = 11	
1+3 = 4	2+4 = 6	3+5 = 8	4+6 = 10		
1+4 = 5	2+5 = 7	3+6 = 9			
1+5 = 6	2+6 = 8				
1+6 = 7					

Totals	2	3	4	5	6	7	8	9	10	11	12	
ways	1	1	2	2	3	3	3	2	2	1	1	= 21

So horses 6, 7, 8 are the most likely to win and that is what happened.

Jessica's Reasoning



These are the outcomes:

(there are 36)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$7 \rightarrow \frac{6}{36} = \frac{1}{6}$ so 7 will win one sixth of the time.

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
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with
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We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org