PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Estimating Volume: *The Money Munchers*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Estimating Volume: *The Money Munchers*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Model a situation.
- Make sensible, realistic assumptions and estimates.
- Use assumptions and estimates to create a chain of reasoning, in order to solve a practical problem.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 3, and 4:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the work of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics:*

7.G: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

INTRODUCTION

In this unit, students choose and use mathematics to model a problem situation:

- Before the lesson, students attempt *The Money Munchers* task individually. You then review their solutions and create questions for them to answer in order to improve their work.
- At the start of the lesson, students work individually, answering your questions about the task.
- Next, students work collaboratively in small groups. Their task is to produce a better solution to *The Money Munchers* problem than they did individually. Then, in the same small groups, students analyze responses to *The Money Munchers* task written by students in another class.
- In a whole-class discussion, students compare and evaluate the solution methods they have seen and used.
- In a follow-up lesson, students review their individual and joint solutions and write about what they have learned.

MATERIALS REQUIRED

- Each student will need a copy of the task sheet *The Money Munchers* and the *How Did You Work*? questionnaire.
- Each small group of students will need a new copy of the task sheet *The Money Munchers,* a sheet of poster paper, and a copy of the *Sample Responses to Discuss*.
- Provide calculators for students who choose to use them.
- There is a projector resource to support whole-class discussions and to help introduce activities.

TIME NEEDED

15 minutes before the lesson, a 1-hour lesson, and 10 minutes in a follow-up lesson. Timings given are approximate and will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *The Money Munchers* (15 minutes)

Have the students do this task, in class or for homework, a day or more before the lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

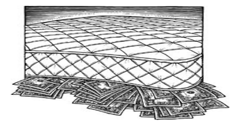
Give each student a copy of *The Money Munchers*. Introduce the task briefly, helping the class to understand the problem and its context:

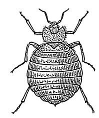
> *Today you're going to work on a Fermi problem.*
>
> *Fermi was a twentieth-century Italian physicist. He loved setting estimation problems for his colleagues and students.*
>
> *A Fermi problem is a question for which you produce a rough, but sensible estimate, without knowing exactly all the measures involved.*

The Money Munchers

Emily doesn't trust banks with her money.

She has stored $24,400 in one-dollar bills under her mattress.





Emily's daughter tries to persuade her to take her money to the bank.

"Just think of all those little bedbugs munching through your money, mom."

The thought of millions of bedbugs eating her money is too much for Emily.

She decides to take the money to the bank.

1. Emily removes the money from under the mattress.
   By how many inches will the mattress be lowered?

The Money Munchers (continued)

2. Emily is taking the $24,400 to the bank in a suitcase.
   The suitcase measures 14" wide, 19" long and 7" deep.
   Will Emily have enough space for all the money in her suitcase?

Ask students to read the scenario about Emily and the money carefully. If students in your class have literacy issues, it may help to read this information aloud.

Now explain what you are asking students to do.

> *I want you to work individually for 15 minutes.*
>
> *Your work on this task will help me see how good you are at estimating quantities like length and using your estimates to calculate approximate solutions to problems.*

It is important that, as far as possible, students answer the questions without assistance. If students are struggling to get started, ask questions that help them understand what they are being asked to do, but do not do the problem for them. The first few questions in the *Common issues* table on page T-4 were found to be helpful in trials of this lesson.

Students should not worry too much if they cannot understand or do everything, because there will be a lesson using the same task, which should help them. Explain to students that by the end of the next lesson they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their problem solving strategies.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on page T-4.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues:	Suggested questions and prompts:
Does not identify missing information For example: The student states they cannot calculate an answer because the size of a bed/dollar bill is unknown.	• In many problems, you have to find or estimate the information you need to solve the problem. How long do you think a dollar bill is? How could you make a good estimate?
Does not identify or justify the assumptions that shape the calculation strategy For example: The student does not say why they have assumed there can be just ten piles of dollar bills. Or: The student assumes that orientation and packing of the notes into the available space makes no difference.	• You have assumed that … Explain why you have made this assumption. • Is it reasonable to assume that …?
Makes poor estimates For example: The student estimates the length of a bed as 5 feet, or its width as 2 feet. Or: The student estimates that a pile of ten one-dollar bills is an inch high.	• How tall are you? How does this help you estimate the length of a bed? • Find a book. How many pages are there? How many sheets of paper are used to make the pages? How high is that pile of pages? How does this help you estimate the height of a pile of one hundred $1 bills?
Provides a poor explanation For example: The student writes calculations without showing which quantities the numbers refer to. Or: The student makes estimates but does not justify them.	• Your solution is difficult to follow. What does the number […] stand for in this calculation? Explain what each number is in turn. • Imagine you have to explain this solution to another student. How could you make your solution easy to understand?
Makes inappropriate calculations and errors For example: The student multiplies rather than divides to find the number of dollar lengths that fit into the length of the case. Or: The student makes an arithmetic error.	• Explain what this calculation is for. • You are calculating the number of dollar lengths that fit into the length of the suitcase. Which operation do you need to use: add, subtract, multiply or divide? Why? • How can you check your calculations to make sure they are accurate?
Provides a complete and adequate solution	• How accurate are your assumptions? • Estimate how accurate your answer is, taking into account the accuracy of your assumptions. • Find another way of answering this problem, with different plausible estimates and assumptions. • Make up a new Fermi problem of your own and answer it.

SUGGESTED LESSON OUTLINE

Individual review (10 minutes)

Return to students their solutions to *The Money Munchers* task.

If you have chosen not to write questions on individual student papers, display your list of questions on the board.

> *[Last lesson] you worked on The Money Munchers problem. Do you recall what the task was about?*
>
> *I read your solutions and have some questions I'd like you to think about.*
>
> *Work individually for 10 minutes, answering my questions to improve your work.*

Students can either make a note of their responses to the questions on the back of their task sheet or you may prefer to give them some plain paper to work on.

Collaborative small-group work (20 minutes)

Organize students into groups of two or three. Give each group a new copy of *The Money Munchers* task and a large sheet of poster paper.

> *I'd like you to put your solutions to one side now.*
>
> *Start afresh on the same Fermi problem. I want you to work together in your groups, to produce a better solution together than you each did individually.*
>
> *I'd like you to make your solution into a poster. Make sure you write down all your reasoning and label everything clearly.*
>
> *To begin, I'd like you to take turns in your groups to share your assumptions. Think about when you were working alone. What extra information did you need to identify, to solve this problem?*

You have two roles while students are working: to find out about student methods and to support student problem solving.

Find out about student methods

Listen and watch to find out about the assumptions students make about the context and about the quantities they identify in their rudimentary mathematical models.

Note students' estimates of quantities such as the length of a bed and the height of a dollar bill. Do students explain and justify their estimations? If so, do they write their reasons down?

Notice whether students are naming and writing down the quantities with which they are working and if they draw diagrams and label them.

Do they notice when different units of measurement arise, such as feet and inches in lengths, and if so, do they convert between them?

Do they make sense of packing dollars into the shape of the mattress, or the dimensions of the case, or do they work with area/volume only?

Do they justify their calculation methods to each other?

Do they check their solutions to see if they make sense in the context of the problem?

Support student problem solving

Try not to prompt students into using a particular problem solving method and try not to point out the difficulties with their chosen methods to them. Instead, ask questions to prompt students to justify and evaluate their own solution strategies.

The questions in the *Common issues* table were found to be useful in trials of this lesson.

Prompt students to write their solutions so that other groups can understand what is written.

If any group finishes their solution, ask them to consider the accuracy of their solution and then to develop a solution using a different method.

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

Give a copy of the three *Sample Responses to Discuss* to each small group of students. Ask students to read the solutions and to answer the questions together.

> *Looking at your posters, I can see you have used a range of different methods to solve this problem.*
>
> *I'm giving you some work produced by students from another class on this same problem.*
>
> *I would like you to answer these questions about each student's work:*
>
> > *Read through the solution and make sense of how the student is solving the problem.*
> > *Figure out what assumptions the student makes.*
> > *Are the estimates reasonable, or way off the mark?*
> > *Figure out how the student calculates an answer.*
> > *Then decide what is good about the solution and how you might improve it.*

The instructions for this task are reproduced on Slide P-2 *Analyzing Sample Responses to Discuss*.

During small-group work, support students as they work.

If students find it difficult to get started, suggest they read the solution aloud, slowly.

> *Stop after one sentence and check everyone understands which numbers stand for which quantities.*
>
> *Explain what the calculation is. What assumptions are being made?*
>
> *Why is Mattie calculating that at this point? How does it help him?*

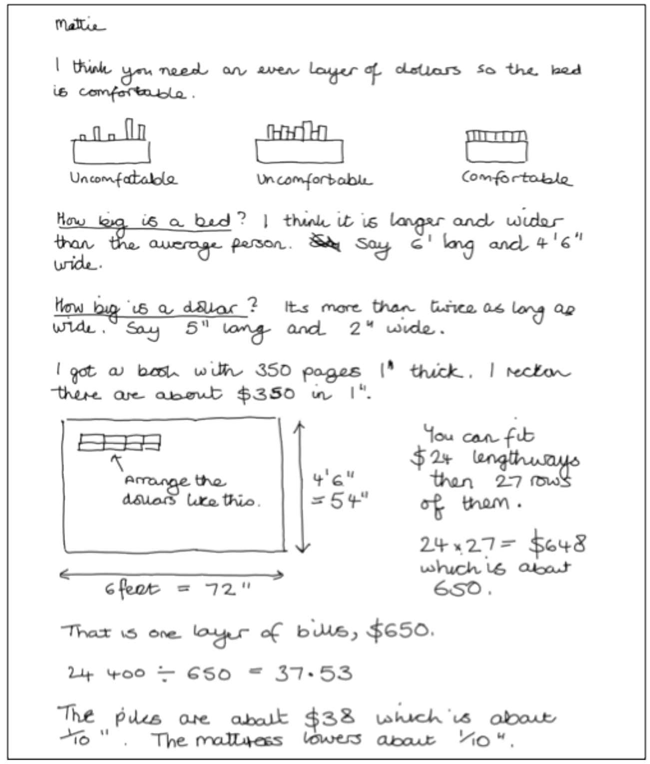
Encourage students to write their reasoning in full.

The *Sample Responses to Discuss* show three methods of solving the first part of *The Money Munchers* task. The three solutions involve the use of different estimates and approximation strategies. Some students explain their reasoning and calculations more clearly than others. A sense of appropriate accuracy is an important part of estimation.

Mattie has made explicit the assumption that, in order to make the mattress comfortable, the money needs to be stacked in piles of equal height.

Mattie explains some of his estimates. He makes a reasonable estimate for the size of a bed. He underestimates the size of a dollar by about 1inch in length and over ½ an inch in width. He uses a book to estimate the thickness of a stack of dollars. The pages of a book are double-sided, so the number of sheets of paper used would be 350 ÷ 2 = 175. He does not notice that used bank notes make thicker piles than new paper.

Mattie thinks about how a single layer of dollars would fit onto the mattress. He divides the length of the bed by the length of a dollar and the width of the bed by the width of a dollar. He makes a mistake: you could fit 14.4 lengths of 5 inches into 72 inches, not 24.



Mattie

I think you need an even layer of dollars so the bed is comfortable.

Uncomfatable Uncomfortable Comfortable

How big is a bed? I think it is longer and wider than the average person. Say 6' long and 4'6" wide.

How big is a dollar? Its more than twice as long as wide. Say 5" long and 2" wide.

I got a book with 350 pages 1" thick. I reckon there are about $350 in 1".

Arrange the dollars like this.

4'6" = 54"

6feet = 72"

You can fit $24 lengthways then 27 rows of them.

24×27 = $648 which is about 650.

That is one layer of bills, $650.

24 400 ÷ 650 = 37.53

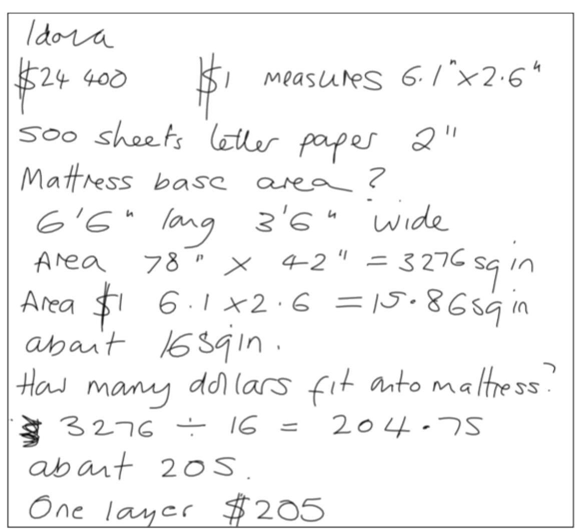
The piles are about $38 which is about 1/10". The mattress lowers about 1/10".

He then rounds the number of dollars in a single layer to the nearest ten. He divides the total number of dollars by the number of dollars in a layer, but does not explain this, or say why. Mattie rounds $37.53 to $38 and states how high each pile of dollars would be. Apart from the size of a dollar bill, Mattie's estimates are quite reasonable and his assumptions legitimate for the context. The strength of his solution method is that he makes assumptions explicit and explains his estimates. His calculations are appropriate. His first diagrams help illustrate his assumptions clearly. His second diagram helps show the orientation of the dollar bills on the mattress base and shows the dimensions of the bed clearly.

To improve his work Mattie could correct his estimate of dollar size and correct the arithmetic error about the number of dollar lengths that would fit into the length of the mattress. With these errors corrected, he would find fewer dollars in a layer and the height of each pile would be increased.

Idora has not made explicit her assumption that the dollars are to be spread evenly, in equal piles. Nor does she explain her assumption that she needs only to think about how many times one area fits into another and that she does not need to consider how to pack the dollars onto the shape of the mattress.

Idora has measured, rather than estimated, the size of a dollar. However, measuring correct to one decimal place is inappropriately accurate, given the other figures with which she works.



Idora

$24 400 $1 measures 6.1"×2.6"

500 sheets letter paper 2"

Mattress base area?

6'6" long 3'6" wide

Area 78" × 42" = 3276 sq in

Area $1 6.1×2.6 = 15.86 sq in

about 16 sq in.

How many dollars fit onto mattress?

3276 ÷ 16 = 204.75

about 205.

One layer $205

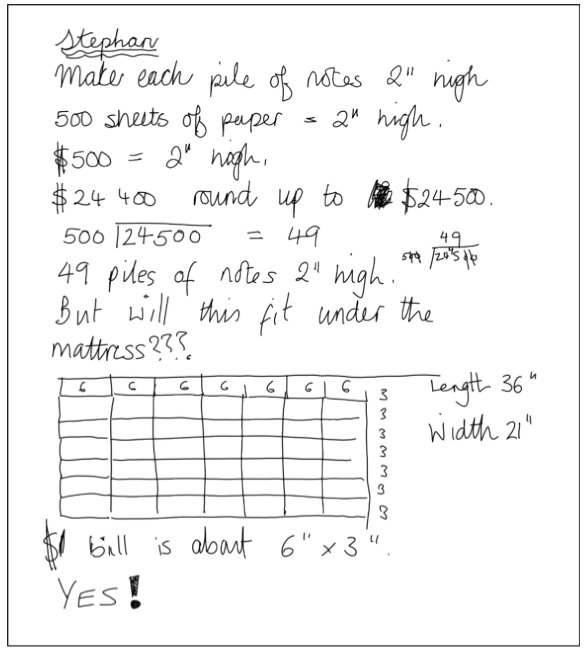
She makes a reasonable estimate of the height of a pile of dollars, using the height of a ream of paper, a sensible thing to do because it is an easy method. She estimates the size of a bed fairly accurately, but does not explain how she made that estimate.

Idora finds the area of the mattress and the area of a dollar bill. She divides one area by the other to find the number of dollars that fit in one layer. She does not complete her estimate.

Idora's calculation method is quite simple and quick.

To improve her solution, Idora could explain her estimates and assumptions more fully. She also needs to complete a solution to the question. At this stage, we only know how many bills she thinks fit on the mattress in a single layer. A diagram might also help a reader understand her method.

Like Mattie, Idora does not consider the uncertainty of her estimate.



Stephan makes the assumption that bills are to be stacked in piles, but does not say why. He also makes the assumption that it does not matter whether the bills are spread evenly under the mattress: he seems to think that stacking all the bills at one end is acceptable.

He estimates the size of a dollar reasonably, at 6 inches by 3 inches. He seems to estimate the size of a bed, but does not make his reasoning explicit.

Stephan works backwards towards a solution. He uses the height of a stack of paper to estimate that a 2-inch stack of bills is $500. This part of his solution is clearly explained, although inaccurate.

Stephan rounds $24,400 to the nearest $500, which is sensible, as it makes the calculation simple.

He calculates how many $500 piles there are in $24,500 and finds there would be 49 piles. He then finds easy factors of 49, 7 × 7 and uses these as the dimensions for the number of rows and columns of bills. He calculates that the 49 $500 stacks would measure 36 inches by 21 inches in total and says this would fit under a mattress, but he does not explain how he knows how big a mattress is. He draws a diagram to show how the stacks fit together. He does not seem concerned that the bed would be very bumpy.

The strength of Stephan's solution is its simplicity: working backwards from something easy to figure out a more complex solution. This is often a good problem solving strategy. His diagram helps show how the bills are arranged.

To improve his solution, Stephan should make an explicit estimate of bed size and make his assumptions about how to stack the bills under the bed explicit. He could then refine his approximation to make better use of the whole bed size, by, say, halving the height of the piles to double the number of piles.

Whole-class discussion: comparing different approaches (10 minutes)

Organize a whole-class discussion of the *The Money Munchers* task. Focus the discussion on the methods students have seen and used during the lesson, rather than discussing who has the 'best' or a 'correct' solution.

In particular, ask students to discuss the strengths and weaknesses of the different approximation methods seen in the *Sample Responses to Discuss*: Mattie's shape packing approach, Idora's division of one area by another, or Stephan's approach of working backwards.

Is there only one reasonable estimate for the width/length of a bed?

Can we say which of the sample estimates is best? Which are good enough? How would we decide?

Is it important to make the bed comfortable by providing equal layers?

How could you improve Mattie's [Idora's / Stephan's] solution?

Ask students to contribute, with reference to their own posters. Try to avoid making evaluative comments yourself. Instead, encourage students to respond to other students' explanations.

Try to help students understand that different adequate solutions can arise from very different, but still reasonable, assumptions.

If you have time, begin to address the issue of accuracy in estimation. None of the respondents make a serious attempt to estimate the accuracy of their answers, taking into account the uncertainties in their assumptions.

The thickness of a pile of dollar bills, in particular, is difficult to estimate. It might be much greater than the same number of pages in a book, or sheets in a new pack of paper. Gaps between piles are another source of inaccuracy. Overall their estimates, when done correctly, are probably accurate to ±20%. This is fine for the mattress, but might cause problems with the suitcase.

Follow-up lesson: review solutions to *The Money Munchers* (10 minutes)

Give out the *How Did You Work?* questionnaire. Ask students to re-read their original solution and think about their joint response to the task before, completing the questionnaire.

Read through your original work and think about your joint solution to the task.

What have you learned during the lesson?

On your own, answer the review questions as carefully as you can.

The questionnaire should help students review their progress and write about what they have learned during the lesson.

Some teachers give this task for homework.

SOLUTIONS

Assessment task: *The Money Munchers*

1. When solving the problem of how many inches the mattress will be lowered, students need to make estimates for the length and width of the mattress and for the dimensions (including the height) of a $1 bill. These approximations should be stated clearly and explained and justified in the solution.

There are also a number of assumptions that students will need to make about how the dollar bills are positioned on the mattress. For example, the orientation of the dollar bills and the evenness of the piles may need to be considered.

A possible solution may look similar to this:

Dimensions of a double bed are taken to be 54 inches by 75 inches.

Dimensions of a dollar bill approximately 6 inches by 2.5 inches and 0.005 inches high.

9 $1 dollar bills will fit lengthways along the width of the mattress with 30 rows filling the length of the mattress.

So one layer is 9 × 30 = 270 dollar bills.

There are 24,400 dollar bills altogether so this works out at 24,400 ÷ 270 ≈ 91 layers.

91 layers × 0.005 = 0.455 inches.

The mattress will therefore be lowered by approximately half an inch.

When completing a solution, a diagram showing the layout of the dollar bills on the mattress may be a useful way of explaining the method used to calculate how many inches the mattress will be lowered.

2. To figure out whether the money will fit into the suitcase, students need to explain, or show in a diagram, how they plan to pack the notes into the available volume. They then need to work with their estimates for the size of a dollar to calculate whether the total amount can be packed into the available space. Two sample solutions are given below.

Solution 1:

My estimate for the size of a dollar: 6 inches by 2.5 inches, because its length is wider than a hand, not so long as one of my feet and it's more than twice as long as it is wide.

If I lie the case flat, the base is 19 inches long and 14 inches wide. I'm going to put piles of dollars over the base.

19 ÷ 6 = 3, remainder 1. I can fit 3 dollars lengthways.

14 ÷ 2.5 = 5, remainder 1.5. I can fit 5 dollars widthways.

This will work even if I've underestimated a bit, because of the remainders.

I get a layer of 3 x 5 = $15 over the base of the case.

24,400 ÷ 15 = 1626.666 = $1,627 (nearest dollar). I get $1,627 in each pile.

There are about $250 in one inch. I got this from measuring 500 sheets of paper, which is 2 inches.

So I can figure out the approximate height of a stack of $1,627. It is 1627 ÷ 250 = 6.5 inches.

The case is 7 inches deep so the money fits.

This solution would be improved were the student to notice that used notes do not fit together as neatly as new paper.

Solution 2:

The volume of the case is $14 \times 19 \times 7 = 1{,}862$ cubic inches.

A dollar is as long as my hand in length and narrower than my hand in width. I estimate a dollar measures about 6 inches by 3 inches.

The height of a pile of \$100 is more than ½ an inch, as this is the height of a book with 100 pages. So \$1 > $\frac{1}{200}$ inch.

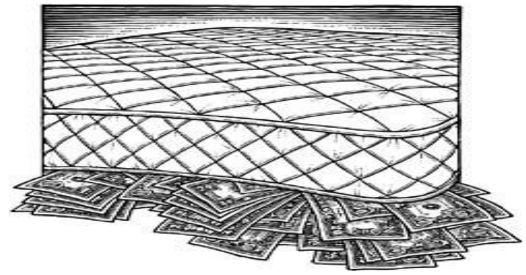
So the volume of a dollar is $6 \times 3 \times \frac{1}{200} > \frac{18}{200}$.

So the volume of \$24,400 is greater than $\frac{24400 \times 18}{200} = \frac{244 \times 18}{2} = 122 \times 18 = 2196$ cubic inches.

Even if you packed it carefully, this volume of money would not fit into this suitcase.

In student solutions to the problem check to see (a) whether their assumptions, estimations and calculation strategies are made clear and explained, (b) whether estimations and rounding are appropriate, (c) whether the solution is reasonable and checked. A really strong solution could also include comments about the margins of error, decisions for rounding, and whether there are ways of improving the solution.

The Money Munchers



Emily doesn't trust banks with her money.

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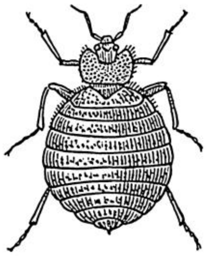


Emily's daughter tries to persuade her to take her money to the bank.

"Just think of all those little bedbugs munching through your money, mom."

The thought of millions of bedbugs eating her money is too much for Emily.

She decides to take the money to the bank.

1. Emily removes the money from under the mattress.

   By how many inches will the mattress be lowered?

The Money Munchers (continued)

2. Emily is taking the $24,400 to the bank in a suitcase.

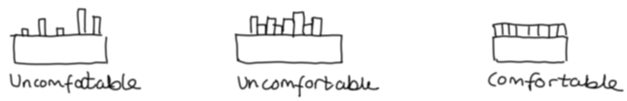
   The suitcase measures 14” wide, 19” long and 7” deep.

   Will Emily have enough space for all the money in her suitcase?

Sample Responses to Discuss: Mattie

Mattie

I think you need an even layer of dollars so the bed is comfortable.



How big is a bed? I think it is longer and wider than the average person. ~~Say~~ Say 6' long and 4'6" wide.

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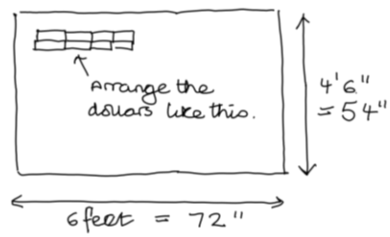
I got a book with 350 pages 1" thick. I reckon there are about \$350 in 1".



You can fit \$24 lengthways then 27 rows of them.

24 × 27 = \$648 which is about 650.

That is one layer of bills, \$650.

24 400 ÷ 650 = 37.53

The piles are about \$38 which is about 1/10". The mattress lowers about 1/10".

What assumptions does Mattie make?

Are his estimates reasonable?

How does Mattie calculate his approximate solution?

What are the good qualities of his work?

How would you improve his solution?

Sample Responses to Discuss: Idora

Idora

\$24 400   \$1 measures 6.1" × 2.6"

500 sheets letter paper 2"

Mattress base area?

6'6" long 3'6" wide

Area 78" × 42" = 3276 sq in

Area \$1 6.1 × 2.6 = 15.86 sq in

about 16 sq in.

How many dollars fit onto mattress?

3276 ÷ 16 = 204.75

about 205.

One layer \$205

What assumptions does Idora make?

Are her estimates reasonable?

How does Idora calculate her approximate solution?

What are the good qualities of her work?

How would you improve her solution?

Sample Responses to Discuss: Stephan

Stephan

Make each pile of notes 2" high

500 sheets of paper = 2" high.

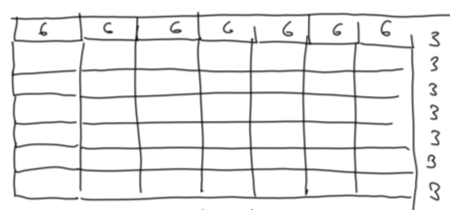
\$500 = 2" high.

\$24 400 round up to \$24 500.

500) 24500 = 49

49 piles of notes 2" high.

But will this fit under the mattress???



Length 36"

Width 21"

\$1 bill is about 6" × 3".

YES!

What are Stephan's assumptions?

Are his estimates reasonable?

How does he calculate his approximate solution?

What are the good qualities of Stephan's work?

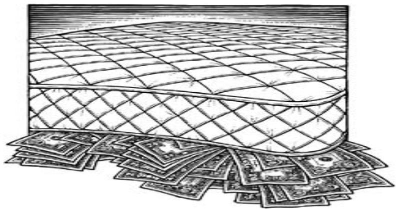
How would you improve his solution?

How Did You Work?

Complete the boxes and sentences that apply to your work:

1. The amount that I estimated the mattress would be lowered by in my individual solution was: ______ inches.
2. The amount that we estimated the mattress would be lowered by in our group solution was: ______ inches.
3. When producing a group solution we improved our work by:
4. In my individual solution I concluded that there **would / would not** be enough space in the suitcase. *(Delete as appropriate)*
5. When producing a joint solution we came to the **same /different** conclusion because: *(Delete as appropriate)*
6. Looking at the sample student work was helpful because:
7. My advice to a student new to this task would be:

The Money Munchers



Emily doesn't trust banks with her money.

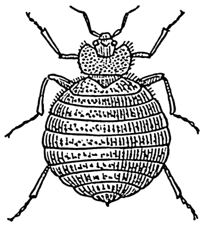
She has stored $24,400 in one-dollar bills under her mattress.

Emily's daughter tries to persuade her to take her money to the bank.

"Just think of all those little bedbugs munching through your money, mom."

The thought of millions of bedbugs eating her money is too much for Emily.

She decides to take the money to the bank.



1. Emily removes the money from under the mattress.
   By how many inches will the mattress be lowered?
2. Emily is taking the $24,400 to the bank in a suitcase.
   The suitcase measures 14" wide, 19" long and 7" deep.
   Will Emily have enough space for all the money in her suitcase?

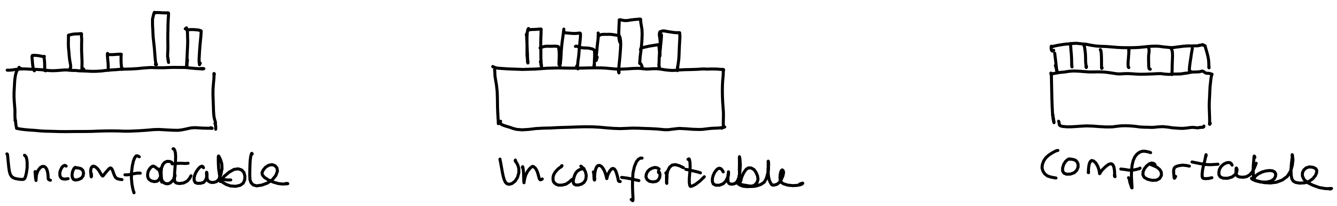
Analyzing Sample Responses to Discuss

- Read through the solution and make sense of how the student is solving the problem.
- Figure out what assumptions the student makes.
- Are the estimates reasonable, or way off the mark?
- Figure out how the student calculates an answer.
- Decide what is good about the solution and how you might improve it.

Sample Responses to Discuss: Mattie (1)

Mattie

I think you need an even layer of dollars so the bed is comfortable.

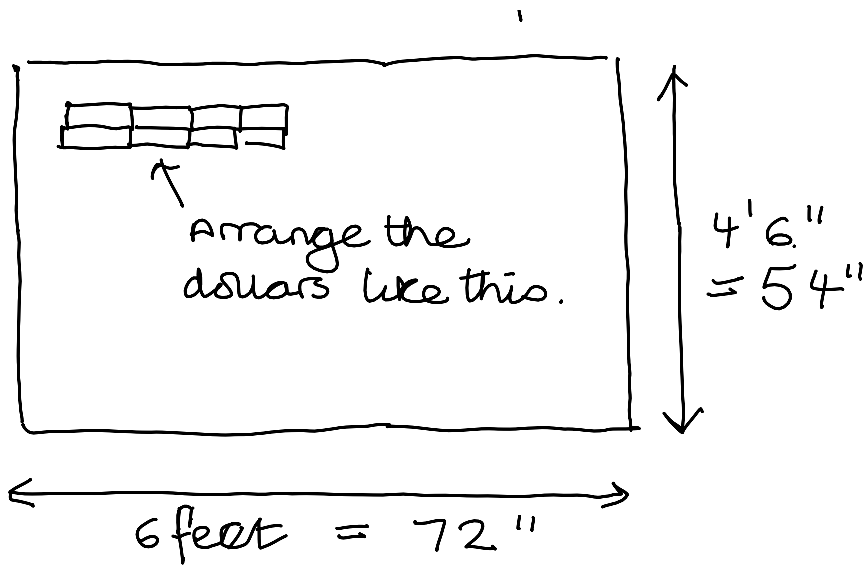


How big is a bed? I think it is longer and wider than the average person. ~~Say~~ Say 6' long and 4'6" wide.

How big is a dollar? Its more than twice as long as wide. Say 5" long and 2" wide.

I got a book with 350 pages 1" thick. I reckon there are about $350 in 1".

Sample Responses to Discuss: Mattie (2)



You can fit $24 lengthways then 27 rows of them.

24 × 27 = $648 which is about 650.

That is one layer of bills, $650.

24 400 ÷ 650 = 37.53

The piles are about $38 which is about 1/10". The mattress lowers about 1/10".

Sample Responses to Discuss: Idora

Idora

\$24 400 \$1 measures 6.1" x 2.6"

500 sheets letter paper 2"

Mattress base area?

6'6" long 3'6" wide

Area 78" x 42" = 3276 sq in

Area \$1 6.1 x 2.6 = 15.86 sq in

about 16 sq in.

How many dollars fit onto mattress?

~~\$~~ 3276 ÷ 16 = 204.75

about 205.

One layer \$205

Sample Responses to Discuss: Stephan

Stephan

Make each pile of notes 2" high

500 sheets of paper = 2" high.

$500 = 2" high.

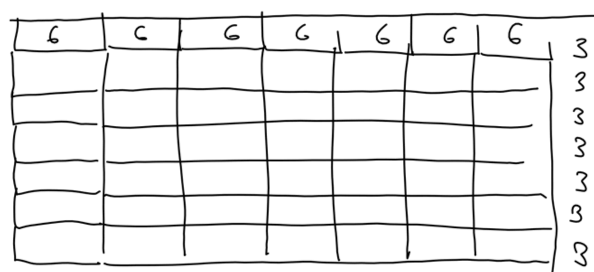
$24 400 round up to $24 500.

$500 \overline{)24500} = 49$

49 piles of notes 2" high.

$500 \overline{)24500}$ 49

But will this fit under the mattress???



Length 36"

Width 21"

$1 bill is about 6" × 3".

YES!

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org