PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Designing 3D Products: *Candy Cartons*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Designing 3D Products: *Candy Cartons*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Select appropriate mathematical methods to use for an unstructured problem.
- Interpret a problem situation, identifying constraints and variables and specify assumptions.
- Work with 2- and 3-dimensional shapes to solve a problem involving capacity and surface area.
- Communicate their reasoning clearly.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics,* with a particular emphasis on Practices 1, 3, 4, and 5:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6.G: Solve real-world and mathematical problems involving area, surface area, and volume.

INTRODUCTION

This activity will take two lessons. The lessons are structured in the following way:

- Before the first lesson, students tackle the problem individually. You review their work and write questions to help students improve their solutions.
- At the beginning of the first lesson, students respond to your questions. They then work collaboratively in pairs to produce a better solution, using their designs to construct a carton.
- To launch the second lesson there is a whole-class discussion. Then in small groups students evaluate and comment on sample solutions, followed by a whole-class discussion about the work.
- Finally, in a follow-up lesson students review and evaluate their work on the problem.

MATERIALS REQUIRED

- Each student will need a mini-whiteboard, a pen, an eraser, a copy of the task sheet, *Design a Candy Carton*, and a copy of the *Design Review* questionnaire.
- Each small group of students will need a large sheet of paper for making a poster, a sheet of light, letter-sized cardboard, scissors, a glue stick, highlighter pens, and a copy of each of the *Sample Responses to Discuss*.
- Provide calculators, rules, protractors, pencils, grid and plain papers, compasses, and cylindrical counters (such as checkers pieces) for students who wish to use them.
- You may also want to bring a couple of cartons to the class.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

20 minutes before the lesson, two one-hour lessons, and 15 minutes in a follow-up lesson. Timings are approximate; exact timings will depend on the needs of your students.

BEFORE THE LESSONS

Assessment task: *Design a Candy Carton* (20 minutes)

Have students work on the assessment task a few days before the formative assessment lessons. This will give you an opportunity to assess the work, find out about students' different problem-solving approaches, and notice any difficulties students experience with the task. You should then be able to target your help more effectively in the lessons that follow.

Give each student a copy of the assessment task *Design a Candy Carton*.[1]

Introduce the task and help students to understand what you are asking them to do.

Design a Candy Carton

A candy company wants you to design a carton that will contain 18 candies.

Your design must take into account the following:

- The candies are each 1 cm deep and 2 cm in diameter.
- The carton must be made from a net that fits on a single sheet of letter-sized cardboard.
- The design should require as little cutting as possible.
- The sides of the carton will be fixed together using glue flaps.
  Show where these will be on your net.

Produce two possible designs for nets of the candy carton.

Compare your two designs.

Record in writing which carton you think is best and why.

Today you are going to design a carton to contain some candy.

There are many ways to tackle the problem and there is more than one 'right answer'. You are to decide on the math to use.

Read through the task carefully and answer the questions.
Make sure you record all your reasoning really carefully – explain all your decisions.

I have more paper and other materials you might wish to use.

Make sure your carton works practically, but also be creative!

Slides P-1 and P-2 of the projector resource may help to support this discussion.

If available, show students an example of an ordinary cardboard box to demonstrate how tabs are used to glue sides together. Do not show them the full net as this might direct them to imitate this.

It is important that, as far as possible, students are allowed to answer the questions without assistance. Some students may find it difficult to get started: be aware that if you offer help too quickly, students will merely do what you say and will not think for themselves. If, after several minutes, students are still struggling, try to help them understand what is required. The first few questions on the *Common issues* table may be useful.

Students who sit together often produce similar answers so that when they come to compare their work, they often have little to discuss. For this reason we suggest that, when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions because students then have varied approaches to discuss.

When all students have made a reasonable attempt at the task, reassure them that they will have time to revisit and revise their solutions later.

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[1] Note that the task specifies particular values for the dimensions of the candy. A few teachers have wanted to use real candy in the lesson and adapt the measurements accordingly. While this may seem a good idea, the lesson materials and sample work that follow will need considerable adaptation.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. Research shows that this will be counter-productive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the start of the first lesson.

Common issues	Suggested questions and prompts
Has difficulty getting started For example: The student writes or draws little. Or: The student draws a net of a right rectangular prism with dimensions not relevant to the problem. Or: The student says that not enough information is given to solve the problem.	• What are you trying to do? Explain the task in your own words. • What math have you worked on before that might help you? • What do you know? What other information do you need? • If information is not given, make your own decision about how to work and explain your decision. • What shapes of carton could you make? • How big does the carton need to be to fit 18 candies? • Try sketching the carton in 3-D. Now use this diagram to draw your net.
Creates only one design	• Can you find a different way of packing the candies together closely? • Can you find a better design?
Does not justify assumptions about packing the candies For example: The student assumes the carton has to be bigger in volume than the volume of all the candies and does not explain why they ignore packing issues. Or: The student assumes the volume of the carton is always the same as the volume of all the candies.	• Use math to justify your net. • Which way of packing the candies have you chosen? Why? • What is the smallest volume of carton needed? For this volume, how can the candies be placed in the carton? • Can you create a carton that uses less card?
Makes errors in drawing the net For example: The net does not have enough faces. Or: The net would not fold to form a closed box. Or: The net is not drawn to the correct size.	• Imagine folding up your net. Which shape is the base? Which shapes are the sides? Which shape is the top? • Does your net fold to make a closed box? • What are the planned dimensions of your carton? Is your net a scale drawing or is it the actual size of the carton?
Does not include flaps for gluing the carton together	• How will you glue your carton closed? • Which edges meet?
Does not use the measurements correctly For example: The student does not take into account the differences in height and diameter.	• What is the diameter of one candy in centimeters? What is the height of one candy in centimeters? • Figure out the dimensions of your carton in centimeters. • How can you show your carton holds 18 candies?

Common issues	Suggested questions and prompts
Carton design requires extra construction For example: The net is in more than one piece.	• Read the task again. Can you make a net that needs less gluing?
Provides inadequate justification For example: The student does not explain their assumptions. Or: The student provides irrelevant justification or lacks the appropriate mathematics. Or: The student does not refer back to the problem situation in their justification.	• What kind of carton do you think the candy company would like? Why? • What features would make one carton better than another? Why? • Can you find a carton that requires less cardboard?
Provides a complete, well-justified response	• Can you produce a different solution by changing some of your assumptions?

SUGGESTED LESSON OUTLINE: DAY 1

Individual work (10 minutes)

Give students back their papers, along with a mini-whiteboard, a pen, and an eraser. Remind students of their work on *Design a Candy Carton*.

Do you recall the work you did on designing a carton for candy?

I have your solutions here. I have read through them and have written some questions to help you improve your work.

Write your responses to my questions on your mini-whiteboards.

If you have chosen to write questions on the board rather than individual student solutions, display those questions now. Explain to students how to use the questions on the board:

Choose one or two of the questions I've written that apply to your work.
Answer those questions on your mini-whiteboard.

The purpose of this is to re-engage students with the task and to encourage them individually to begin to critique their own work.

Collaborative small-group work: *Designing a Candy Carton* (15 minutes)

Bring the class back together and explain the next activity:

Now I want you to work in pairs on the same task, to try to design a better candy carton than either of you produced individually.

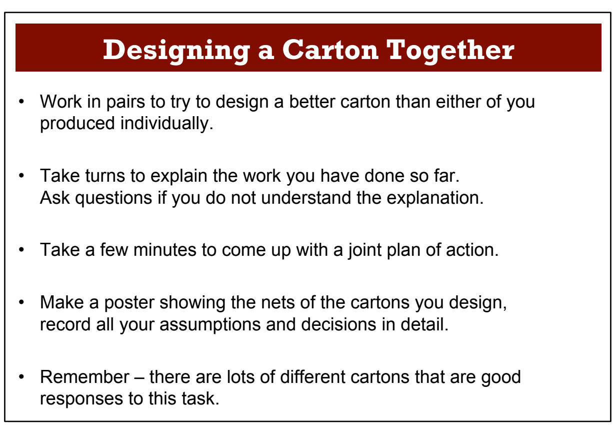
In your pairs, take turns to explain the work you have done so far. Then take a few minutes to come up with a joint plan of action.

Make a poster showing the nets of the two cartons you design, recording all your assumptions and decisions in detail.

Make sure you use math to explain your design.

Remember – there are lots of different cartons that are good responses to this task.

Slide P-3 of the projector resource summarizes these instructions:



Organize students into pairs or groups of three. Give each group a large sheet of paper for making a poster and a couple of pens. Mention that counters, rules, pencils, protractors, pairs of compasses, and grid paper are also available on request.

Whilst students are working in their groups, you have two tasks: to notice students' approaches to problem solving and to support them in their work.

Notice students' approaches to problem solving

By carefully listening and watching students as they work together you will get a better idea of students' understanding, be in a better position to ask questions to help them progress and in a whole-class discussion be more purposeful in who you select to explain solutions.

Do students begin by thinking about packing arrangements for the candy? If so, do they systematically identify factors of 18, or do they work experimentally with counters? Do students explore different carton shapes based on their exploration of packing? If so, do they justify and explain the choice of one carton shape rather than another? Do students use math to justify their nets? Do students first sketch the 3-D carton and then draw its net?

Do students consider minimizing the area of card used? How flexible and protective will the packaging be? How will cartons stack on shelves? Do they identify and use any other contextual constraints?

Do students interpret this task as a volume problem and calculate the volume of the candies? Do they recognize that how the candies are stacked may produce gaps inside a carton? Do they notice that this will necessitate a carton with greater capacity than the volume of the candies?

What assumptions do students make about constraints on the shape and size of the carton? Do they justify any assumptions they make?

Do students sketch/draw/construct accurate nets? Would their nets fold to form a closed 3-D carton? Do they figure out how to position the flaps correctly?

Support students' problem solving

Try to support students in their work without pushing them towards any particular solution method. Students will not necessarily use the mathematics you think relevant from what you have taught them and may generate a variety of approaches and solutions. Students may need to be reassured that it is acceptable to try an approach that is different from those being used by others in the class and that this might lead to a different, but valid, conclusion.

One way to do this is to provide general or strategic rather than technical hints. Avoid simplifying the problem for pupils by breaking it down into steps. Instead, ask more strategic questions such as:

What do you know?
What are you trying to do?
Don't ask for help too quickly. Try to think this out for yourself.
What have you tried so far?
Can you give a specific example?
Re-read the problem. Look at the last two sentences.
Try first drawing the carton in 3-D and then use this diagram to draw the net.

Try to assist pupils that are really stuck without adding structure that reduces the amount of reasoning they will be required to do. If you intervene too quickly, students have no chance to experience the satisfaction of puzzling out a solution for themselves. Follow up student responses to your questions with further questions, such as:

What makes you say that? Can you use math to justify your answer?

You might then ask questions to help them improve the net they have made.

What do you think about your carton?

Where will the bottom of your carton be? Could it be anywhere else?
Would this affect the shape of the carton? How? Would all the candies still fit into the carton?

How are you going to improve this net?

Are there sides missing or too many sides? Or are the sides in the wrong places? What change to the net would that involve?

How will the sides of your net stay together? Where will you put glue flaps?

What did you consider when selecting the 'best' design?

Where is the math in your solution?
[Sometimes students have difficulty recognizing what is and what is not math.]

If students are having difficulty in understanding how a net works, offer them a copy of *Hint Sheet A*. This offers a selection of nets and asks them to decide which make a closed carton.

If students are having difficulty drawing a net because they cannot decide on dimensions for the box, provide them with *Hint Sheet B*. This asks them to produce a net for a given carton. They can then try to adjust the dimensions of the net for that carton to fit 18 pieces of candy.

Would the candies fit into that carton? How do you know?
How could you change the dimensions so the candy would fit?

Challenge students who are finding the problem straightforward by asking them to come up with a different design by changing some of their assumptions or arranging the candy differently.

You may want to use the questions in the *Common issues* table to support your questioning. If the whole class is struggling on the same issue, you could write one or two questions on the board, and hold a brief whole-class discussion. You could also give any struggling students one of the sample responses.

Collaborative small-group work: *Making a Candy Carton* (20 minutes)

As students finish their posters, ask them to make one of their candy carton designs.

Give each group a sheet of light, letter-sized cardboard.

There is equipment and paper if you choose to use it.

Again, monitor and support students as in the first collaborative activity.

Notice if students are able to construct their 3-D models accurately. Do any students have difficulty cutting/gluing? What do students do if their net does not have flaps?

Encourage students to explain their reasoning:

Of the two nets on your poster, why did you decide to construct this one?

Has your carton turned out as you envisaged? If not, why?

Did your 3-dimensional model turn out as you expected? If not, where do you think you went wrong?

Sharing posters (15 minutes)

Once students have finished making their cartons, ask them to share their work with another group.

Take turns to explain your designs.

Ask questions if you do not understand or disagree.

Encourage students to check whether the 3-dimensional carton made matches the design on the poster.

SUGGESTED LESSON OUTLINE: DAY 2

Whole-class introduction (15 minutes)

The main purpose of this introduction is to encourage students to compare and evaluate different approaches to the problem.

Begin by comparing two contrasting designs, or two designs with different underlying assumptions. Emphasize again that there are many alternative ways to respond to this task: you are not looking for a single right answer.

I want us to share some of the different ways you've approached the task.

Ask two groups with contrasting designs to show their finished cartons and to say why they think that their idea is a 'good' design for the candy carton. Make sure that other students understand what is being explained to them.

What is 'good' about the design? What do we mean by 'good' in this context?
[E.g. Will hold 18 candies, doesn't waste card, easy to make, fits in pocket, strong and rigid …]

Andy, can you explain what Ben just said about his carton again, but in your own words?

Has anyone designed a carton similar to Ben's? What is the same/different about your carton?

Focus on the assumptions students have made:

I want you to think about the assumptions you made when doing the task. What choices did you make when the task didn't say exactly what to do? Those are your assumptions.

Lucia, tell me an assumption that Ben made. Ben, why did you make that assumption?

Did anyone make a different assumption?

Encourage students to challenge assumptions that have been made.

You may also have noticed a particular difficulty or misunderstanding that occurred in the previous lesson and you may wish to spend some time discussing it.

Collaborative small-group work: *Sample Responses to Discuss* (30 minutes)

Organize students into pairs or groups of three. Provide each small group of students with a set of the *Sample Responses to Discuss*.

Explain how you would like students to work:

In your group, choose one of the sample responses to work on together.

Read through the sample response carefully. Notice any assumptions the student has made.

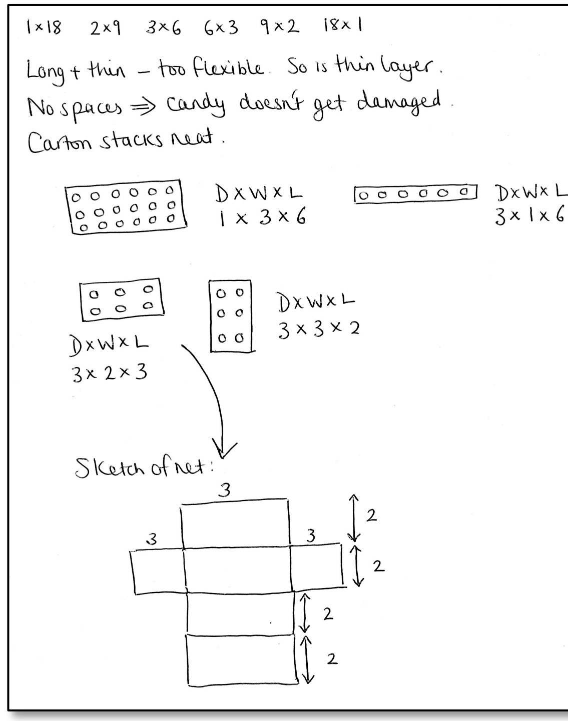
There are some questions for you to answer as you look at the work.
You may want to add notes to the work to make it easier to follow.

When you have worked on one solution, choose another to work on together.

The projector resource Slide P-4 summarizes these instructions.

While students are working, support them in identifying mathematical issues in the work and in writing clear notes to explain these issues. It is more important that students work on one or two *Sample Responses to Discuss* thoroughly, than work superficially on them all.

Ewan identifies different ways of factorizing 18 but does not explain that this is what he is doing. He makes some reasonable assumptions about the type of product required and identifies some practical constraints on box design. He does not explain this part of his work very well. For example, he does not explain which of the arrangements are 'long and thin' or 'thin layer'.



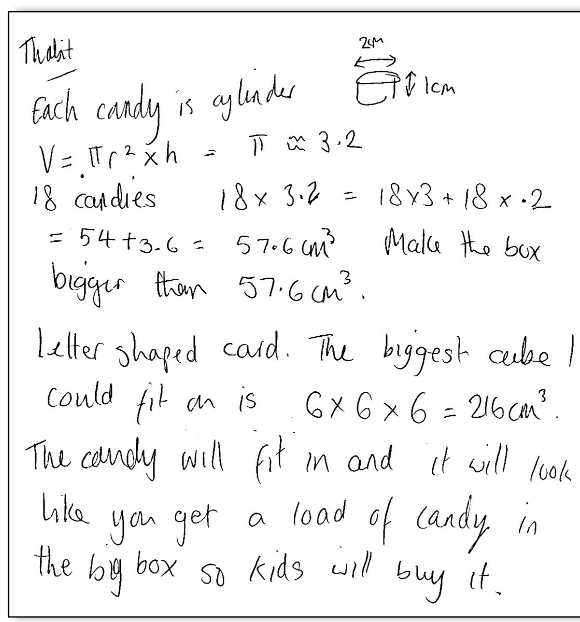
Ewan draws some diagrams showing different packing arrangements in plan view. He makes a decision to produce the net of an arrangement of sweets that is 3 deep, 3 wide, and 2 long. He uses sweets as a measure for sketching out a net, but ignores the dimensions of each sweet.

The net is very inaccurate and not to scale. He does not provide a good scale drawing or full-size net. He does not include flaps for joining edges together.

Ewan could improve his work by explaining his early work better, by accurately figuring out dimensions of his chosen box, and by drawing an accurate net.

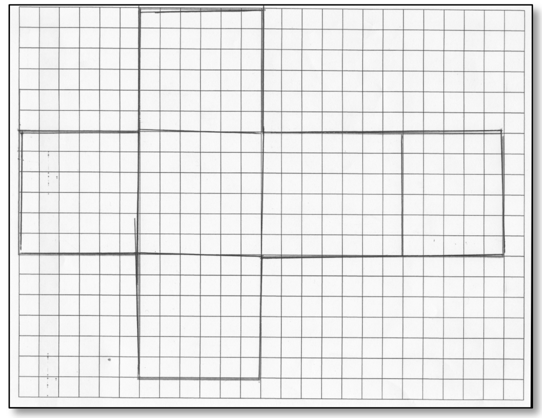
How would adding the diameter of each candy to a diagram help Ewan? [It may help Ewan to understand the number of candies in a row does not equate to the length of the row in cm.]

Thabit makes a different assumption about the carton required. He assumes that he needs a carton big enough to fit the candy but the candy does not need to be tightly packed. Thabit is free to make this assumption because the design constraints do not specify otherwise.



Thabit correctly calculates the volume of the sweets to an appropriate degree of accuracy (π is taken as 3.2!). He has not included the dimensions for the height and radius as they are both 1 cm and so do not affect the answer; however this makes his work difficult to follow. He does not consider how cylindrical candy pieces fit together – only if it was stacked as a single column would all the candy fit into a carton of this capacity.

Thabit does make a big enough box though, with plenty of room for the candy. This can be seen by thinking about how the pieces would fit onto one of the faces, as two layers of nine candies. There would be plenty of space to spare, more than enough room for the candies to move about. Thabit does not justify his assumption that the extra space is acceptable.

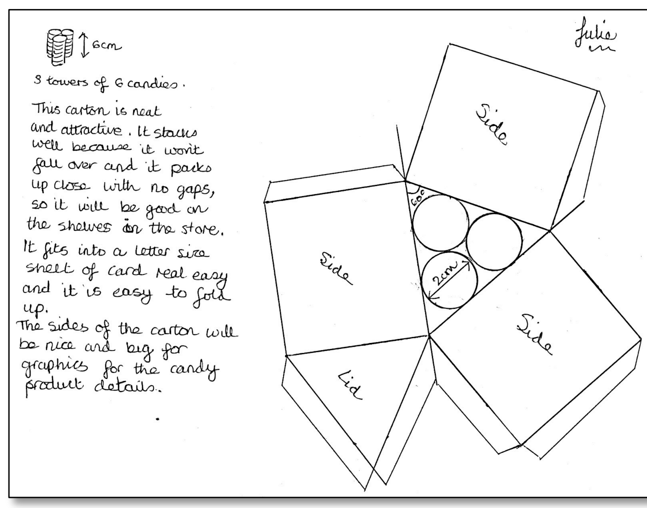


Thabit's net is roughly and inaccurately drawn and there are no flaps to join sides. He has drawn two faces incorrectly (one is 7×6 and one is 5×6).

Thabit's solution is quite simple, and although his approach to the task is quite different to Ewan's, his proposed carton will do the job, with a few changes.

Thabit could improve his solution by considering packing and how the candies fit into a given space, by stating and explaining more of his assumptions, by providing a comparative justification for his choice of carton, and by drawing an accurate net.

Julie has chosen to produce a triangular prism. She shows how the candies would be arranged, but she has provided no explanation of how she reached that arrangement. She does list some of the good features of her packaging. Julie's explanations of the benefits of her carton are quite clear, but there is no comparison with other possible choices. She also identifies a contextual constraint – the need to put graphics on the sides of the carton.



Drawing round candy sized pieces enabled Julie to produce a fairly accurate base for the net. Julie has not, however, drawn a net that will fold up into a triangular pyramid. The lid is joined incorrectly to one of the sidepieces. The flaps will not all work, either.

Julie could improve her solution by providing comparative justification for her choice of carton and by improving the net.

Whole-class discussion (15 minutes)

Bring the class back together to compare and contrast the different responses including any assumptions the students have made when designing the cartons.

There are projector resources of each sample response on Slides P-5, P-6, and P-7.

Which design did you like best? Why?

Which design do you think will produce the:

- *strongest carton?*
- *carton with the least wastage of card?*
- *carton that will stack neatly on shelves?*

Which approach did you find most difficult to understand? Why?

Did any of the students make assumptions? What were they? Do you agree with them?

Finally, evaluate the cartons discussed relative to their own cartons with regard to assumptions made. Try to choose different assumptions so that different cartons come out 'best' each time!

If you assume that [the carton has to fit neatly around the candies], which design do you think is best? Why?

If you assume that [the carton needs to pack neatly] which design do you think is best? Why?

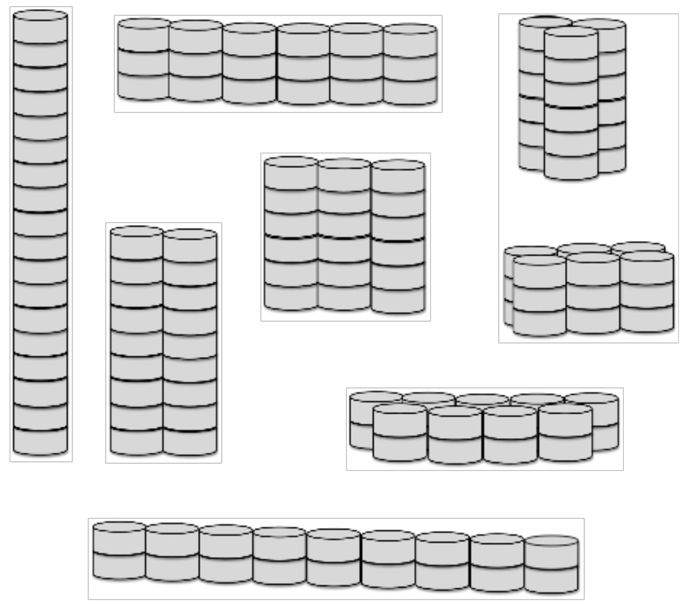
If you assume that [the carton must protect the candies] which design do you think is best? Why?

Follow-up lesson: individual review (15 minutes)

Students will need to sit with their partners for this part of the task but will be working individually. Make sure each student has their individual solution and can see the joint solution they worked on.

Ask students to read through their individual solution and joint work on the task and then complete the *Design Review* questionnaire. The questionnaire should help students review their work and the math they used.

SOLUTIONS



Students may assume that the candy has to fit closely into a box or that the candy can be loosely packed.

If students assume that the candy can be loosely packed, they will still need to consider volume and packing constraints. Check that the student does not simply use the volume of the 18 pieces of candy to decide on the size of the box.

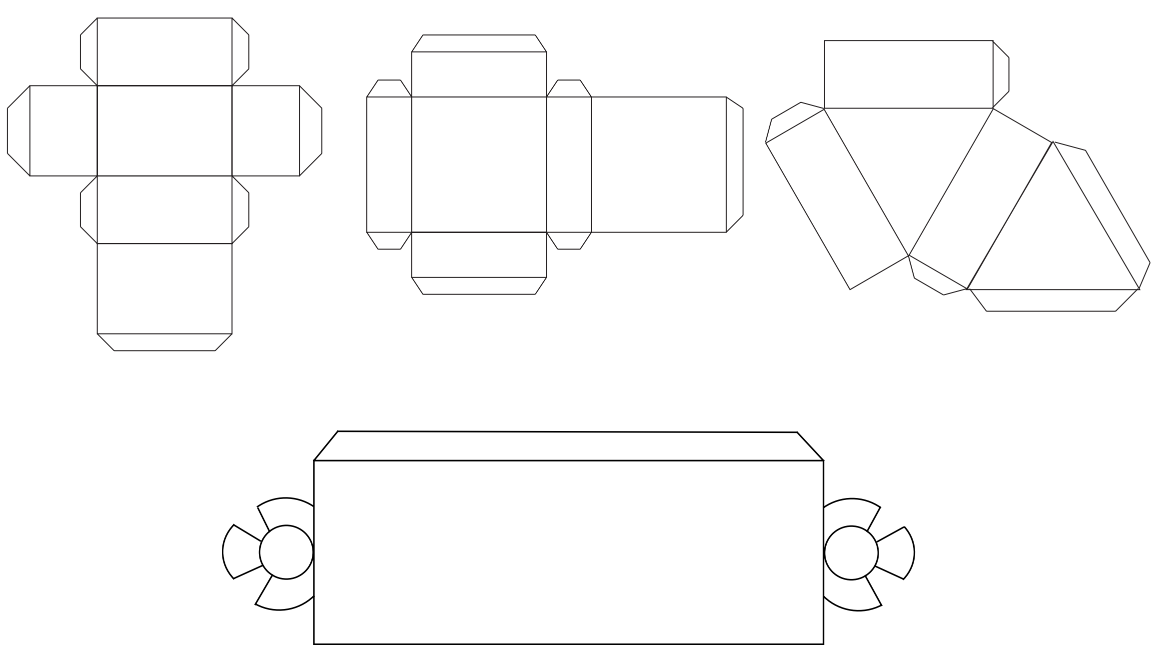
If the student considers packing the candy neatly, they could explore different options systematically. Students may choose different ways of arranging the pieces of candy and provide some contextual justification for their choices. For example, choosing a large flat package might be justified with reference to space for graphics, or customer perceptions of the amount of candy. This task is not set as an optimization task, so minimal card area is not required.

Each arrangement will lead to a different box design. Their dimensions may be calculated theoretically, or a more concrete approach may be adopted by stacking objects.

With suitably sized objects, a student could draw round the objects to form the base of a net. This is the approach taken by Julie in the *Sample Responses to Discuss*.

Whichever approach students take, they should draw an accurate full sized or reduced-scale drawing of the net, including enough flaps to join each pair of conjoining edges securely.

Some sample nets are shown below:



Design a Candy Carton

A candy company wants you to design a carton that will contain 18 candies.

Your design must take into account the following:

- The candies are each 1 cm deep and 2 cm in diameter.
- The carton must be made from a net that fits on a single sheet of letter-sized cardboard.
- The design should require as little cutting as possible.
- The sides of the carton will be fixed together using glue flaps.
  Show where these will be on your net.

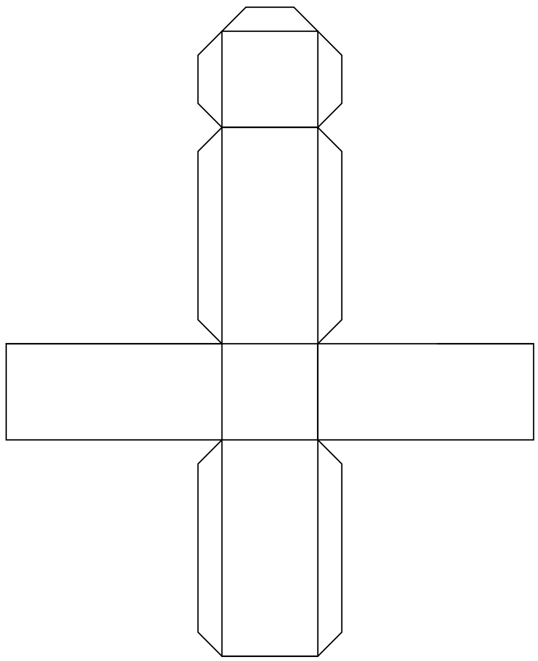
Produce two possible designs for nets of the candy carton.

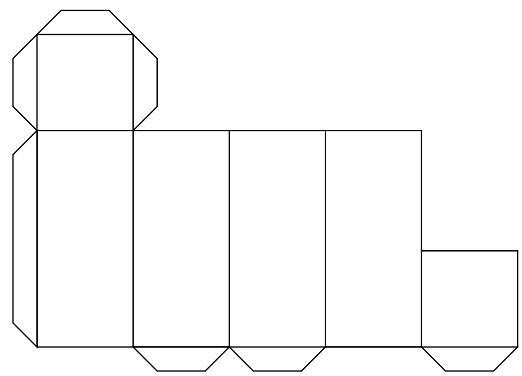
Compare your two designs.

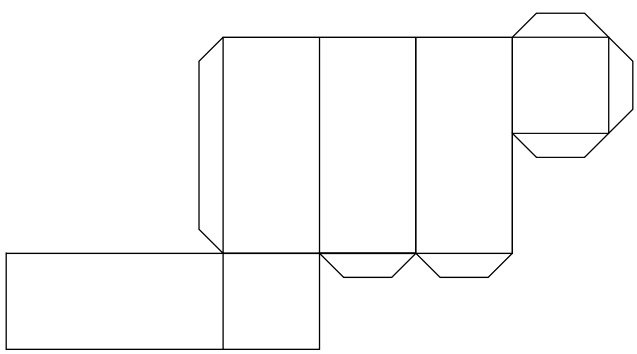
Record in writing which carton you think is best and why.

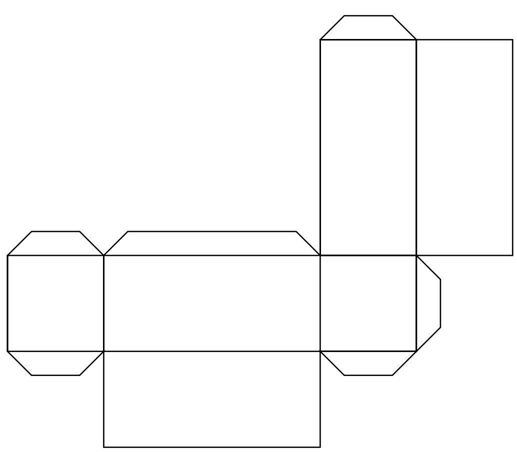
Hint Sheet A

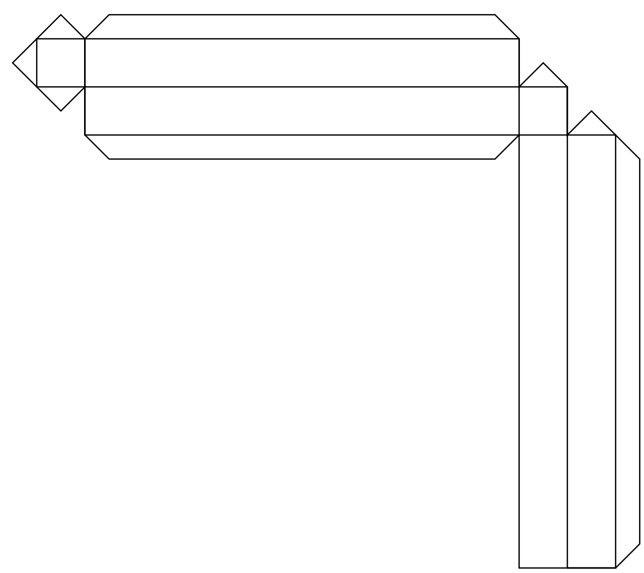
Which nets will fold up to make a closed carton?





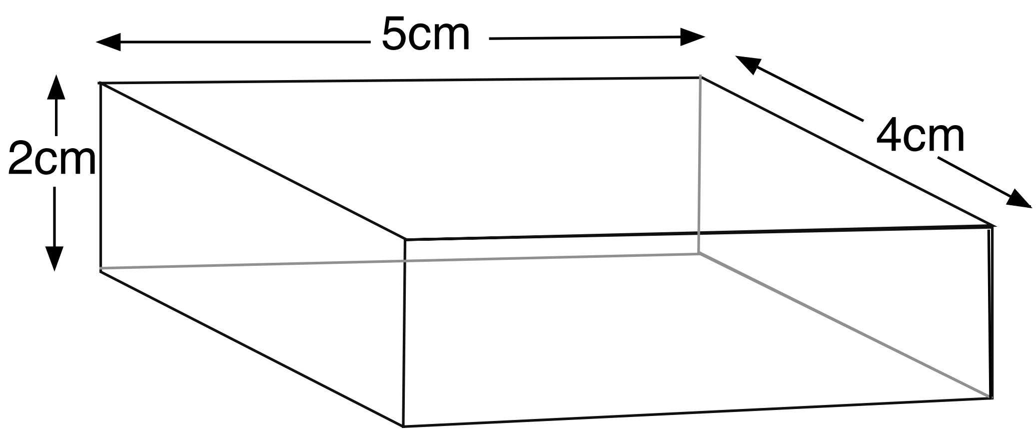






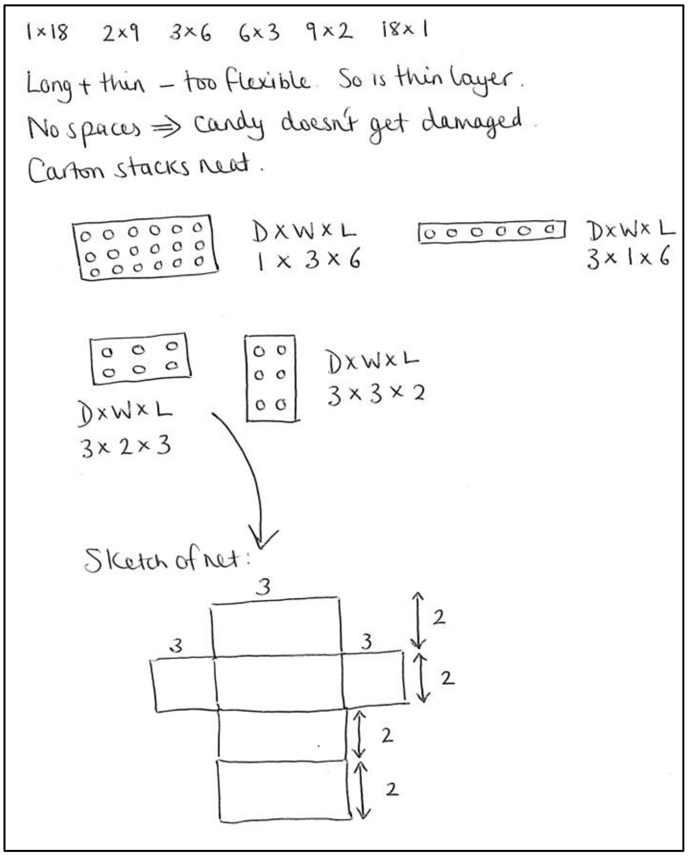
Hint Sheet B

Draw an accurate net for this right rectangular prism candy carton on grid paper.



Would the 18 pieces of candy fit into this carton? Explain your answer.

Sample Responses to Discuss: Ewan



What did Ewan do well?

How could Ewan's design be improved?

If you think Ewan's sketch of the net is incorrect, correct it.
Figure out the area of card used for the correct net.

Sample Responses to Discuss: Thabit

Thabit

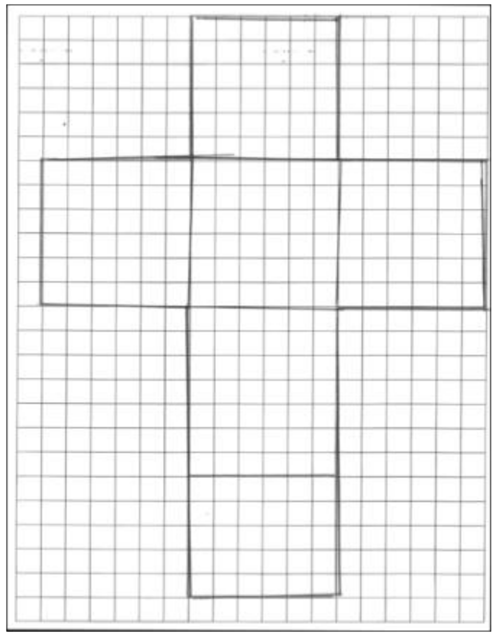
Each candy is a cylinder 2cm 1cm

$V = \pi r^2 \times h = \pi \approx 3.2$

18 candies $18 \times 3.2 = 18 \times 3 + 18 \times .2$

$= 54 + 3.6 = 57.6\ cm^3$ Make the box bigger than $57.6\ cm^3$.

Letter shaped card. The biggest cube I could fit on is $6 \times 6 \times 6 = 216 cm^3$. The candy will fit in and it will look like you get a load of candy in the big box so kids will buy it.

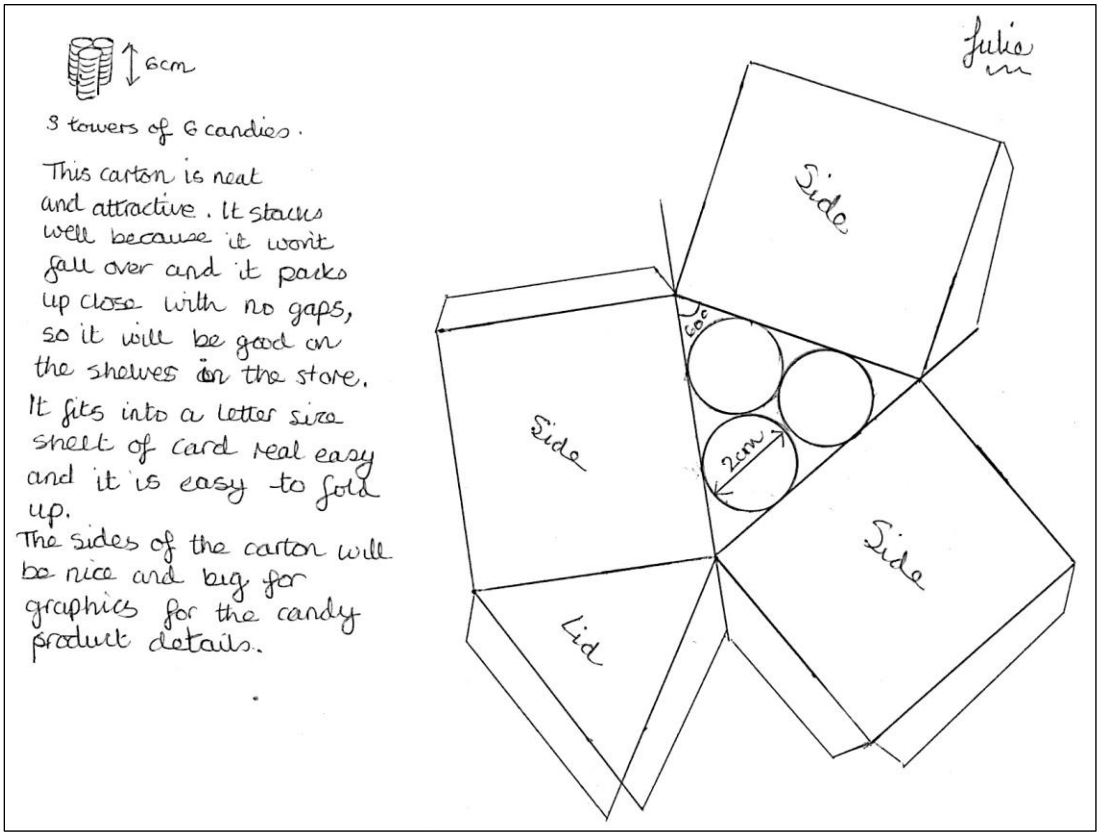


What did Thabit do well?

How could Thabit's design be improved?

If you think Thabit's sketch of the net is incorrect, correct it.
Figure out the area of card used for the correct net.

Sample Responses to Discuss: Julie



What did Julie do well?

How could Julie's design be improved?

If you think Julie's sketch of the net is incorrect, correct it.
Figure out the area of card used for the correct net.

Design Review

1. Which carton was your group's design similar to? (Circle one answer.)

Ewan's carton	***Thabit's carton***	***Julie's carton***	***None of these cartons***

2. Which do you think was the best carton design? (Circle one answer.)

My carton	***Our group's carton***	***Ewan's carton***	***Thabit's carton***	***Julie's carton***	***Another carton***

Explain your reasons:

---

3. In our group, we made these assumptions when designing the candy carton:

---

4. Circle all the aspects of math you used in your design work. Add more activities if you need to.

Drawing and constructing:	Drawing accurately on grid paper	Drawing lengths accurately with pencil and rule	Drawing angles accurately
	Drawing the net for a right rectangular prism	Drawing the net for a triangular prism	Constructions with compasses and rule
	Accurately constructing a 3-D model of the net		
Arranging the design	Packing congruent 2-D shapes	Finding congruent 3-D shapes	Stacking congruent 3-D shapes
	Finding different arrangements of 18 objects		
Measurements	Using metric units for area	Using metric units for volume	Measuring angles
	Using metric units for length	Measuring lengths	
Calculations	Calculating measures using the number of candies	Calculating the volume of a right rectangular prism	Calculating the volume of a cylinder
	Calculations using The Pythagorean Theorem	Calculations using trigonometry	

Other math I used:

Design a Candy Carton

- A candy company wants a carton that will contain 18 candies.

Your design must take into account the following:

- The candies are each 1 cm deep and 2 cm in diameter.
- The carton must be made from a net that fits on a single sheet of letter-sized cardboard.
- The design should require as little cutting as possible.
- The sides of the carton will be fixed together using glue flaps. Show where these will be on your net.

Produce two possible designs for nets of the candy carton.
Compare your two designs. Record in writing which carton you think is best, and why.

A Piece of Candy

Designing a Carton Together

- Work in pairs to try to design a better carton than either of you produced individually.
- Take turns to explain the work you have done so far.
  Ask questions if you do not understand the explanation.
- Take a few minutes to come up with a joint plan of action.
- Make a poster showing the nets of the cartons you design, record all your assumptions and decisions in detail.
- Remember – there are lots of different cartons that are good responses to this task.

Sample Responses to Discuss

- Read through each Sample Response together carefully.
- Highlight and explain what the student did well.
- Highlight and explain what the student could improve.
- What questions would you like to ask the student?
- When you have worked on one solution, choose another to work on together.

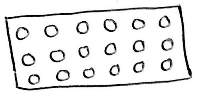
Sample Responses to Discuss: Ewan

1×18 2×9 3×6 6×3 9×2 18×1

Long + thin – too flexible. So is thin layer.

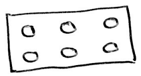
No spaces ⇒ candy doesn't get damaged.

Carton stacks neat.

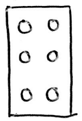


D×W×L
1 × 3 × 6

D×W×L
3 × 1 × 6

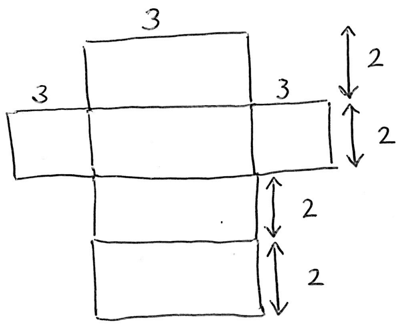


D×W×L
3 × 2 × 3



D×W×L
3 × 3 × 2

Sketch of net:



Sample Responses to Discuss: Thabit

Thabit

Each candy is cylinder

2cm

1cm

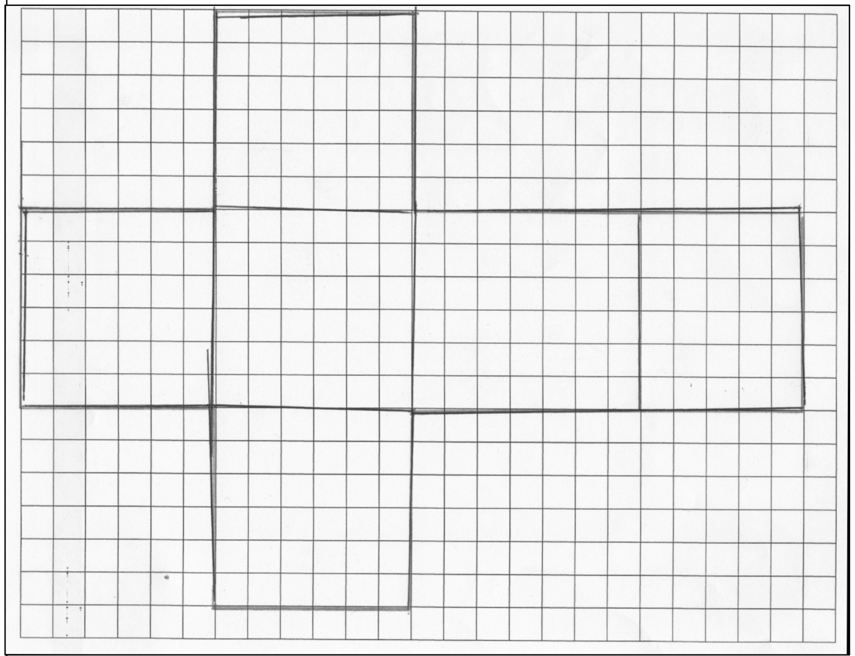
$V = \pi r^2 \times h = \pi \approx 3.2$

18 candies $18 \times 3.2 = 18 \times 3 + 18 \times .2$

$= 54 + 3.6 = 57.6\,cm^3$ Make the box bigger than $57.6\,cm^3$.

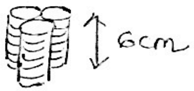
Letter shaped card. The biggest cube I could fit on is $6 \times 6 \times 6 = 216\,cm^3$.

The candy will fit in and it will look like you get a load of candy in the big box so kids will buy it.



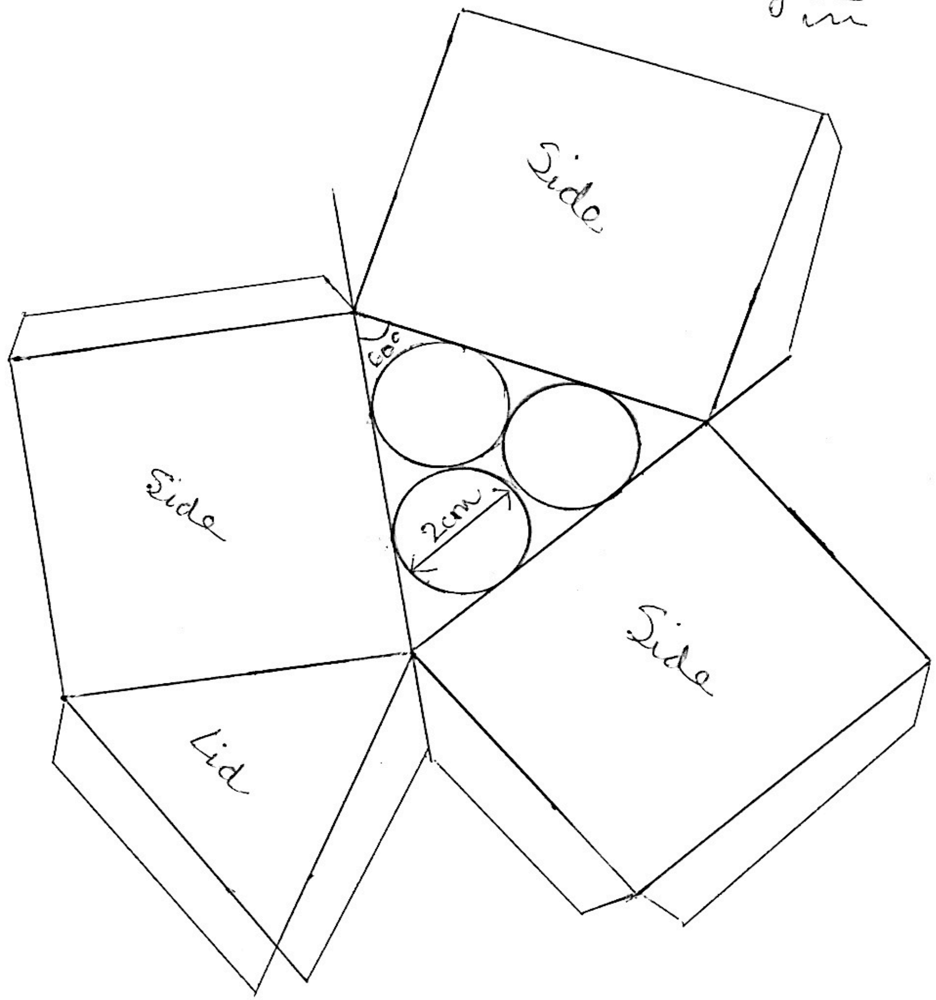
Sample Responses to Discuss: Julie

Julie



3 towers of 6 candies.

This carton is neat and attractive. It stacks well because it won't fall over and it packs up close with no gaps, so it will be good on the shelves on the store. It fits into a letter size sheet of card real easy and it is easy to fold up.
The sides of the carton will be nice and big for graphics for the candy product details.



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

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Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
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The central task in this lesson was originally designed for
Bowland Maths (http://www.bowlandmaths.org.uk)
and appears courtesy of the **Bowland Charitable Trust**

We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org