

Mathematics Assessment Program *CCR-C2*

College and Career Readiness Mathematics

Time Allowed: 3 hours

These tasks give you a chance to show what you know and how you reason, and to solve mathematical problems.

Please show your work and reasoning in the spaces provided. Explain any assumptions you make.

**Try as many tasks as you can in the time given.
If you get stuck on a task, move on to the next task.**

Name: _____	Male Female
School: _____	City: _____
Teacher: _____	Grade: _____
Date: _____	

Do not write in the box below:

CCR-C2	Short Tasks	Leaky Faucet	A Golden Crown?	Birds' Eggs	Cubic Graph	Floor Pattern	Fruit Boxes	Sidewalk Patterns	Pythagorean Triples	Circle Pattern	Fearless Frames	Total
11												

These tests were developed with support from the Bill and Melinda Gates Foundation

Short Tasks

1. Simplify: $\sqrt{x+2} + \sqrt{4x+8}$.

2. Using the Remainder Theorem, decide whether $x - 5$ and $x + 2$ are factors of the polynomial $f(x) = 2x^3 - 5x^2 - 28x + 15$.

3. Find the volume of a cone with height 7cm and base diameter 6cm.

4. If the slope of a line is $\frac{1}{2}$ and the y-intercept is 3, what is the x-intercept of the same line?

5. Joe has a bag containing 8 red sweets, 9 yellow ones and 11 green.
Without looking, he takes out a sweet and eats it, then, he takes out a second sweet.
What is the probability that both the sweets are red?

Short Tasks (continued)

6. By completing the square in $x^2 - 2x + 3$, find the minimum value of $f(x) = x^2 - 2x + 3$.

7. If $3x + y = 19$, and $x + 3y = 1$. Find the value of $2x + 2y$.

8. Use the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ to find the value of $\cos(x)$ if $\sin 2(x) + 2\cos(x) - 2 = 0$

9. If $\frac{5c}{4} - \frac{2c}{3} = \frac{7}{10}$, find c .

10. If the equation of a line p in the coordinate plane is $y = 3x + 2$, what is the equation of line q which is a reflection of line p in the x -axis?

Leaky Faucet

Jan estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds.

1. Estimate how many times the faucet drips in a **week**.
Show your calculations.



Jan estimates that approximately 575 drops fill a 100 milliliter bottle.

2. Estimate how much water her leaky faucet wastes in a **year**. _____
Show how you figured it out.

A Golden Crown?

The King asks Archimedes if his crown is made from pure gold.

He knows that the crown is either pure gold or it may have some silver in it.



Archimedes figures out that the volume of the crown is 125 cm^3 and that its mass is 1.8 kilograms.

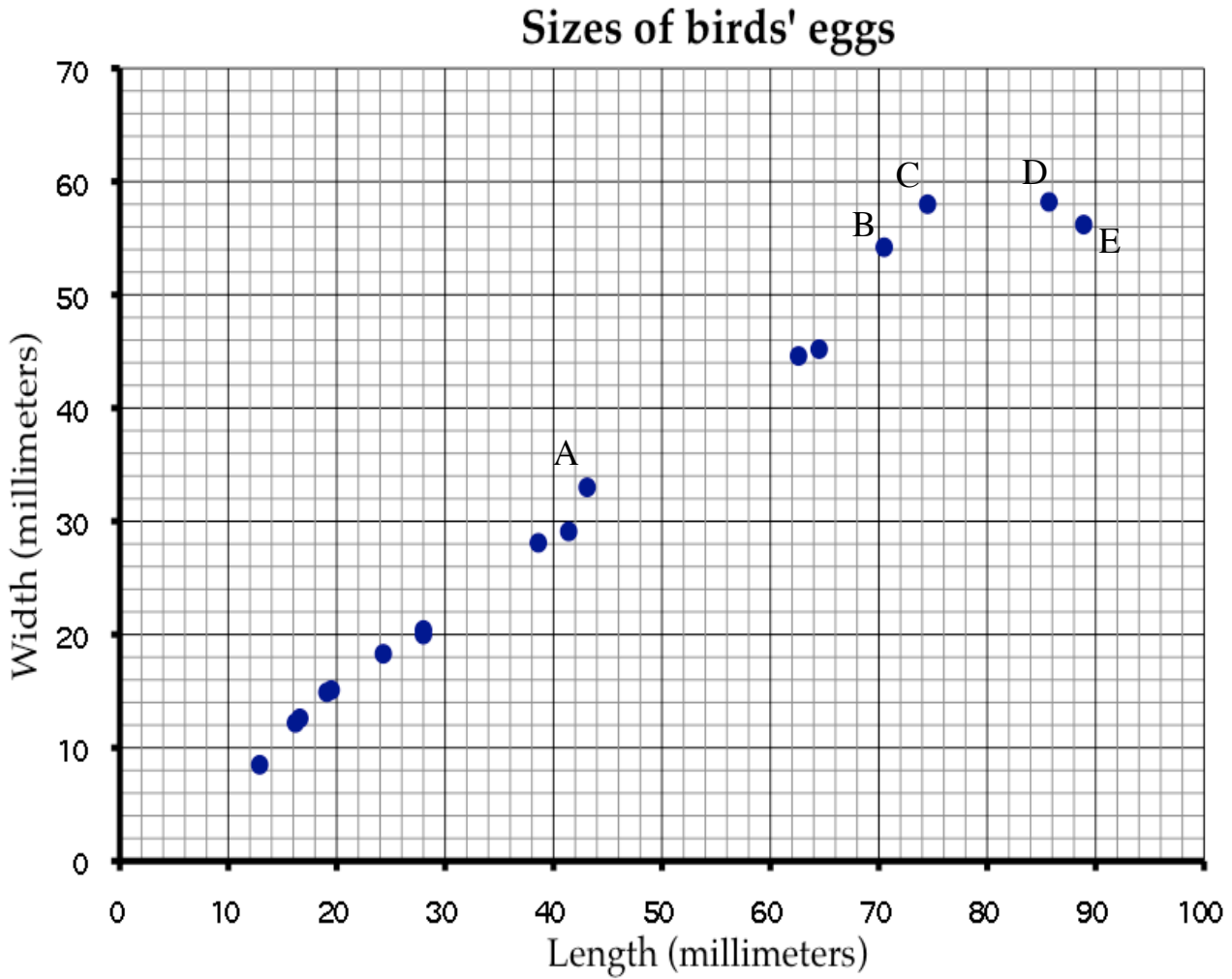
He also knows that 1 kilogram of gold has a volume of about 50 cm^3 and 1 kilogram of silver has a volume of about 100 cm^3 .

1. Is the crown pure gold? Explain how you know.

2. If the crown is not pure gold, then how much silver is in it?
Show all your work.

Birds' Eggs

This scatter diagram shows the lengths and the widths of the eggs of some American birds.



1. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width 41.6 millimeters.

Use a **X** to mark a point that represents this on the scatter diagram.

2. What does the graph show about the connection between the lengths of birds' eggs and their widths?

3. Another sample of similar birds has eggs with a length of 35 millimeters on average. If these birds follow the trend in the scatterplot, about what width would you expect these eggs to be, on average?

4. Describe the differences in shape of the two eggs C and D.

5. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

Cubic Graph

1. a. Show that $x = 2$ is a solution of the equation $x^3 - x - 6 = 0$.

b. The diagram opposite shows the graph of $y = x^3 - x - 6$.

i. Write down the coordinates of point A. _____

ii. Use the graph to explain why there is only one solution to the equation. $x^3 - x - 6 = 0$.

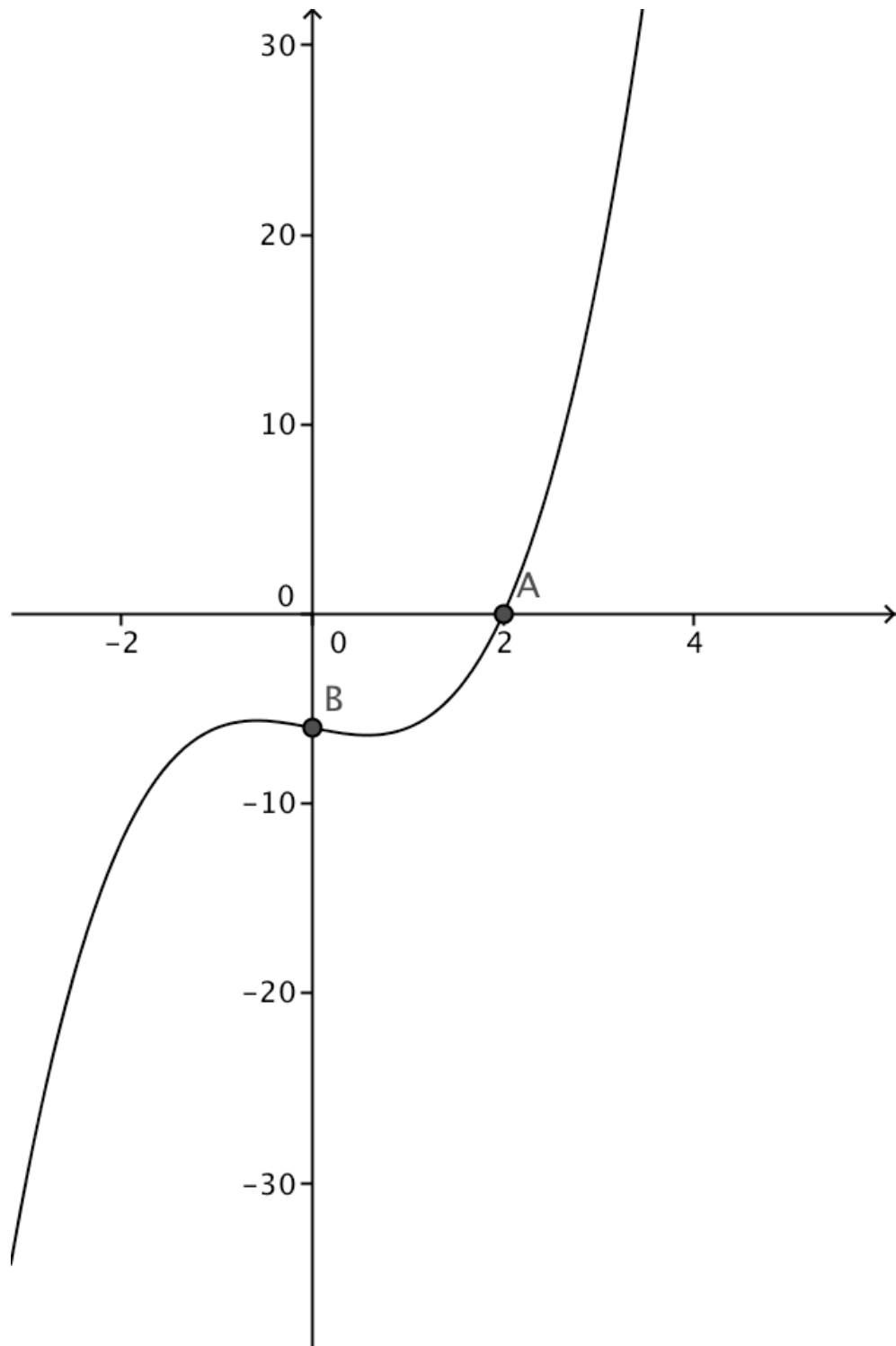
2. a. Find the coordinates of point B. _____

b.

i. What transformation changes the graph of $y = x^3 - x - 6$ into the graph of $y = x^3 - x$?

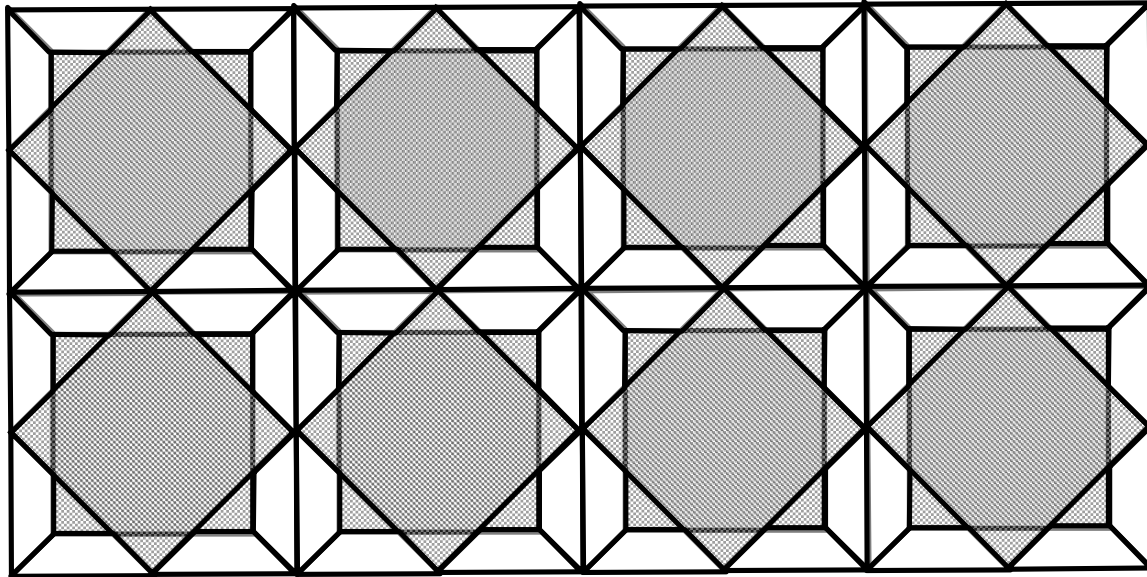
ii. Sketch the graph of $y = x^3 - x$ on the diagram.

iii. What are the solutions of the equation $x^3 - x = 0$?

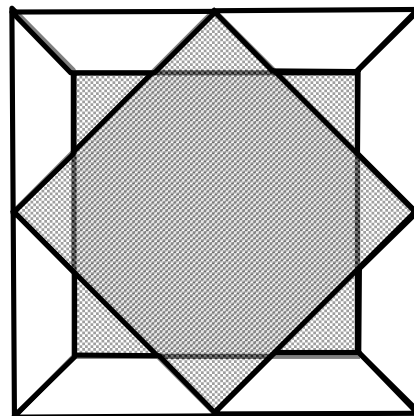


Floor Pattern

The diagram shows a floor pattern.



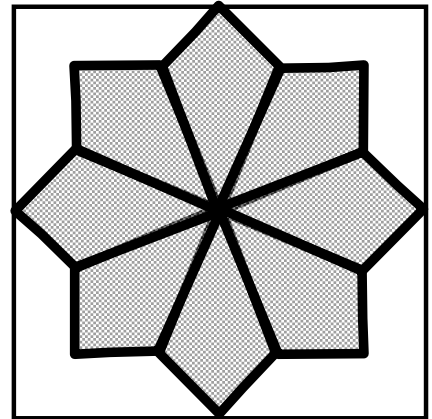
In the floor pattern, the shaded part is made by overlapping two equal squares.



The shaded shape can also be seen as a set of eight equal kites.

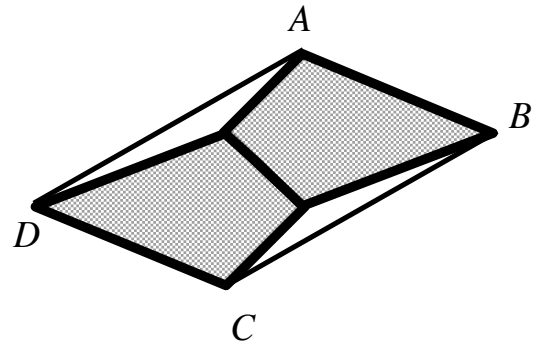
1. Find the measures of all four angles of the kites.

Explain how you obtained your answers.



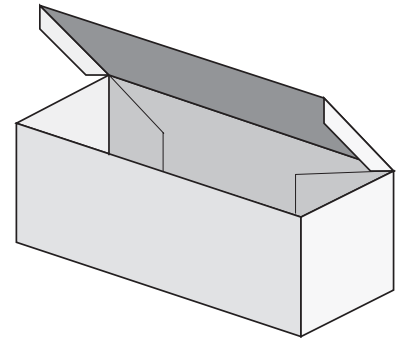
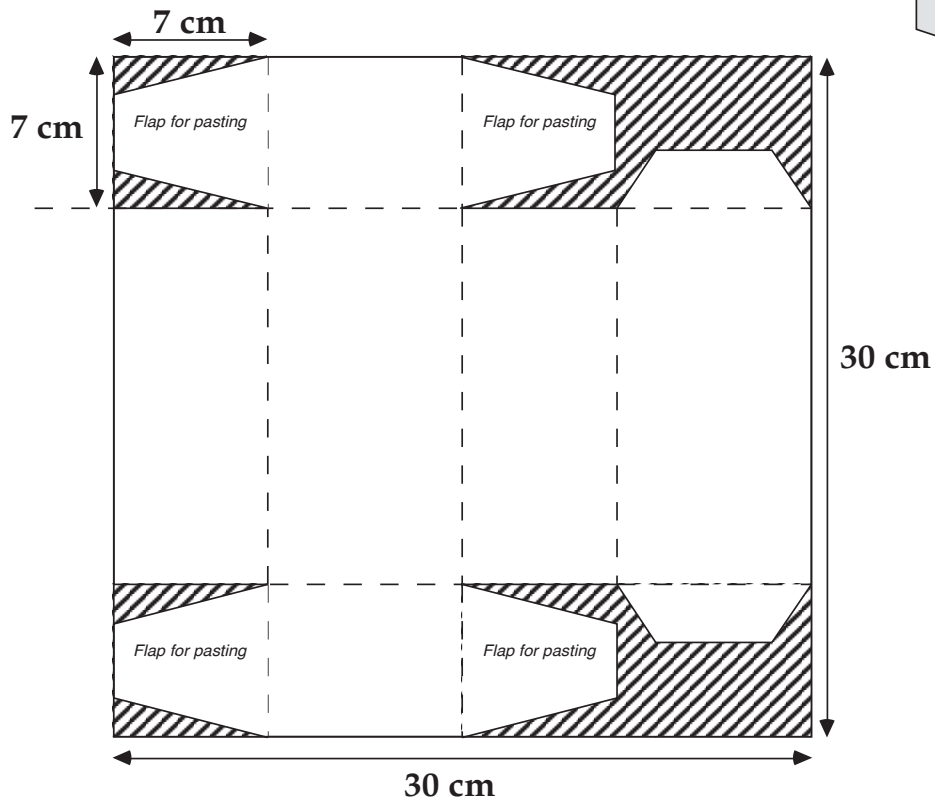
2. Two of the kites can fit together to make a hexagon.

Prove that the quadrilateral ABCD is a parallelogram.



Strawberry Boxes

A grocer wants to sell strawberries in small boxes.
He wants to make the boxes from square card 30 cm long
and 30 cm wide as shown below:



The shaded areas are cut away and the rest is folded along the dashed lines.
The sides are folded up and stuck together using flaps.
The lid has two flaps that are not glued.

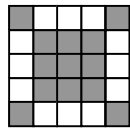
1. Calculate the volume of this box. Show your work.

Please continue your work on the page opposite.

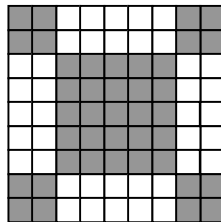
Sidewalk Patterns

In Prague some sidewalks are made of small square blocks of stone.

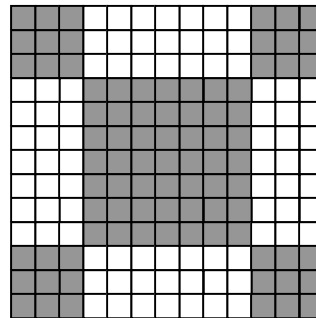
The blocks are in different shades to make patterns that are in various sizes.



Pattern #1

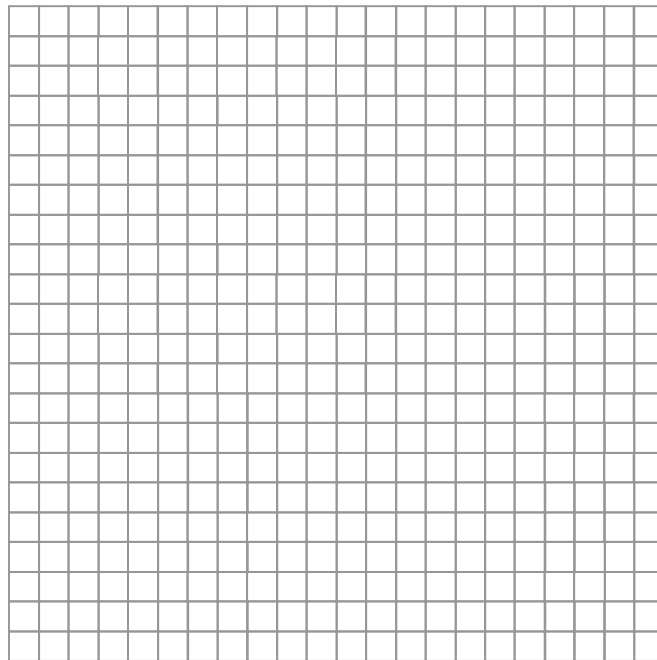


Pattern #2



Pattern #3

Draw the next pattern in this series. You may not need to use all the small squares.



Pattern #4

1. Complete the table below

Pattern number, n	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

2. What do you notice about the number of white blocks and the number of gray blocks?

3. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.

a. Fill in the blank spaces in this list.

$$25 = 5^2 \quad 81 = \underline{\hspace{2cm}} \quad 169 = \underline{\hspace{2cm}} \quad 289 = 17^2$$

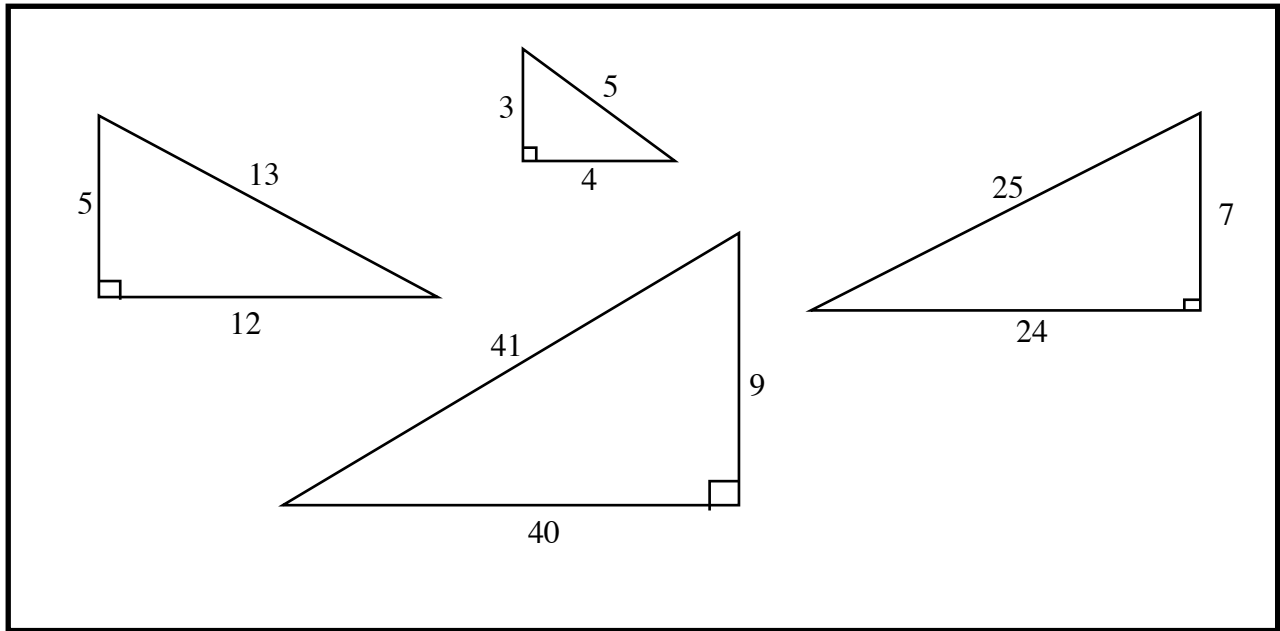
b. How many blocks will pattern #5 need? _____

c. How many blocks will pattern # n need? _____

4. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

b. Pattern # 6 has a total of 625 blocks.
How many white blocks are needed for pattern #6? _____
Show how you figured this out.

Pythagorean Triples



(3, 4, 5), (5, 12, 13), (7, 24, 25) and (9, 40, 41) are called Pythagorean Triples because they satisfy the condition

$$c^2 = a^2 + b^2$$

1. Investigate the relationships between the lengths of the sides of triangles that belong to this set.

Pythagorean Triples (continued)

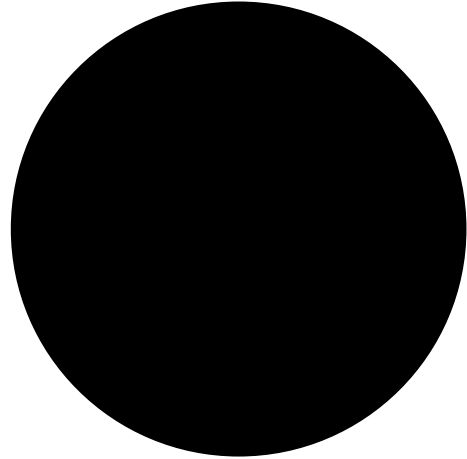
2. Use these relationships to find the numerical values of at least two further Pythagorean Triples that belong to this set.

3. Investigate rules for finding the perimeter and area of triangles that belong to this set when you know the length of the shortest side.

Circle Pattern

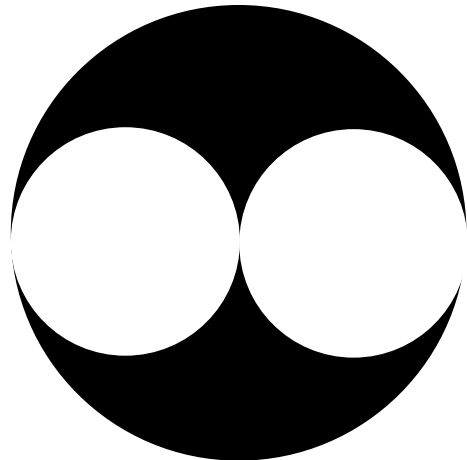
Here is a developing circle pattern.

Here is one black circle.



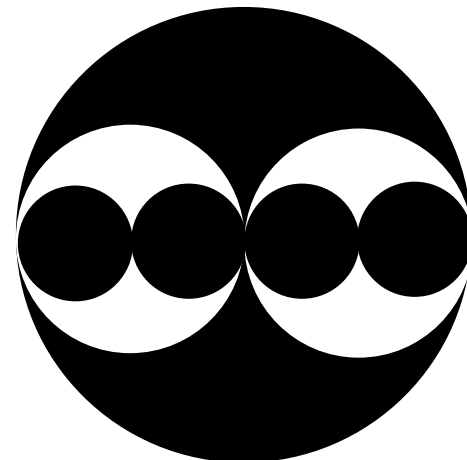
Two white circles of half the radius have been added to the diagram.

1. Show that the fraction of the diagram that is now black is one half.

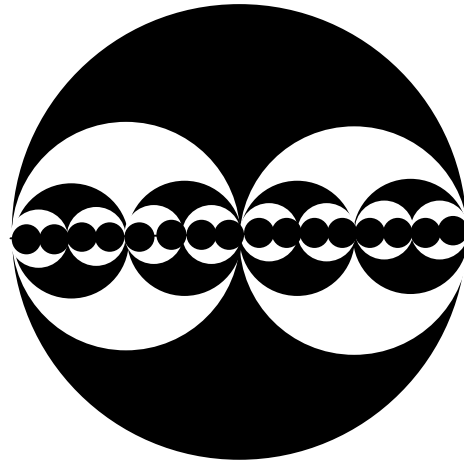
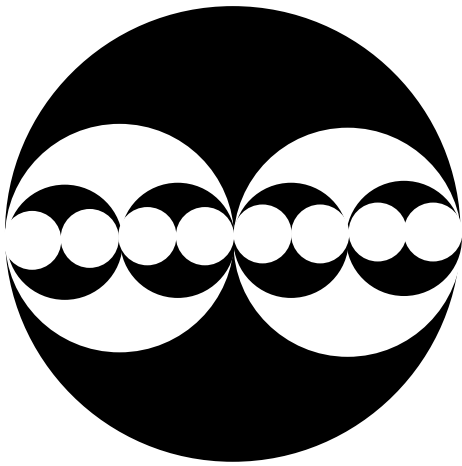


Four black circles have now been added.

2. What fraction of the diagram is now black?



3. Fill in the table to show what happens as the pattern continues.



Pattern	Black fraction	White fraction
One black circle	1	0
Two white circles	$\frac{1}{2}$	$\frac{1}{2}$
Four black circles		
Eight white circles		
Sixteen black circles		

4. Write a description of what is happening to the black and white fractions as the pattern continues.

Fearless Frames

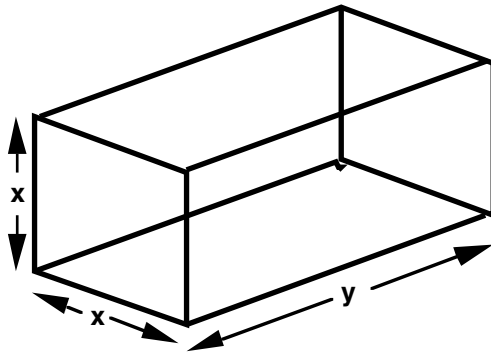
Fearless Frames Inc makes metal frames for containers.

1 A client asks Fearless Frames to make a large container which is a rectangular prism with a square cross section.

The company has only 60 meters of suitable metal tubing in stock.

Find the dimensions of the container which holds the maximum volume the company can make using 60 meters of tubing.

Show how you figured it out.

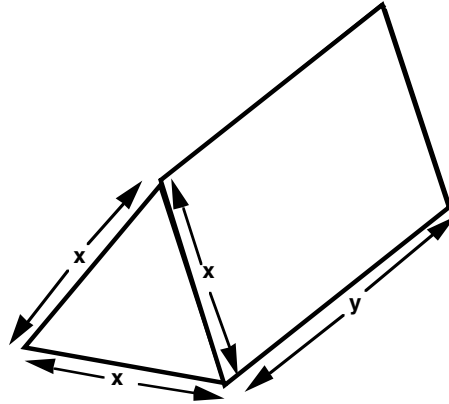


Fearless Frames: (continued)

2. The client changes his mind!

He asks for a container that is a prism with a cross-section which is an equilateral triangle.

Investigate the maximum volume of the container that can be made using 60 meters of tubing for the frame.



3. What advice do you think Fearless Frames should offer to this customer?
Show all your calculations.
