**Introduction**

Mathematics teaching assumes that students do not arrive at sessions as ‘blank slates’, but as actively thinking people with a wide variety of skills and conceptions. Research shows that teaching is more effective when it assesses and uses prior learning so that the teaching may be adapted to the needs of students (Black & Wiliam, 1998). Prior learning may be uncovered through any activity that offers students opportunities to express their understanding and reasoning. It does not require more testing. For example, it can take the form of a single written question given at the beginning of a session to elicit a range of explanations that may then be discussed. This process, often referred to as formative assessment, may be defined as:

…all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes ‘formative assessment’ when the evidence is actually used to adapt the teaching work to meet the needs. (Black & Wiliam, 1998)

This module considers the different ways this can be done and focuses on the following questions:

- How can problems be used to assess performance?
- How can this assessment be used to promote learning?
- What kinds of feedback are most helpful for students and which are unhelpful?
- How can students become engaged in the assessment process?

**Activities**

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**Acknowledgement:**

In preparing this material, we acknowledge the permissions given by the Bowland Charitable Trust to adapt the professional development resources, *Bowland Maths*, that they had previously funded us to produce for the UK. This includes many of the handouts and most of the video extracts. Additional resources were also adapted from *Improving Learning in Mathematics*, a government funded program in the UK. The original sources are:


Related research to help your planning for this module.

Now published by GL Assessment: [http://shop.gl-assessment.co.uk](http://shop.gl-assessment.co.uk)

This short booklet offers a summary of the extensive research literature into formative assessment. It shows that there is clear evidence that improving formative assessment raises standards, and offers evidence showing how formative assessment may be improved. This booklet is essential reading for all teachers.

Now published by GL Assessment: [http://shop.gl-assessment.co.uk](http://shop.gl-assessment.co.uk)

In this booklet, the authors describe a project with teachers in which they studied practical ways of implementing formative assessment strategies and the effect this had on learning. The section on feedback and marking (pp. 8-9) are particularly relevant to this module.


This book gives a fuller account of the earlier booklets *Inside the black box* and *Working inside the black box*. It discusses four types of action: questioning, feedback by marking, peer- and self-assessment and the formative use of summative tests. The section on feedback and marking (pp. 42-49) is particularly relevant to this module, while the section on peer and self-assessment (pp. 49-53) is relevant for the next CPD module.


This booklet applies the above findings specifically to Mathematics. It considers some principles for Mathematics learning, choice of activities that promote challenge and dialogue, questioning and listening, peer discussion, feedback and marking, and self and peer assessment. This booklet is essential reading for all mathematics teachers.
**Activity A: Introducing formative assessment**

**Minimum time needed: 10 minutes.**

**The different types and purposes of assessment.**

Invite participants to discuss the following issues:

- Why do you assess students?
- What different purposes do your assessments serve? Make a list.

Their list of reasons might include: to diagnose difficulties; to celebrate achievement; to motivate students; to select students for classes; to maintain records to keep teachers and parents informed of progress; to assess teaching methods.

To summarize, there are two main purposes of assessment:

- **Summative assessment** – to summarize and record overall achievement at the end of a course, for promotion and certification. Most ‘high stakes’ tests and external examinations are designed for this purpose. Summative assessment is also used to evaluate the relative effectiveness of a particular course, teaching method, or even an institution.
- **Formative assessment** – to recognize achievements and difficulties at the beginning or during a course, so that teachers and students can take appropriate action. This type of assessment forms an integral part of all learning.

**The potential of formative assessment to improve learning.**

Briefly mention the research evidence that sets out the case for formative assessment. This is summarized by Black and Wiliam in several accessible publications for teachers (see p. 2). These researchers set out to find out whether or not improving formative assessment improves learning.

We checked many books and nine years’ worth of more than 160 journals, and earlier reviews of research. This process yielded 580 articles or chapters to study. We prepared a review using material from 250 of these sources. All... studies show that... strengthening... formative assessment produces significant, and often substantial, learning gains. These studies range over ages, across several school subjects, and over several countries... (Black and Wiliam, 1998).\(^1\)

This module will examine the implementation of formative assessment, based on this and other research.

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Activity B: Teachers’ own experiences of formative assessment

Minimum time needed: 10 minutes.

What do teachers know about their students and what consequential action do they take?

Ask participants to work in pairs, considering the following questions.

- Think of two students in your class, one who is particularly strong and one who is finding the work very difficult. Take it in turns to describe the students’ strengths and difficulties to your partner, in as much detail as possible.
- How did you become aware of these strengths and difficulties? On what evidence do you base your judgments? Test results? Memories of oral responses during lessons? Observations of the student working? Written work?
- In what ways do your assessments of these students affect your lesson planning? Give examples.

What difficulties do teachers encounter?

Issue participants with copies of Handout 1.

- How far are the criticisms on the handout valid in your context?
- If any are, then what may be done about them?
Handout 1: Difficulties in formative assessment

The research literature suggests that formative assessment practices are beset with problems and difficulties. These are summarized in the extensive review by Black and William (1998)¹ as follows:

**Effectiveness of learning:**

- Teachers’ tests encourage rote and superficial learning.
- The questions and methods used are not shared between teachers, and they are not critically reviewed in relation to what they actually assess.
- There is a tendency to emphasize quantity of work and to neglect its quality in relation to learning.

**Impact of assessment**

- The giving of scores and the grading function are overemphasized, while the giving of useful advice and the learning function are underemphasized.
- Approaches are used in which students are compared with one another, the prime purpose of which seems to them to be competition rather than personal improvement; in consequence, assessment feedback teaches low-achieving students that they lack “ability,” causing them to come to believe that they are not able to learn.

**Managerial role of assessment**

- Teachers’ feedback to students seems to serve social and managerial functions, often at the expense of the learning function.
- Teachers are often able to predict students’ results on external tests because their own tests imitate them, but at the same time teachers know too little about their learning needs.
- The collection of marks to fill in records is given higher priority than the analysis of students’ work to discern learning needs; furthermore, some teachers pay no attention to the assessment records of their students’ previous teachers.

**Activity C: Principles for formative assessment**

Minimum time needed: 20 minutes.

Issue participants with a copy of Handout 2. The ideas presented here are all drawn from research into formative assessment.

- Bearing in mind the difficulties discussed in Activity B, how would you suggest that your formative assessment practices be improved?
- Discuss the principles outlined on Handout 2.
  - Which of these do you currently use in your own teaching?
  - Which do you find most difficult? Why?
- What other principles do you think are important?

Issue copies of Handout 3.

"It's all very well telling us to assess our students, but how can a busy teacher know what is going on inside 30 individual heads?"

- How would you answer this teacher?
- What strategies do you have for finding out what students are thinking in your lessons?
- Discuss the two suggestions shown on Handout 3, and watch the two Activity C Videos: ‘Mini-Whiteboards’ and ‘Posters’ to see these in action.
- Suggest some further strategies for making reasoning more evident.

The two strategies described on Handout 3 and illustrated in the Activity C Videos may help to make reasoning more ‘visible’.

**Mini-whiteboards** are an indispensable resource for the following reasons:

- When students hold their ideas up to the teacher, it is possible to see at a glance what every student thinks.
- During whole class discussions, they allow the teacher to ask new kinds of question (typically beginning: ‘Show me an example of...’).
- They allow students to, simultaneously, present a range of written and/or drawn responses to the teacher and to each other.

**Posters** are also a powerful way of helping students to externalize their thinking. This use does not require ‘polished’, ‘complete’, ‘attractive’ products but rather they should be seen as working documents. Perhaps the simplest way of using a poster is for students to solve a problem collaboratively, explaining the thought processes involved at every step. A second use of posters is to find out what they already know about a given topic. In the diagram shown on Handout 2, the teacher asked students to write down all they knew about \( y = 2x - 6 \). As a class, the diagram was developed on the whiteboard. Students were then given a variety of equations (the level of challenge was varied appropriately) and were asked to produce their own poster. The discussion enabled the teacher to assess how much learners knew about equations and how well they were able to link the ideas together.
Handout 2: Principles for formative assessment

Formative assessment may be defined as:

... all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet the needs. (Black & Wiliam, 1998)

Make the objectives of the lesson explicit

Share the objectives with students and from time to time ask students to produce evidence that they can achieve these objectives.

"Make up an example to show me that you know and understand Pythagoras' theorem."

"This lesson was about you deciding what methods to use. Show me where you did this."

Students may find it difficult to appreciate that some lessons are concerned with understanding concepts, while others are more concerned with developing mathematical practices. Making objectives explicit doesn't mean writing them on the board at the beginning of the lesson, but rather referring to them explicitly while students are working. If the objectives are to develop mathematical practices then in plenary sessions, ask students to share and compare approaches, rather than answers.

Assess groups as well as individual students

Group activities allow many opportunities to observe, listen, and question students. They help to externalize reasoning and allow the teacher to see quickly where difficulties have arisen.

Watch and listen before intervening

Before intervening in a group discussion, wait and listen. Try to follow the line of reasoning that students are taking. When you do intervene, begin by asking them to explain something. If they are unsuccessful then ask another student to help.

Use divergent assessment methods ("Show me what you know about ... ").

Convergent assessment strategies are characterized by tick lists and can-do statements. The teacher asks closed questions in order to ascertain whether or not the student knows, understands or can do a predetermined thing. This is the type of assessment most used in written tests.

Divergent assessment, in contrast, involves asking open questions that allow students opportunities to describe and explain their thinking and reasoning. These questions allow students to surprise us - the outcome is not predetermined.

Handout 3: Making reasoning visible.

Use questioning with mini-whiteboards

One difficulty with normal classroom questioning is that some students dominate while others are too afraid to participate. In this strategy, every student presents a response simultaneously. When open questions are used, students are able to give different responses to those around them. The teacher is able to immediately assess which students understand the ideas and which are struggling.

Ask students to produce posters

Ask each small group of students to work together to produce a poster:
- showing their joint solution to a problem
- summarizing what they know about a given topic, or
- showing two different ways to solve a given problem
- showing the connections between a mathematical idea and other related ideas.

Adapted from: Improving Learning in Mathematics, Department for Education and Skills, 2005.
Activity D: Analyze students’ responses to problem-solving tasks

Minimum time needed: 20 minutes

Handout 4 presents three problems together with four student responses on each. The tasks are: Counting Trees, Cats and Kittens, Security Cameras.

These tasks use a variety of mathematical content, but all three focus on the problem solving and modeling skills described in the Common Core Standards for Mathematical Practice. Often, such tasks prove more difficult than the individual elements of mathematical content would suggest, as students are required to choose and combine techniques in non-routine ways. Traditional summative assessments often focus on isolated content standards and fail to test these ‘process skills’. Formative assessment is an effective way to ensure that students are developing these practices.

The following phases of problem solving are a useful tool to analyze students’ work on these tasks:

A. Formulate questions, choose appropriate representations and techniques.
B. Reason logically, construct hypotheses and arguments, compute accurately.
C. Interpret and evaluate results obtained.
D. Communicate and reflect.

These are closely related to the phases of the modeling cycle described in the CCSS High School standards for modeling, but also relate to language found throughout the practices and are generally applicable to most problem-solving activities.

- Read through all three tasks then choose one task that will be most suitable for a class you will soon teach. If you are working on this module in a group, it will be helpful if each participant chooses the same problem, as this will facilitate the follow-up discussion.

- Consider the four student responses. What does each student’s response tell you about his or her capacity to use each of the phases of problem-solving: formulate, reason, interpret and evaluate, communicate and reflect?

Handout 5 offers some comments on students’ responses to each of the tasks.

- If you were the teacher of these students, what feedback would you give them to help them improve their responses? Try to frame this help in the form of oral questions you could ask in the classroom. You may find it helpful to refer to the generic questions given on Handout 6.

- Watch Activity D Video: ‘Teachers Discussing Feedback.’ This shows three teachers discussing the feedback they gave on the three problems.
Handout 4: Assessment tasks and sample responses

Handout 5: Improving students’ responses through questioning

Handout 6: Suggestions for questions

<table>
<thead>
<tr>
<th>Phases of problem-solving</th>
<th>Suggested Questions</th>
</tr>
</thead>
</table>
| Formulate questions, choose appropriate representations and techniques.  
  MFL, MPN | • What questions might you ask about this situation?  
  • How can you get started on this problem?  
  • What topics might be useful here?  
  • What sort of diagram might be helpful?  
  • Can you invent a simple situation for this?  
  • How can you simplify this problem?  
  • What is known and what is unknown?  
  • What assumptions might you make? |
| Reason logically, construct hypotheses and arguments, compute accurately  
  MFL, MP2, MP4, MPN | • Where have you seen something like this before?  
  • What is fixed here, and what can you change?  
  • What is the same and what is different here?  
  • What would happen if I changed this, to this?  
  • Is this approach going anywhere?  
  • What will you do when you get that answer?  
  • Is this just a special case of...what?  
  • Can you form any hypotheses?  
  • Can you think of any counterexamples?  
  • What mistakes have you made?  
  • Can you suggest a different way of doing this?  
  • What conclusions can you make from this data?  
  • How can you check this calculation without doing it all again?  
  • What is a sensible way to record this? |
| Interpret and evaluate results obtained  
  MFL, MP3, MPN, MP6, MP7, MPN | • How can you best display your data?  
  • Is it better to use this type of chart or that one?  
  • What patterns can you see in this data?  
  • What reasons might there be for these patterns?  
  • Can you give me a convincing argument for that statement?  
  • Do you think that answer is reasonable?  
  • How can you be 100% sure that is true?  
  • How do you think of Anna’s argument?  
  • What method might be best to use here?  
  • What method might be best to use here?  
  • What method did you use?  
  • What other methods have you considered?  
  • Which of your methods was the best?  
  • Which method was the quickest?  
  • What method did you use last time?  
  • Would they have worked here?  
  • What helpful strategies have you learned for next time? |

Counting Trees

Sample response: Laura

Laura attempts to estimate the number of old and new trees by multiplying the number along each side of the whole diagram and then adding. She does not account for gaps or does she realize that there are an unequal number of trees at each level.

What questions could you ask Laura that would help her improve her response?

Sample response: Jenny

Jenny realizes that sampling is needed, but she multiplies the number of young trees and old trees in the left hand column by the number of trees on the bottom row. She guesses the number in the bottom row, in her method she underestimates the total number of trees. She does, however, take account of the different numbers of old and new trees.

What questions could you ask Jenny that would help her improve her response?

Sample response: Woody

Woody uses a sample of two columns and counts the number of old and young trees. He then multiplies by 26 (half of 52) columns to find an estimate of the total number.

What questions could you ask Woody that would help him improve his response?

Sample response: Amber

Amber chooses a representative sample and counts through her work to get a reasonable answer. She correctly uses proportion/reasoning. She checks her work as she goes along by counting the gaps in the trees. Her work is clear and easy to follow.

What questions could you ask Amber that would help her improve her response?
Activity E: Observe formative assessment in action

Time needed: 15 minutes.

In this activity, you are provided with video extracts of Andrew, Dominic and Amy exploring how formative assessment may be used to promote students learning. They are using the three tasks from Activity D.

In an earlier lesson, these teachers had asked students to sit in different places and attempt one of the tasks individually, with no help. They then collected in their students’ responses, assessed the work qualitatively and prepared written feedback in the form of questions. The film clips you are about to see are taken from the follow-up lesson. Students have returned to their normal places and most have solutions that are different to those of their partners.

Watch Activity E Video: ‘Observing Formative Assessment’

and consider:

- What different kinds of assessment can you see?
- What is the purpose of each kind of assessment?
- What do both the teachers and students learn?

In the video, you will see:

- Andrew exploring how students respond to his feedback on the ‘counting trees’ problem;
- Amy listening to, then questioning individuals as they try to share their ideas and produce joint solutions to the ‘security camera’ problem;
- Dominic listening to presentations from students on their methods and reasoning for the ‘cats and kittens’ problem;
- Amy concluding her lesson by asking students to describe how they have used her feedback to improve their work.
**Activity F: Plan and report back on an assessment lesson**

*Minimum time needed: 30 minutes before the lesson.*
*20 minutes for the pre-lesson assessment.*
*30 minutes to prepare feedback.*
*60 minutes for the lesson.*
*15 minutes for reporting back.*

**Planning the lesson**

Plan your own lesson using one of the problems.

- Plan a time for students to tackle the problem on their own without help.
- Plan how you will assess this work, give feedback and conduct a follow up lesson.
- Collect samples of students’ work to show how their thinking has changed. These will be discussed during the follow-up session.

To help with your planning, you may now like to watch one of the three 10-minute Activity F Videos showing Andrew, Dominic and Amy teaching the three problems from Activity D. They are each following the lesson plan on **Handout 7**.

The pattern of activities on **Handout 7** is as follows:

- Give the problem before the lesson and ask students to attempt it. (20 minutes)
- Collect in the work and prepare some constructive, qualitative feedback. (20 minutes)
- In the follow-up lesson, reintroduce the problem to the class. (5 minutes)
- Students work alone, responding to the feedback using mini whiteboards. (5 minutes)
- Students work in pairs to improve their solutions. (10 minutes)
- Students share their approaches with the class. (15 minutes)
- Students continue with the problem or extend the problem. (20 minutes)

**Handout 7: A formative assessment lesson plan**

**Before the lesson**

- Plan the problem for the lesson. (60 minutes)
- Model the problem to the class. (60 minutes)
- Prepare the feedback for the follow-up lesson. (60 minutes)
- Plan how you will collect evidence of your students’ work. (60 minutes)
- Prepare the feedback you will give your students. (60 minutes)
- Prepare the feedback you will give your colleagues. (60 minutes)

**During the lesson**

- Collect samples of students’ work to show how their thinking has changed. These will be discussed during the follow-up session. (20 minutes)

**After the lesson**

- Plan how you will assess this work, give feedback and conduct a follow up lesson. (60 minutes)
- Plan a time for students to tackle the problem on their own during the follow-up lesson. (60 minutes)
- Plan and report back on an assessment lesson plan. (60 minutes)

**Reporting back on the lesson**

After you have taught the lesson, reflect on what happened with a group of colleagues.

Take it in turns to share stories of the assessment strategies you used.

- How did you collect and assess evidence of your students’ work?
- What did you learn from this evidence?
- What did students learn from the follow-up lesson?
- What are the more general implications for your Mathematics teaching?
Activity G: Consider the effects of feedback on student learning

Time needed: 20 minutes.

So far we have focused on the teachers’ role in providing assessment feedback to students. In this activity we will consider the use students make of different types of feedback and the impact this has on their learning.

Watch Activity G Video: ‘Student’s Views’ showing Andrew’s students as they discuss the impact of assessment feedback on their learning.

- Which of their comments strike you as particularly perceptive and important?
- What are the implications of their comments?

Handout 8 presents some results of research from Black and Wiliam into the relative merits of feeding back assessment information to students in different forms. In particular it compares the effects of feeding back quantitative information in the form of marks, levels and rankings with the effects of offering qualitative information in the form of specific, content-focused feedback.

Compare the students’ comments with the research quotes given on Handout 8.

**The dangers of giving marks, levels, rewards and rankings**

- What are the implications of this for your practice?
- What would happen if you stopped giving marks or levels on pupils’ work?
- Why are so many teachers resistant to making this change?

**The advantages of giving clear, specific, content-focused feedback**

- What are the implications of this for your practice?
- Does this kind of feedback necessarily take much longer to give?

Research shows that students benefit most from feedback that:

- Focuses on the task, not on grades or scores.
- Is detailed rather than general.
- Explains why something is right or wrong.
- Is related to objectives.
- Makes clear what has been achieved and what has not.
- Suggests what the student may do next.
- Offers specific strategies for improvement.

Conclude this module by discussing some ways of applying what you have learned in this PD module to the other mathematics lessons that you teach.

- How could you involve pupils in improving your assessment practices?
Handout 8: The effects of feedback on student learning

Read the following two extracts from Black and Wiliam and respond to the questions that follow:

The dangers of giving marks, levels, rewards and rankings

Where the classroom culture focuses on rewards, ‘gold stars’, grades or place-in-the-class ranking, then pupils look for the ways to obtain the best marks rather than at the needs of their learning which these marks ought to reflect. One reported consequence is that where they have any choice, pupils avoid difficult tasks. They also spend time and energy looking for clues to the ‘right answer’. Many are reluctant to ask questions out of fear of failure. Pupils who encounter difficulties and poor results are led to believe that they lack ability, and this belief leads them to attribute their difficulties to a defect in themselves about which they cannot do a great deal. So they ‘retire hurt’, avoid investing effort in learning which could only lead to disappointment, and try to build up their self-esteem in other ways. Whilst the high-achievers can do well in such a culture, the overall result is to enhance the frequency and the extent of under-achievement.

• What are the implications of this for your practice?
• What would happen if you stopped giving marks or levels on pupils’ work?
• Why are so many teachers resistant to making this change?

The advantages of giving clear, specific, content-focused feedback

What is needed is a culture of success, backed by a belief that all can achieve. Formative assessment can be a powerful weapon here if it is communicated in the right way. Whilst it can help all pupils, it gives particularly good results with low achievers where it concentrates on specific problems with their work, and gives them both a clear understanding of what is wrong and achievable targets for putting it right. Pupils can accept and work with such messages, provided that they are not clouded by overtones about ability, competition and comparison with others. In summary, the message can be stated as follows:

Feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils.

• What are the implications of this for your practice?
• Does this kind of feedback necessarily take much longer to give?

Further Materials: MAP Formative Assessment Lessons

The Mathematics Assessment Project has produced a suite of Formative Assessment Lessons based on the principles discussed here. Each lesson provides materials and comprehensive guidance for teaching a lesson on a particular topic.

Concept development lessons aim to strengthen students’ understanding of a specific topic and help them connect it with their other mathematical knowledge. Problem-solving lessons focus on improving students’ ability to apply mathematical practices as they work together to improve their initial responses to a problem.

A typical lesson unit comprises:

- An assessment task for students to attempt before the main lesson
- Teacher notes describing common issues seen in student work on the assessment task and suggesting formative feedback in the form of questions
- Collaborative learning activities to help students develop their understanding of the mathematics contained in the task and to expose and challenge common misunderstandings. Typical activities include:
  - Classifying mathematical objects & challenging definitions
  - Interpreting multiple representations
  - Evaluating conjectures and assertions
  - Modifying situations & exploring their structure
  - Critiquing supplied examples of other students’ work
- A follow-up assessment task, closely related to the original task, in which students re-engage with the original challenge.

Each lesson comes with a comprehensive lesson plan including whole-class introductions and discussions, group and individual work and further suggestions for probing questions and formative feedback.

The three tasks in this module appear in the lessons:

- **Modeling: Having Kittens** (High School)
- **Estimating: Counting Trees** (Middle School)
- **Optimizing: Security Cameras** (Middle School)

The lesson units are available from [http://map.mathshell.org/](http://map.mathshell.org/)

More about the pedagogical issues arising from these lessons can be found in PD modules 2 and 3.

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2 *Having Kittens* and *Counting Trees* are currently available to Math Development Collaborative partners, but will be publicly released in early 2012. *Security Cameras* is under development and will appear later in 2012.