PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Representing Probabilities: *Medical Testing*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

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Representing Probabilities: *Medical Testing*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Make sense of a real life situation and decide what math to apply to the problem.
- Understand and calculate the conditional probability of an event A, given an event B, and interpret the answer in terms of a model.
- Represent events as a subset of a sample space using tables, tree diagrams, and Venn diagrams.
- Interpret the results and communicate their reasoning clearly.

COMMON CORE STATE STANDARDS

This lesson relates to **all** the *Standards for Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, 4, 7, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

S-CP: Understand independence and conditional probability and use them to interpret data.
Use the rules of probability to compute probabilities of compound events.

INTRODUCTION

- Before the lesson, students work individually on an assessment task designed to reveal their current levels of understanding and difficulties. You then review their work and create questions for students to answer to help them improve their solutions.
- At the start of the lesson, students work alone answering your questions about the same problem. They are then grouped and engage in a collaborative discussion of the same task.
- In the same small groups, students are given sample solutions to comment on and evaluate.
- In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- In a follow-up lesson, students review what they have learned.

MATERIALS REQUIRED

- Each student will need the *Medical Testing* task, the *How Did You Work?* questionnaire, a mini-whiteboard, pen, eraser, and calculator.
- Each small group of students will need the *Sample Responses to Discuss*, some poster paper and a felt-tipped pen. Have a few copies of the *Hint Sheet* available in case some groups of students struggle to start the task. There is a projector resource to support whole-class discussions.
- Optionally, demonstrate the *Medical Testing* applet and the interactive spreadsheet *Medical_Testing.xls* or *Medical_Testing.xlsx* using a data projector or interactive whiteboard.

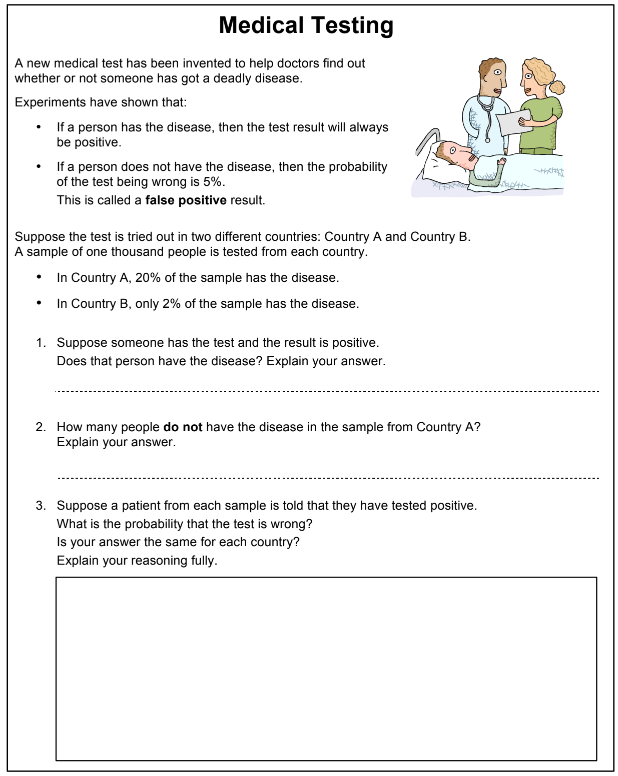
TIME NEEDED

20 minutes before the lesson, a 110-minute lesson (or two 55-minute lessons), and 20 minutes in a follow-up lesson. Timings are approximate and depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Medical Testing* (20 minutes)

Have students complete this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work, to find out the kinds of difficulties students have with it and to see the mathematics they choose to use. You should then be able to target your help more effectively in the next lesson.



Medical Testing

A new medical test has been invented to help doctors find out whether or not someone has got a deadly disease.

Experiments have shown that:

- If a person has the disease, then the test result will always be positive.
- If a person does not have the disease, then the probability of the test being wrong is 5%.
  This is called a **false positive** result.

Suppose the test is tried out in two different countries: Country A and Country B.
A sample of one thousand people is tested from each country.

- In Country A, 20% of the sample has the disease.
- In Country B, only 2% of the sample has the disease.

1. Suppose someone has the test and the result is positive.
   Does that person have the disease? Explain your answer.
   ....
2. How many people **do not** have the disease in the sample from Country A?
   Explain your answer.
   ....
3. Suppose a patient from each sample is told that they have tested positive.
   What is the probability that the test is wrong?
   Is your answer the same for each country?
   Explain your reasoning fully.

Give each student a copy of the *Medical Testing* task, a mini-whiteboard, pen, and eraser. Students are to write their answers on their whiteboards. Try to ensure that they understand the context and what the task is asking:

> *Have any of you ever had a medical test? Have you ever heard of tests being wrong?*
>
> *There is sometimes a small chance of someone being told they have an illness when they haven't.*
>
> *This is called a false positive result. The test is positive but the result is false: the person doesn't have the disease.*

Show students the *Medical Testing* applet. Select Interactive Sample and press Hide Details. This screen will appear:

> *These faces represent 15 people who have been tested for a disease. The smiley faces represent healthy people; the sad faces represent people that have the disease. The faces framed in red have tested positive for the disease. The faces framed in green have tested negative.*
>
> *On your whiteboards, show me answers to the following:*
>
> *How many people are healthy? [11.]*
> *How many people have the disease? [4.]*
> *How many people have a false positive result? [3.]*

Ask students to show you their mini-whiteboards and then press Show Details.

The icons can be sorted, rather than randomly arranged. You may want to hide the details and click on the icons to change the state of each face. Then ask students again about the situation.

Once students understand the meaning of a false positive, ask them to read through the questions and try to answer them as carefully as they can.

If students are struggling to get started, then ask questions that help them understand what is required, but make sure you do not do the task for them. It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students who sit together often produce similar responses and then when they come to compare their work, they have little to discuss. For this reason we suggest that, when students do the task individually you ask them to move to different seats. At the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on the next page. We suggest that you make a list of your own questions, based on your students' work, using the ideas on the following page. We recommend you:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students at the beginning of the lesson.

Common issues	Suggested questions and prompts
Has trouble getting started	• What information are you given? What do you need to find out? • How many people in the sample have the disease in Country A?
A suitable representation is not chosen	• Can you think of a diagram that will show all the possible outcomes?
No recognition that the figures in the diagram do not make sense For example: The total number of people who have not got the disease is greater than 1,000 (Country A.)	• Do the figures in your diagram make sense? • For Country A, how many people, in total, take the test? Does your diagram show this?
Confusion about the probability being calculated For example: To find the probability of a positive result being wrong, the student divides the number of people who test positive but do not have the disease by the total number of people who have the disease, rather than by the total number of positive results.	• What does the phrase: 'the probability that a positive result is wrong' mean? • What figures do you need to know in order to calculate the probability of a false positive (a positive result being wrong)? • Can you use any of the figures in your diagram to calculate this probability? • How can you figure out the total number of people who tested positive? How can you figure out the number of people who incorrectly tested positive? • You are calculating the probability as a fraction that the test is wrong. What number do you need for the numerator of the fraction? What number forms the denominator of the fraction? Why?
Incomplete work For example: The student does not fully label the table, Venn diagram, or tree diagram.	• Would someone unfamiliar with this type of work understand your reasoning?
All answers are correct The student needs an extension task.	• Given that the test is incorrect, what is the probability that a person does not have the disease? Use the rules of probability to check your answer.

SUGGESTED LESSON OUTLINE

Throughout this lesson unit the phrase 'probability of a false positive' means that given a positive result, the probability that this result is false.

Improve individual solutions to the assessment task (10 minutes)

Give each student a mini-whiteboard, pen, and eraser. Students are to write their answers on their whiteboards.

You may want to start the lesson by reminding students of the context of the assessment task. Show the *Medical Testing* Applet.

To establish that students understand the concept of a false positive, show them different scenarios.

Then return the assessment task scripts to the students.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students are to select questions appropriate to their own work and spend a few minutes answering them.

Recall what we were looking at in a previous lesson. What was the task?

I read your solutions and have some questions about your work.

I would like you to work on your own to answer my questions for about 10 minutes.

Collaborative small group work (30 minutes)

Organize the class into small groups of two or three students. Give each group a large piece of paper and a felt-tipped pen.

Deciding on a Strategy

Ask students to share their ideas about the task and plan a joint solution:

I want you to share your work with your group.

Take turns to explain your method and how you think your work could be improved.

Listen carefully to each other and ask questions if you don't understand.

Once students have evaluated the relative merits of each approach, ask them to write their strategy on the large piece of paper.

Once you have each explained your work, plan a joint method that is better than each of your separate ideas.

Write a brief outline of your method on one side of your sheet of paper.

Make sure that everyone in the group can explain the reasons for your chosen method.

Slide P-1 of the projector resource, *Planning a Joint Method*, summarizes how they should work together:

> **Planning a Joint Method**
>
> 1. Take turns to explain your method and how you think your work could be improved.
> 2. Listen carefully to each other.
>   – Ask questions if you don't understand.
> 3. Once everyone in the group has explained their method, plan a joint method that is better than each of your separate ideas.
> 4. Write a brief outline of your method on one side of your sheet of paper.
>
> **Make sure that everyone in the group can explain the reasons for your chosen method.**

Implementing the Strategy

Students are now to turn their large sheet of paper over and write their joint solution clearly in the form of a poster.

While students are working, you have two tasks: to note different student approaches to the problem and to support student problem solving.

Note students' chosen problem solving approaches

For example, how do students figure out the number of people in the sample who have the disease? Or the total number of positive test results? Or the probability that a positive result is wrong? Do the students use a diagram? Are they able to use more than one method?

In particular, note any common mistakes. Do students correctly identify the parts and wholes they are working with when calculating probabilities? For example, when calculating the probability that someone tests positive, do they find the number of people who have a positive result divided by the total number of people in the sample? When calculating the probability that a positive result is wrong, do they find the total number of false positive results divided by the total number of positive results? Are they able to carry out the work accurately, using their chosen method?

You can then use this information to focus the whole-class discussion at the end of the lesson.

Support student problem solving

There are many ways of producing a good solution to this problem. Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking. In particular, focus on their strategies rather than the correct solution. Encourage students to justify their statements, to you and to each other.

You may want to use the questions in the *Common issues* table to support your questioning. If the whole-class is struggling on the same issue, you could write a relevant question on the board, or hold a brief whole-class discussion.

Only if a group is **really** struggling to make any progress give them a copy of the *Hint Sheet*. This provides them with a table on which to record the outcomes of some calculations and a few questions to help them begin to calculate probabilities. We strongly suggest you avoid handing this out if you can, because it will direct students towards a single approach to the problem.

Whole-class discussion (15 minutes)

After students have had enough time to attempt the problem, you may want to hold a brief whole-class discussion.

Have students solved the problem using a variety of methods? Or have you noticed some interesting ways of working or some incorrect methods; if so, you may want to focus the discussion on these. If you have noticed different groups using similar strategies but making different assumptions, you may want to compare solutions.

Extending the lesson over two days

If you are taking two days to complete the unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of the problem before moving on to the collaborative analysis of sample responses.

Collaborative analysis of *Sample Responses to Discuss* (30 minutes)

This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

Give each group a copy of the four *Sample Responses to Discuss* and ask for written comments. However, you may feel that there is not time to address all four sample responses. In that case, use your understanding of your class's difficulties to choose the sample responses that are most appropriate to your students' needs. Groups that have struggled with a particular approach may benefit from seeing a sample version of the same strategy. Groups that have successfully completed the task using one method will benefit from looking at alternative approaches. Any groups that used the *Hint Sheet* could consider Amy's response, in which she uses the same method and compare it to at least one of the other approaches.

In your groups you are now going to look at some student work on the task. Notice in what ways this work is similar to yours and in which ways it is different.

The students in these sample responses have looked at Country A.

Slide P-2 of the projector resource describes how students should work together:

Evaluating Sample Responses to Discuss

1. Take turns to work through a student's solution.
   - Write your answers on your mini-whiteboards.
2. Explain your answer to the rest of the group.
3. Listen carefully to explanations.
   - Ask questions if you don't understand.
4. Once everyone is satisfied with the explanations, write the answers to the questions below the student's solution.
   - Make sure the student who writes the answers is not the student who explained them.

The students in these sample responses have looked at Country A.

If there is enough time, ask students to use the different methods to calculate the probability of a positive test being wrong in Country B.

During the group work, observe, listen, and support the students as before. In particular, note the similarities in and differences between the sample approaches and the approaches students took in their small-group work. Check, also, to find any methods students have difficulties in understanding.

This information can help you focus the next activity, a whole-class discussion.

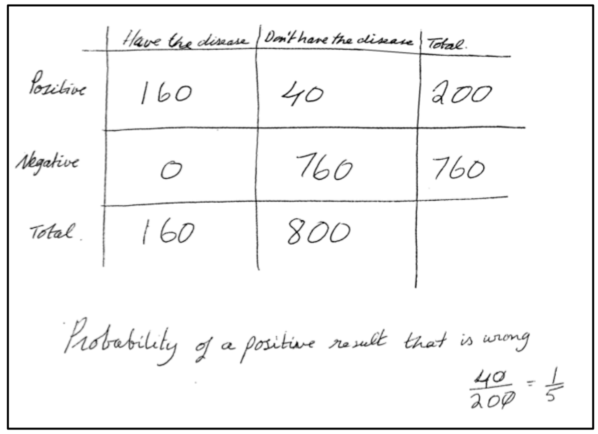
Amy used a table

Amy made a mistake in the table. There should be 200 people that have the disease and test positive, not 160.

Are the figures in the table accurate?

How many people have the disease? How many of the people with the disease test positive?

Amy used the correct formula for figuring out the probability of a false positive though.



	Have the disease	Don't have the disease	Total.
Positive	160	40	200
Negative	0	760	760
Total.	160	800	

Probability of a positive result that is wrong $\frac{40}{200} = \frac{1}{5}$

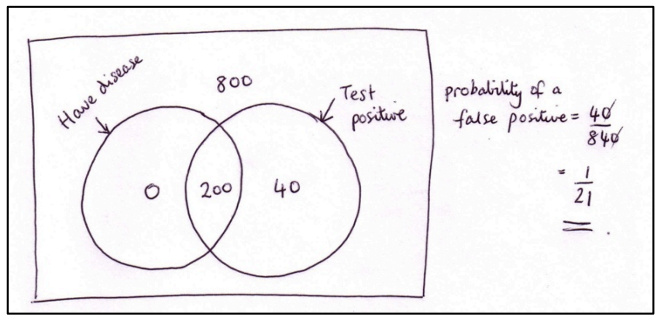
In Amy's probability calculation, what do the figures represent?

Is Amy using the correct formula to figure out the probability?

Noreen used a Venn diagram

Noreen did not explain how she calculated any of the figures in the diagram.

There is also an error: there should be 760 people that do not have the disease and test negative, not 800. The figures sum to 1,040 not 1,000.



How has Noreen calculated the figures in the Venn diagram? Are they correct?

What does the diagram tell you about the number of people who do not have the disease? Is this correct?

How many people do not have the disease and test negative?

Noreen calculated the probability of a false positive incorrectly.

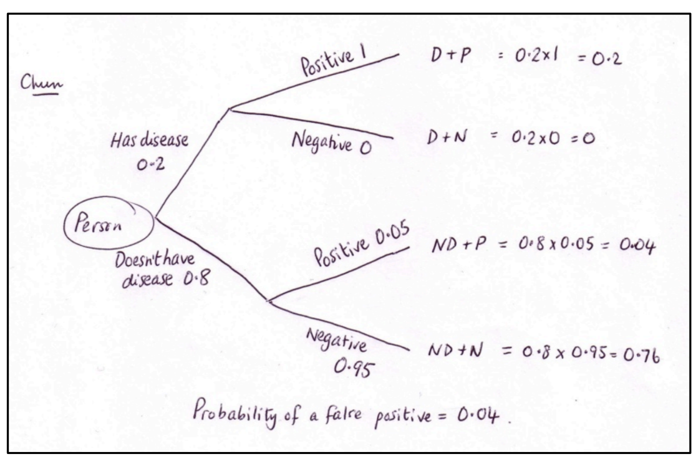
Remind me: What does 'false positive' mean?

If someone tests positive, what is the chance that the result is incorrect?

How many people test positive?
How many of these results are incorrect?

Chun used a tree diagram

The labels on Chun's diagrams are not very helpful to the reader trying to make sense of the work. She could have added more explanation. Chun assumed that the probability of a person not having a disease but having a positive test result is the same as the probability of a false positive. This is not the case. 0.04 is the probability of **anyone in the population** not having the disease and getting an incorrect test result.



The probability of a false positive is the probability of getting an incorrect result **given** that the test is positive, or:

$$\frac{\text{Number of people who tested incorrectly positive}}{\text{Number of people who tested positive}}$$

Is Chun's tree diagram accurate? [Yes.] Is the probability of a false positive correct? Please explain.

What probability has Chun calculated? [Probability of anyone in the population not having the disease and getting an incorrect test result.]

Remind me: What is the definition of the probability of a false positive? [Given a positive result, the probability that the result is wrong.]

What figures do you need to know in order to calculate the probability of a false positive? [Number of people who incorrectly tested positive ÷ Number of people who tested positive.]

How can you use the diagram to figure out the number of people who incorrectly tested positive? [(0.04 × 1000) = 40.]

How can you use the diagram to figure out the total number of people who tested positive? [(0.2 + 0.04 = 0.24) multiplied by 1000 = 240.]

So how do you work out the probability of a false positive result? [40 ÷ 240 or 0.04 ÷ (0.2 + 0.04) = 0.17.]

Rajeev put his answer in words

Rajeev did not explain how he calculated the numbers of people who have and do not have the disease. He calculated the number of false positive results correctly, but the probability of a false positive is incorrect.

200people have the desese, 800 don't.
No of people with a false positive = 5% of 800 = 40
Probality of a wrong positive result = 40/800 = 1/20

How has Rajeev calculated the number of people who have the disease and the number of people who don't have the disease?
[Number of people with the disease: 20% of 1,000 = 200 (these all test positive).
Number of people who don't have the disease: 80% of 1,000 = 800 (5% of these people test positive).]

Why is Rajeev's answer incorrect? [Rajeev should divide the number of false positives (40) by the total number of positive results (240).]

Whole-class discussion: comparing different approaches (25 minutes)

Organize a whole-class discussion to consider the different approaches used in the *Sample Responses to Discuss*. To begin, focus the discussion on parts of the task students found easy. Ask students who struggled with and made progress on a particular sample response to describe their reasoning:

At first, you found it difficult to understand this method. What was the difficulty?

You figured out how to solve that problem. What was your reasoning?

Ask the students to compare the different solution methods:

Which approach did you like best? Why?

Which approach did you find most difficult to understand? Why?

To support the discussion, you may want to use Slides P-3 to P-6 of the projector resource.

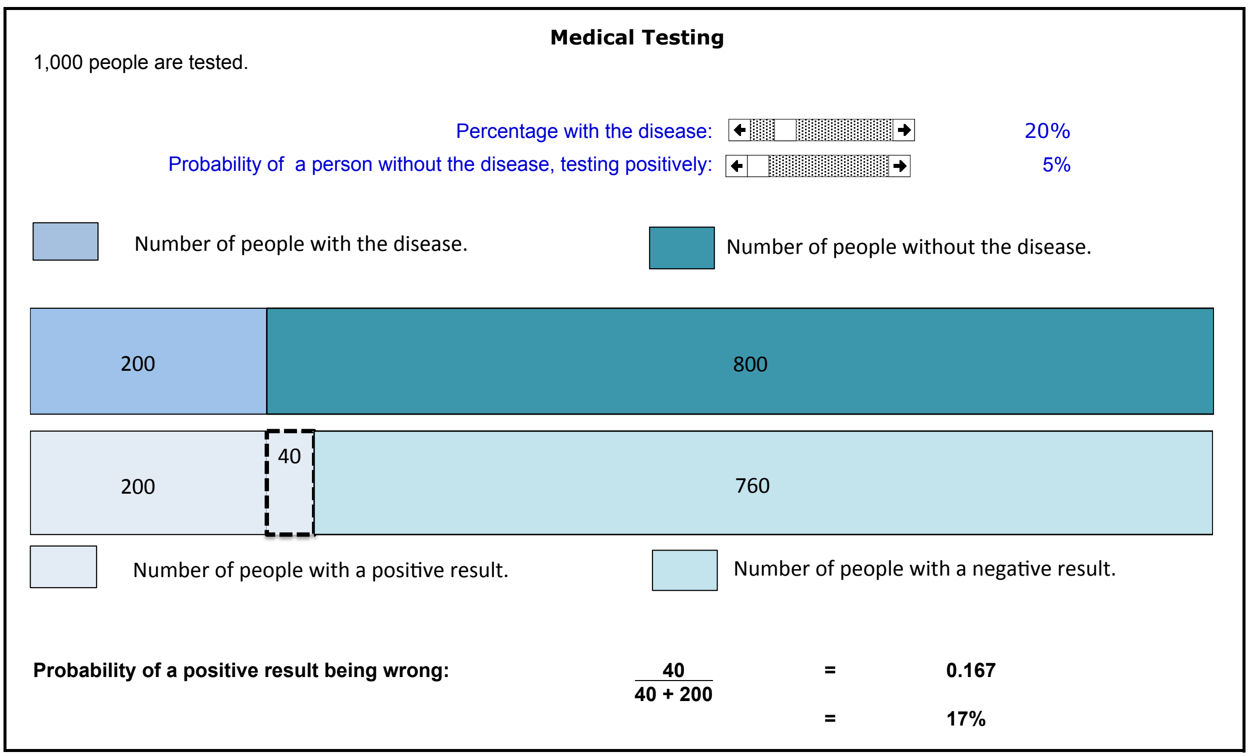
The false positive paradox

During the discussion, do not miss the opportunity to discuss the surprise element of this task. If a patient goes to their doctor and gets a positive test, the chances of it being wrong are much lower in Country A (0.17) than in Country B (0.71)! This is known as the 'false positive' paradox. The probability of a false positive result depends not only on the accuracy of the test itself, but also on the characteristics of the sample population (the size of the population and how many of them have the disease.)

(This has applications also in a legal context, where the context in which the accused has been brought to court is falsely assumed to be irrelevant to judging how confident a jury can be in evidence against them with a statistical measure of doubt.)

You now may want to show students the Sliders page of the *Medical Testing* applet or the sheet *Tests* in the Excel file *Medical Testing*.

The Excel sheet is shown below:



This sheet shows how the probability of a false positive alters as the rarity of the disease changes.

The first bar compares the number of people with the disease to the number without the disease. The second bar compares the number of people with a positive result to the number of people with a negative result. The number of people with a positive result is split into those with a correct result and those with an incorrect result. In the example above, there are 760 people with a correct positive result and 40 people with a false positive result. As the disease becomes more rare, the proportion of the positive tests that are incorrect increases.

Use the arrow keys to change the percentage of the population with the disease and the probability of a person without the disease testing positively.

You may want to ask students to predict some results:

Suppose you got a positive result. Should you be worried?

If the percentage of the population with the disease increases/decreases would you expect the probability of a positive result being wrong to change? Why?

If the probability of a person without the disease testing positive increases/decreases, would you expect the probability of a positive result being wrong to change? Why?

These questions are quite challenging, so make sure you allow students sufficient time to work on their own and then discuss the problem with a partner.

Follow-up lesson: review solutions to *Medical Testing* (20 minutes)

Give out the *How Did You Work?* sheet and ask students to complete this questionnaire. The questionnaire should help students review their progress.

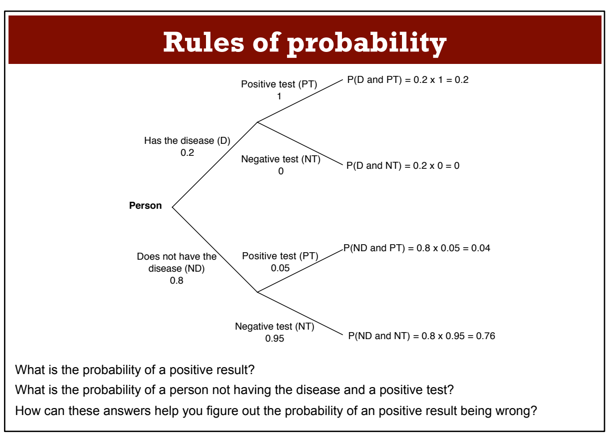
Read through your first solution. Think about all the work you've done on this problem: the first time you tried it, when you used my questions working alone, when you worked with your partner.

Fill in the questions as you reflect on your experience.

If you have time you may also want to ask students to use what they have learned to attempt the task again.

Optional extension: Using the rules of probability

With some classes, you may want to use the multiplication rule of probability to calculate the probability of a false positive. Show Slide P-7 of the projector resource:



Which country does this tree diagram represent? [Country A.]

What is the probability that a test is positive? [0.2 + 0.04 = 0.24.]

What is the probability of a person not having the disease and testing positive? [0.04.]

What is the probability of a positive result being wrong? [0.04 ÷ 0.24 = 0.17.]

With each question, allow students a few minutes to think about the question individually and then ask them to discuss the problem with a partner.

Next ask students to show you their answers. Find students with different answers and ask them to justify them. Encourage the rest of the class to challenge these explanations.

You may want to write these probabilities on the board as:

$P(ND \ \& \ PT) = 0.04$ or $P(ND \cap PT) = 0.04$.

$P(PT) = 0.2 + 0.04 = 0.24$.

$$P(ND \mid PT) = \frac{P(ND \ \& \ PT)}{P(PT)} = \frac{0.04}{0.24} = 0.17 \quad \text{or } P(ND \mid PT) = \frac{P(ND \cap PT)}{P(PT)} = \frac{0.04}{0.24} = 0.17.$$

You could also ask:

Is the probability of a positive test being wrong the same as the probability of a person who does not have the disease testing positive? [No, the probability of a person who does not have the disease testing positive is 5%.]

SOLUTIONS

When reviewing student work, remember to prompt for a full solution, which will include a useful representation of data with its interpretation in the context of the question.

Assessment task: *Medical Testing*

1. Suppose someone has the test and the result is positive. Does that person have the disease?

   This is not necessarily true. It is true that if they have the disease then the result is positive, but the reverse is not necessarily true. Perhaps the easiest way to see this is using a Venn diagram. The number having the disease is a **subset** of those testing positive.

2. How many people do not have the disease in the sample from Country A?

   20% of Country A has the disease. So 80% do not. 80% of 1000 people = 800 people.
   So 800 do not have the disease.

3. This question may be answered using many different representations. We consider several alternatives below, in turn.

(i) Using a Table

Country A

	Person has the disease	Person does not have the disease	Total number
Test is positive	200	40	240
Test is negative	0	760	760
Total number	200	800	1,000

Country B

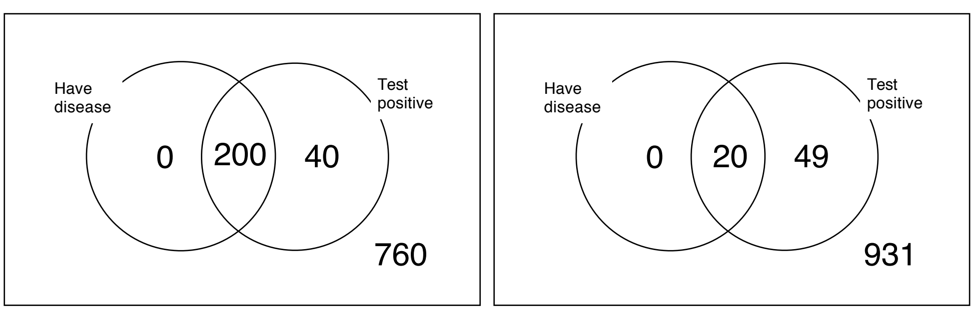
	Person has the disease	Person does not have the disease	Total number
Test is positive	20	49	69
Test is negative	0	931	931
Total number	20	980	1,000

The probability of a positive test being wrong in Country A is $\frac{40}{240} = 0.17$

The probability of a positive test being wrong in Country B is $\frac{49}{69} = 0.71$

(ii) *Drawing a Venn diagram*

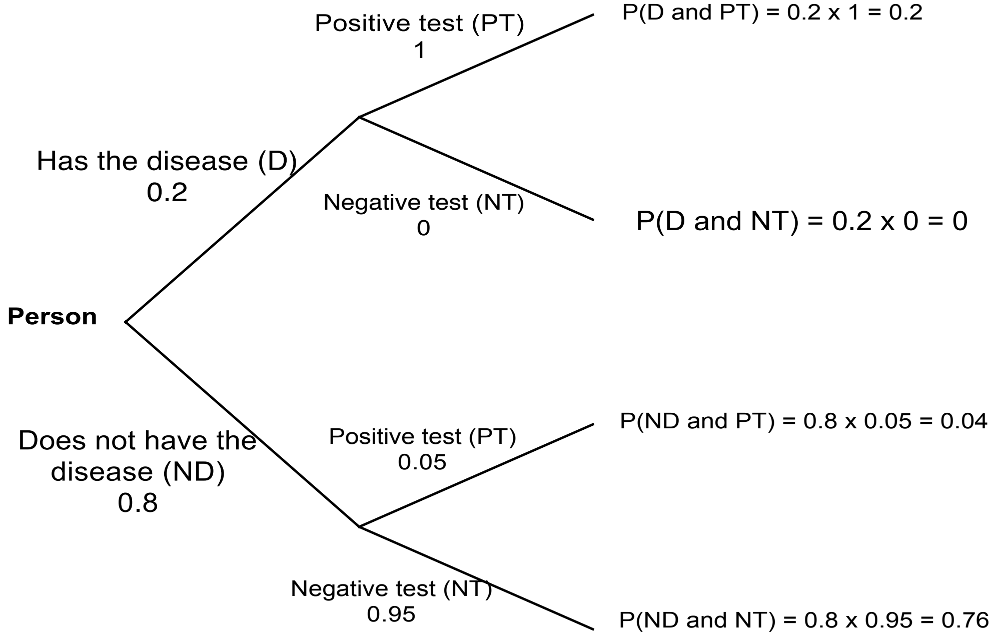
Country A **Country B**



The calculations are the same as for the Table solution.

(iii) *Tree Diagram*

Country A

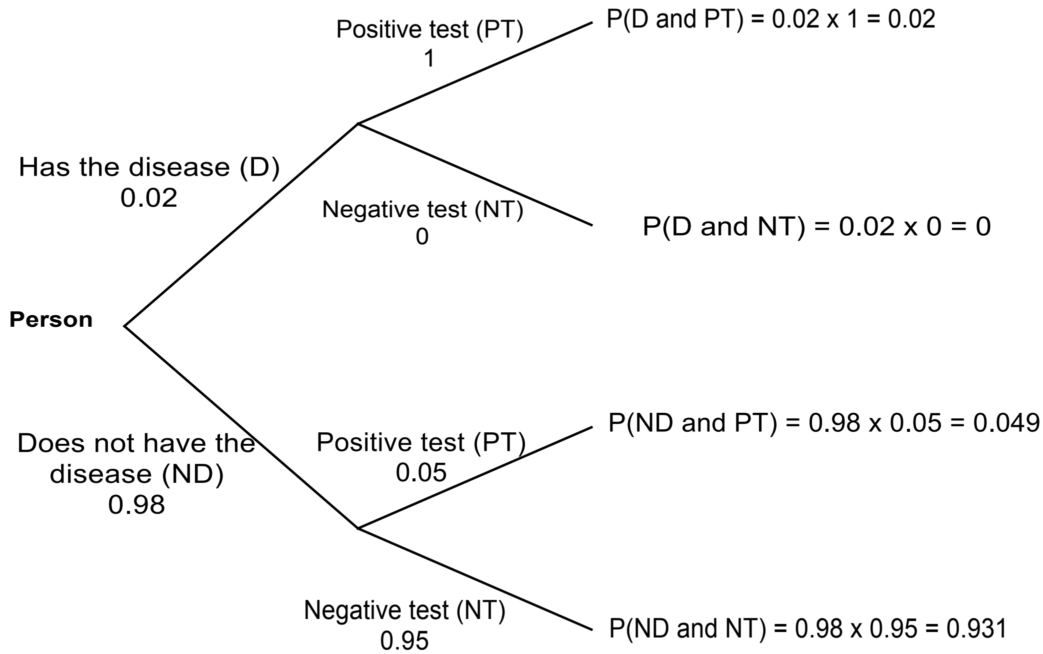


Number of people with a positve result: (0.2 + 0.04) × 1,000 = 0.24 × 1,000 = 240

Number of people with a positive result, but don't have the disease: 0.04 × 1,000 = 40

Probability of a false positive = $\frac{40}{240}$ = **0.17**

Country B



Number of people with a positive result: (0.02 + 0.049) × 1,000 = 0.069 × 1,000 = 69.

Number of people with a positive result, but don't have the disease: 0.049 × 1,000 = 49.

Probability of a false positive: $\frac{49}{69}$ = **0.71**

(iv) *The algebraic rules of probability*

Country A

D: Person has the disease; ND: Person does not have the disease; PT: Positive test; NT: Negative test.

P(ND & PT) = 0.04 or P (ND ∩ PT) = 0.04.

P(PT) = 0.2 + 0.04 = 0.24.

$$P(ND \mid PT) = \frac{P(ND \,\&\, PT)}{P(PT)} = \frac{0.04}{0.24} = 0.17 \quad \text{or } P(ND \mid PT) = \frac{P(ND \cap PT)}{P(PT)} = \frac{0.04}{0.24} = 0.17.$$

Country B

P(ND & PT) = 0.049 or P (ND ∩ PT) = 0.049.

P(PT) = 0.02 + 0.049 = 0.069.

$$P(ND \mid PT) = \frac{P(ND \,\&\, PT)}{P(PT)} = \frac{0.049}{0.069} = 0.71. \quad \text{or } P(ND \mid PT) = \frac{P(ND \cap PT)}{P(PT)} = \frac{0.049}{0.069} = 0.71.$$

Medical Testing

A new medical test has been invented to help doctors find out whether or not someone has got a deadly disease.

Experiments have shown that:

- If a person has the disease, then the test result will always be positive.
- If a person does not have the disease, then the probability of the test being wrong is 5%.
  This is called a **false positive** result.

Suppose the test is tried out in two different countries: Country A and Country B.
A sample of one thousand people is tested from each country.

- In Country A, 20% of the sample has the disease.
- In Country B, only 2% of the sample has the disease.

1. Suppose someone has the test and the result is positive.
   Does that person have the disease? Explain your answer.

   ---

2. How many people **do not** have the disease in the sample from Country A?
   Explain your answer.

   ---

3. Suppose a patient from each sample is told that they have tested positive.
   What is the probability that the test is wrong?
   Is your answer the same for each country?
   Explain your reasoning fully.

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Hint Sheet

Try to complete the table below for each country:

Country ...	Number of people who test positive	Number of people who test negative	Total
Number of people who have the disease			
Number of people who do not have the disease			
Total			

How many people in total take the test? Where is this shown in your table?

Figure out some probabilities from your table.

Use your results to calculate the percentage of false positive results.

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Sample Responses to Discuss: Amy

Country A

	Have the disease	Don't have the disease	Total.
Positive	160	40	200
Negative	0	760	760
Total.	160	800	

Probability of a positive result that is wrong

$\frac{40}{200} = \frac{1}{5}$

Write some comments about Amy's work.
Can you understand what she has done?
Try to correct her mistakes.

Use Amy's method to calculate the probability of a positive test being wrong in Country B.

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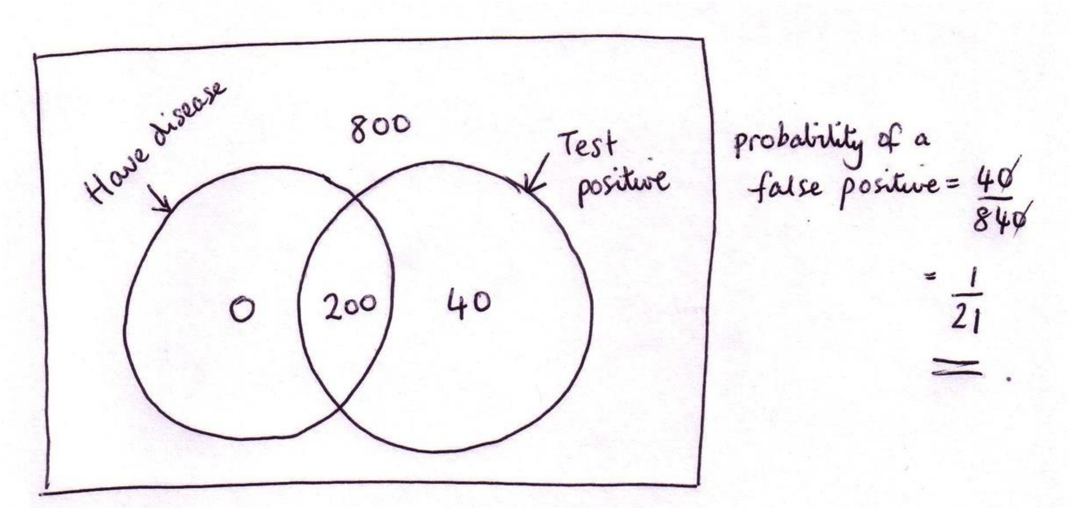
Sample Responses to Discuss: Noreen

Country A



Write some comments about Noreen's work.
Can you understand what she has done?
Try to correct her mistakes.

Use Noreen's method to calculate the probability of a positive test being wrong in Country B.

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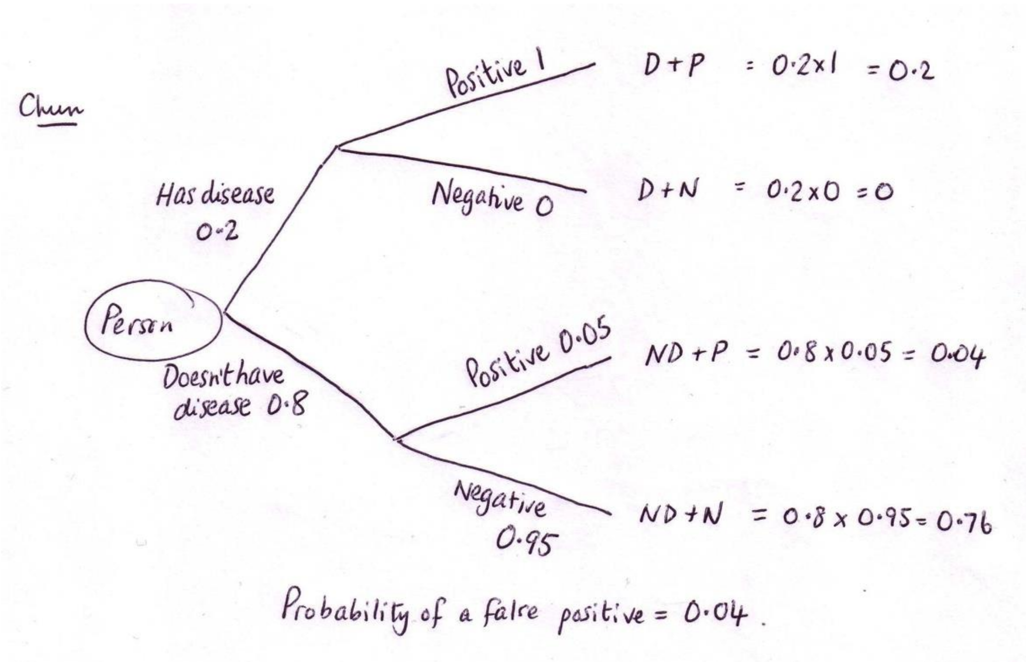
Sample Responses to Discuss: Chun

Country A



Write some comments about Chun's work.
Can you understand what she has done?
Try to correct her mistakes.

Use Chun's method to calculate the probability of a positive test being wrong in Country B.

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Sample Responses to Discuss: Rajeev

Country A

200people have the desese, 800 don't.
No of people with a false positive = 5% of 800 = 40
Probality of a wrong positive result = 40/800 = 1/20

Can you understand what Rajeev has done? What isn't clear about his work?

What has he got right? What mistakes has he made?

Use Rajeev's method to calculate the probability of a positive test being wrong in Country B.

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How Did You Work?

1. Compare the sample responses and your group response. What are the advantages and disadvantages of each approach?

	Advantages	Disadvantages
Noreen		
Chun		
Rajeev		
Our group work		

2. Now that you have seen Noreen's, Chun's, and Rajeev's work, what would you do if you started the task again?

3. What do you think are the difficulties someone new to the task will face?

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Planning a Joint Method

1. Take turns to explain your method and how you think your work could be improved.

2. Listen carefully to each other.
   – Ask questions if you don't understand.

3. Once everyone in the group has explained their method, plan a joint method that is better than each of your separate ideas.

4. Write a brief outline of your method on one side of your sheet of paper.

Make sure that everyone in the group can explain the reasons for your chosen method.

Evaluating Sample Responses to Discuss

1. Take turns to work through a student's solution.
   – Write your answers on your mini-whiteboards.

2. Explain your answer to the rest of the group.

3. Listen carefully to explanations.
   – Ask questions if you don't understand.

4. Once everyone is satisfied with the explanations, write the answers to the questions below the student's solution.
   – Make sure the student who writes the answers is not the student who explained them.

The students in these sample responses have looked at Country A.

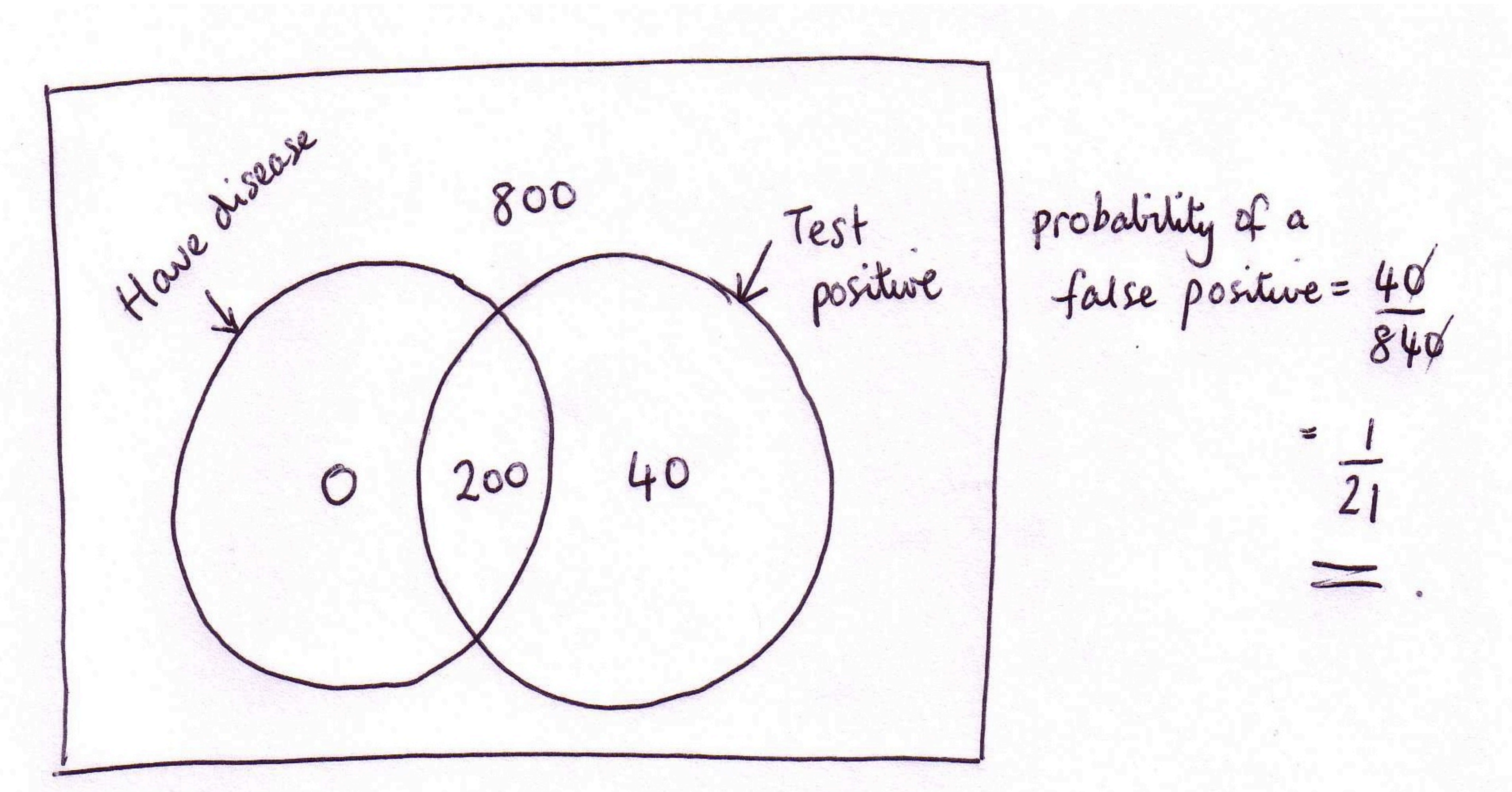
Sample Responses to Discuss: Amy

	Have the disease	Don't have the disease	Total.
Positive	160	40	200
Negative	0	760	760
Total.	160	800	

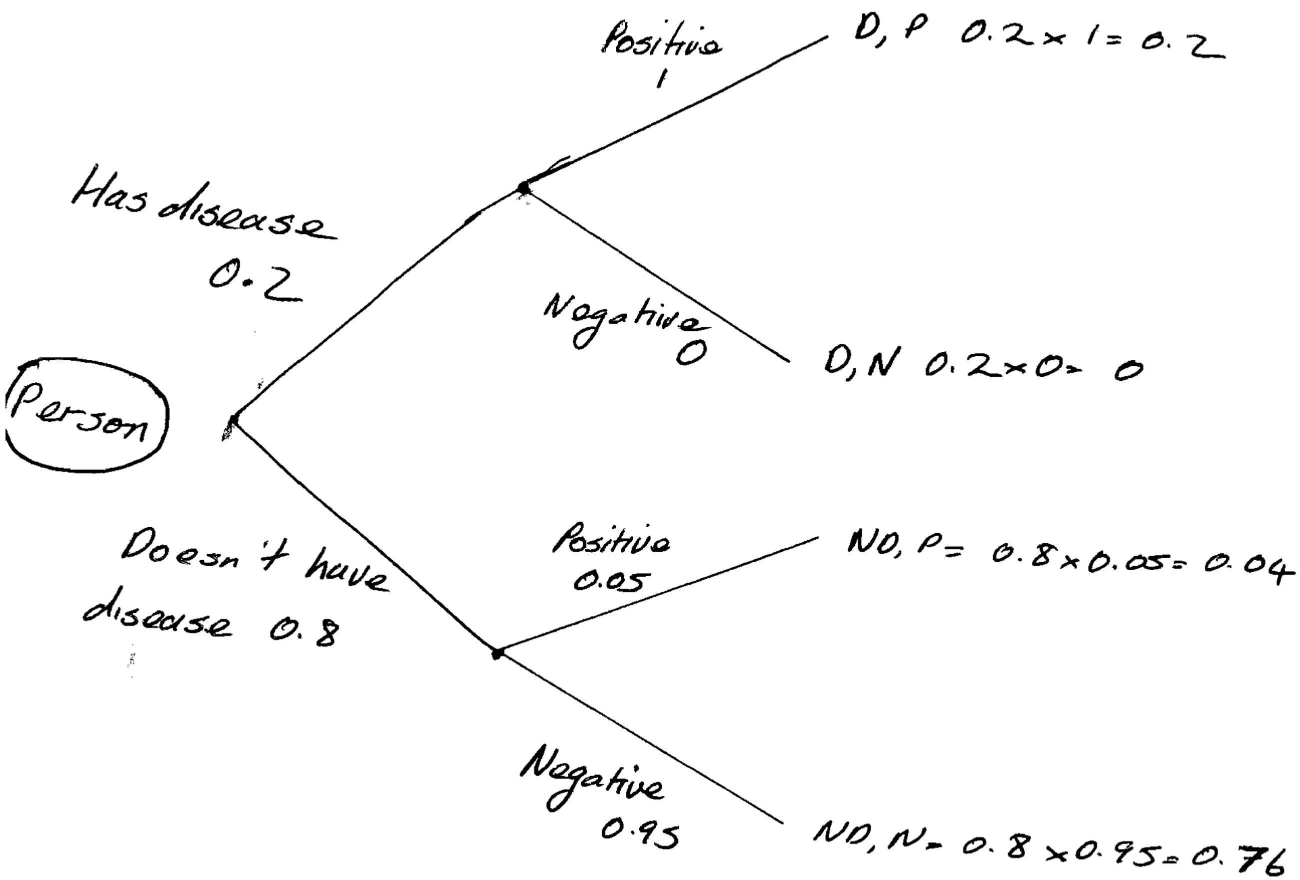
Probability of a positive result that is wrong

$$\frac{40}{200} = \frac{1}{5}$$

Sample Responses to Discuss: Noreen



Sample Responses to Discuss: Chun

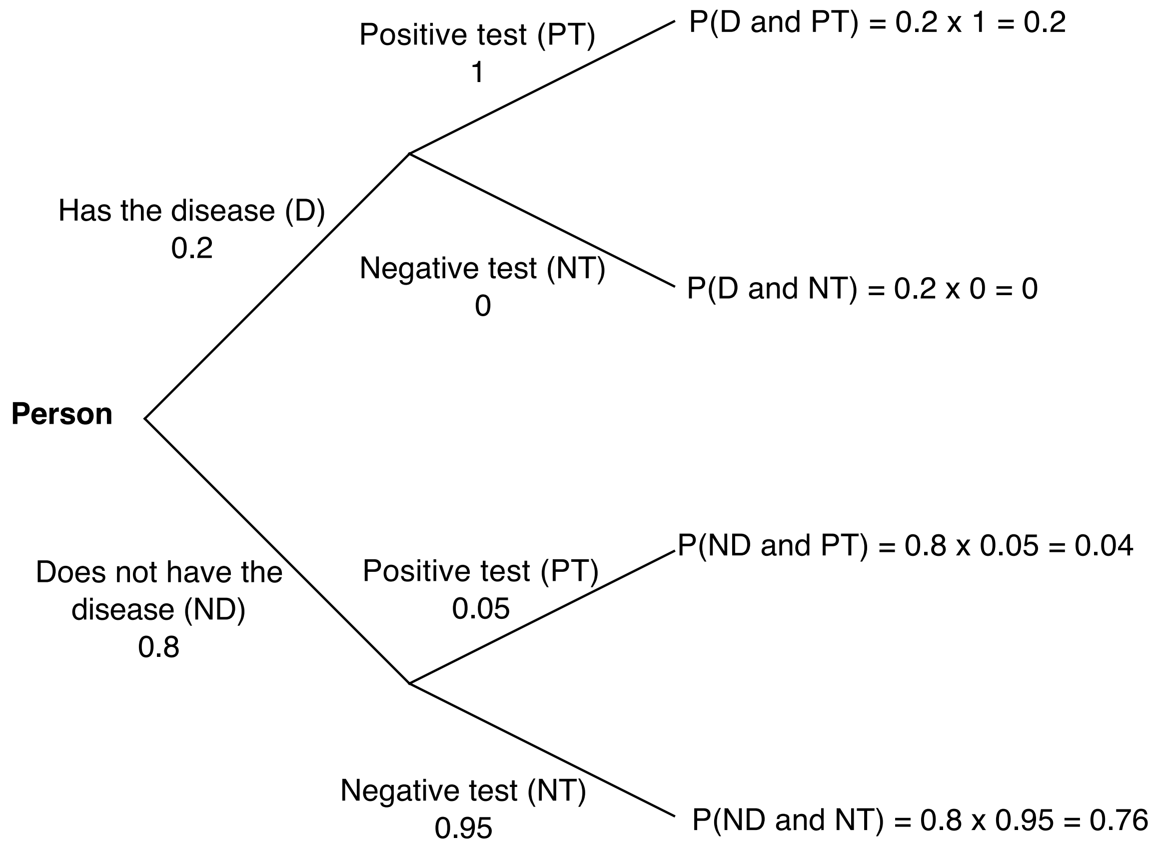


Probability of a false positive = 0.04

Sample Responses to Discuss: Rajeev

200people have the desese, 800 don't.
No of people with a false positive = 5% of 800 = 40
Probality of a wrong positive result = 40/800 = 1/20

Rules of probability



What is the probability of a positive result?

What is the probability of a person not having the disease and a positive test?

How can these answers help you figure out the probability of a positive result being wrong?

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

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with
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who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
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Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org

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