PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Modeling Motion:
Rolling Cups

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Modeling Motion: *Rolling Cups*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Choose appropriate mathematics to solve a non-routine problem.
- Generate useful data by systematically controlling variables.
- Develop experimental and analytical models of a physical situation.

COMMON CORE STATE STANDARDS

This lesson relates to **all** the *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, 4, 5, 7, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

F-BF: Build a function that models a relationship between two quantities.
G-MG: Apply geometric concepts in modeling situations.
G-GMD: Visualize relationships between two-dimensional and three-dimensional objects.
G-SRT: Prove theorems involving similarity.
A-CED: Create equations that describe numbers or relationships.

INTRODUCTION

- Before the lesson, students watch a video of *Rolling Cups*. They then work on a related task designed to assess their current approaches to modeling. You collect and review students' work and write questions to help improve their models.
- During the lesson, students first work individually to review their initial work. They then work in pairs to compare their approaches to the task, choosing a strategy that will produce an improved model and together implement that strategy. Students next review some sample work before discussing as a class the strengths and weaknesses of different approaches to the modeling task.
- In a follow-up lesson, students review their work on the task.

MATERIALS REQUIRED

- Each student will need a copy of the task sheet: *Modeling Rolling Cups*.
- Each pair of students will need a fresh copy of the task sheet, *Modeling Rolling Cups*, copies of each of the *Sample Responses to Discuss,* a large sheet of paper, and a felt-tipped pen.
- It is desirable that each pair of students has access to a computer with the applet *Modeling with Geometry: Rolling Cups*. If this is not possible the lesson can still be conducted with one class computer and a data projector.
- Supply paper and plastic cups, rules, scrap paper, graph paper, protractors, string, chalk, glue, scissors, and calculators for students who choose to use them.
- For the whole-class discussions, you will need a computer with data projector and two plastic/paper cups that have different base diameters and heights.

TIME NEEDED

30 minutes before the lesson, a 90-minute lesson (or two 50-minute lessons), and 20 minutes in a follow-up lesson. All timings are approximate and will depend on the needs of your students.

BEFORE THE LESSON

Introduction to *Modeling Rolling Cups* (15 minutes)

We suggest you take time to explore carefully some different solution strategies to this problem before teaching the lesson. Introduce the problem scenario to students. We describe two ways of doing this:

(i) Practical demonstration

Show students a cup and make sure they understand the terms 'slant height', 'perpendicular height', 'narrow diameter', and 'wide diameter'.

Show the students a paper cup and indicate the **slant side** of the cup.

I'm going to lay this cup on its slant side and push it. What do you think will happen?

Roll the cup.

Tell me what you see.
Was this what you predicted?

Take a second cup. Ask students to make predictions about the 'roll radius', the radius of the circle the top of the cup (widest diameter) rolls.

Do you think the circle this cup rolls will have a bigger or smaller roll radius? Why?

Check students' predictions by rolling the cup.

(ii) Using the computer data projector

Introduce the task using the video extracts on the MAP website:
http://map.mathshell.org/lesson_support/rolling_cups/

Display the introductory slide, *The Task*. Ask students to make some predictions:

Which of these objects do you think will roll the largest circle? Why?
What about the second largest circle?
What will happen when the soup can is rolled?

Accept all student answers without comment.

Show the four videos: *Short Glass*, *Plastic cup*, *Tall Glass*, *Soup can*.

As I play each video, try to guess what will happen!
Were you surprised by any of the results?

Note: At this stage, do **not** show the *Cup Rolling Calculator*. That is for the main lesson.

Individual work on the assessment task: *Modeling Rolling Cups* (15 minutes)

Give each student a copy of the *Modeling Rolling Cups* task.

I would like you to think about how to solve this problem. Think about what you've seen so far.

Explain what you want students to do.

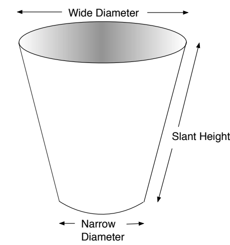
I would like you to spend 15 minutes working on this task.

Make sure you explain your work clearly.

At the top of the sheet is the data you saw in the four videos, together with some more data.

Modeling Rolling Cups

Cup	Dimensions in inches			
	Wide diameter	Narrow diameter	Slant length	Roll radius
A	3½	3	3¾	26¼
B	3	2	3½	10½
C	2½	2	5¾	28¾
D	3	3	4¼	Infinite!
E	3	2	6	18
F	3½	2	3¾	8¾
G	3¾	3	3¾	18¾
H	3½	0	3¾	3¾



Here is a reminder of the data you saw in the video with a few extra cups added.

1. Describe how each of the three lengths on the picture affect the roll radius.
   Show how you used the data to explain your ideas.

2. Show how you can use math to predict the radius of the circle rolled by *any* size of cup.
   Show all your reasoning, including any diagrams and calculations.

Slide P-1 of the projector resource may be projected as you make these points.

It is important that, as far as possible, students are allowed to answer the question without assistance. If students are struggling to get started, help them understand what is required, but make sure you do not do the task for them. The first few questions on the *Common issues* table, on the next page, may be useful.

Students who sit together often produce similar answers, so that, when they come to compare their work, they have little to discuss. For this reason we suggest that, when students do the task individually, you ask them to move to different seats. At the beginning of the formative assessment lesson allow them to return to their usual seats. Experience has shown that this produces more profitable discussions because students have varied approaches to discuss.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

Do not be too concerned if you found this work difficult, as we will have a lesson tomorrow that should help you. You will have a chance to improve your solutions then.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current modeling strategies.

We suggest that you do not score students' work. Research shows that this will be counterproductive, as it will encourage students to compare scores and distract their attention from what they can do to improve their mathematics. Instead, help students make further progress by using a list of questions and prompts that summarize ways to develop their modeling strategies. Some suggestions for these are given in the *Common issues* table on page T-4. We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues:	**Suggested questions and prompts:**
Has some idea of the effect of the variables but has not considered how they interact (Q1) For example: The student says that if the wide diameter is made bigger, then the cup rolls in a smaller circle.	• Is this true whatever the narrow diameter value? • What happens if the narrow diameter is equal to the wide diameter?
Does not control the variables (Q1) For example: The student draws a conclusion by comparing two cups with no dimensions in common.	• What do you think would happen if you fixed the narrow diameter and slant height and varied the wide diameter?
Makes no use of the data (Q1&2) For example: The student makes no reference to the data in their discussion of the problem.	• How might the data on the task sheet be helpful? • Could you reorganize the data from the video to show any patterns? How might that help you?
Does not consider all the variables (Q1&2) For example: The student only considers slant height in their hypothesis about which cup rolls the larger circle. Or: The student omits reference to one variable.	• Which measures occur in the data from the video? • Are there any other measures that might be useful?
Has no modeling strategy (Q2) For example: The student has written/drawn little.	• What do you already know? What are you trying to find? • Could you draw a diagram? Where is the roll radius on your drawing? How is your diagram related to the measures in the problem?
Does not move on from data production For example: The student organizes the data to vary quantities systematically but does not analyze for patterns. Or: The student notices a pattern but does not try to record it algebraically.	• Could you reorganize the data to help you see a pattern? • Can you see any patterns? Describe the pattern you see in words. • How could you record what happens in the pattern every time using algebra? • What are the extreme cases? How would they show in a formula?
Requests further information For example: The student says they cannot solve the problem because there is insufficient information.	• What other data do you want? Describe the experiments you want to do. How will that information help you solve the problem?
Only draws a diagram For example: The student makes a scale drawing without explanation. Or: The student sketches the cup, identifying key variables.	• How will you use your diagram/scale drawing to solve the problem?
Provides a model For example: The student uses scale drawings to predict which cup will roll the larger circle. Or: The student writes a formula.	• You predict that … How could you check this? • Does your model make a correct prediction for each of the cups in the table? • Would your model work for any cup? How do you know?

SUGGESTED LESSON OUTLINE

Introduction and individual work (10 minutes)

Remind students of their work on *Modeling Rolling Cups*.

Your work today will be to produce a mathematical solution to Modeling Rolling Cups. There are lots of different ways of doing this!

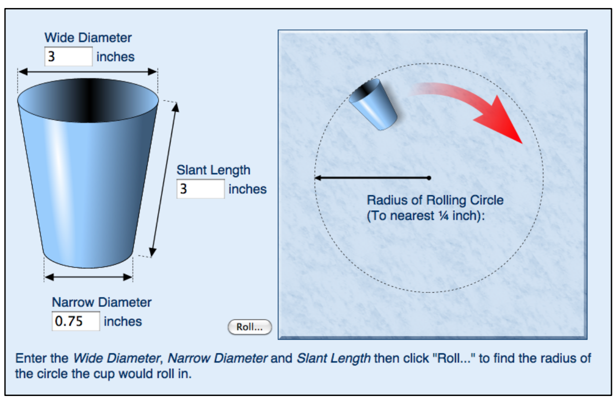
Give the students their scripts from the assessment task. If you have not written your questions on individual scripts, display them on the board for the class to read.

I read your solutions from [last lesson] and wrote some questions that might help you to improve your work. Spend a few minutes answering my questions. Then decide what you will do next to solve this problem.

Ask students to read through your questions carefully, spend 10 minutes answering them, and make a decision about a possible plan of action.

Introducing the *Cup Rolling Calculator* (5 minutes)

Show students the *Cup Rolling Calculator* using a data projector:



To improve your models, you could try rolling some cups. We also have a calculator that will help you produce more data, if you need it.

Demonstrate how to use the calculator:

What values could I choose for the narrow diameter? Where do I put that?

Which number or numbers do you want me to change?

What is the dependent variable?

Teacher note

The students' next job is to work together to produce an improved solution to the task. There are two ways to work on this: using a practical method, and/or using the computer calculator. The practical method has the advantage that students can more readily visualize the 3-dimensional aspects of the situation, which should help them move towards an analytical method. The computer calculator generates data rapidly and helps students to find patterns and make generalizations.

Both approaches have their difficulties. Students may find it difficult to record on paper how the cup rolls and to make accurate measurements. Some students may, therefore, opt to work with real cups, string, chalk, and the sidewalk! The computer approach may lead to students getting buried in data and will not help them develop explanations.

It is worthwhile for students to try both approaches where possible. Everyone should have the experience of rolling one cup and gathering data from it, if possible. The computer can then be used to gather additional data.

Collaborative small-group work: modeling (30 minutes)

Ask students to form pairs and give each pair a fresh copy of the *Modeling Rolling Cups* task, a paper cup, a rule, a protractor, a pencil, a large sheet of paper, and a felt-tipped pen.

You're going to work in pairs today.

To begin, take turns to explain your work to the other student in your pair. When listening to your partner's solution, ask questions so you understand.

When your pair has shared solutions, choose a strategy to work on together. Take about 5 minutes to come up with a plan of work.

Work together to produce a joint solution to the problem that is better than either of your individual solutions.

I have given you a paper cup and some equipment that you can use to help you visualize the problem and collect some data on one cup. There is other equipment if you need it. There is also the Rolling Cup Calculator that you can use to collect additional data.

Plan your data collection before you start, so that you don't collect too much!

These instructions are summarized on Slide P-2 of the projector resource.

If there is only one computer and data projector, explain that when students have discussed their ideas in their pairs for a while, they should write a list of any additional data they want collecting. You will collect this data for them using the computer.

During the collaborative small-group work, you have two tasks: to notice students' approaches to the task and to support their modeling activity.

Notice students' approaches to the task

What kind of models do the students attempt to make? Do they produce an empirical model based on data? Do they analyze the data for patterns and then record those algebraically to produce a formula? Or do they produce an analytical model using geometry?

When using the computer calculator, do students control the variables, keeping some fixed and varying others systematically? Do students recognize the need to go beyond data production? Do they consider extreme cases such as 'wide diameter = narrow diameter' or 'narrow diameter = 0'? Do they choose useful representations, such as tables or graphs? Do they identify patterns in the data and try to record these algebraically? Do they try to identify a general rule or formula from their data?

When using a practical approach by rolling a cup, do students record the path of a cup on a surface using e.g. chalk? Do they attempt to draw scale diagrams? Do they produce an analytical geometric model using similar triangles? Do they distinguish 'slant height' and 'perpendicular height'? Do they recognize what role height plays in their geometrical models?

Support students' modeling activity

There are many ways to tackle this task. Try not to prompt students towards using a particular approach. Instead, ask questions that help students develop and clarify their thinking while following their chosen modeling strategy.

It is important to recognize that after producing data, some students may struggle to identify how to move forward. They may need to be reassured that this struggle is just part of the modeling process.

Students themselves need to figure out the math they know that is relevant to this situation, and formulate a strategy for moving forward. By engaging with that struggle, students can experience being real problem solvers, rather than just solving problems with information and strategies given to them.

> *Why did you choose these values for the narrow/wide diameter? What other data might be useful?*
>
> *What happens if… you make the narrow diameter much bigger than/close in size to/smaller than the wide diameter? …you increase the wide diameter but keep the narrow diameter fixed?*
>
> *Can you see any patterns in the data? Use your patterns to predict new results. Check to see if you are correct.*
>
> *How would you write that pattern using algebra? What does that tell you about a formula for the roll radius?*
>
> *How else could you represent your data to help understand the relationships? [Graphs, tables, diagrams, scale drawings.]*
>
> *How do you think the roll radius depends on the narrow diameter / the wide diameter / the slant length?*
>
> *Try to combine the patterns you have learned about into a formula to help you predict the size of **any** cup roll radius.*
>
> *How can you check that your answer is correct?*
>
> *Think about what you know about real cups rolling. Does your formula make sense?*

Extending the lesson over two days

If you are taking two days to complete the lesson unit then you may want to end the first lesson here. At the start of the second day, allow students time to familiarize themselves with their joint solution before moving on to the collaborative analysis of sample responses.

Collaborative analysis of *Sample Responses to Discuss* (30 minutes)

The sample responses will help students to think further about their own approaches and confront them with approaches they may have not yet considered.

Give each group of students a copy of all three *Sample Responses to Discuss*.

> *Choose one of the sample solutions. Read it carefully and then work together to answer the questions.*
>
> *When you have finished your work on one sample solution, choose another.*

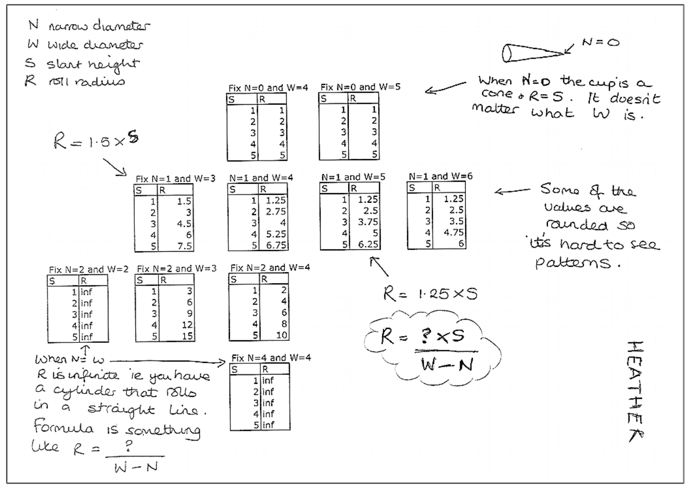
It is important that students work in depth through a couple of the *Sample Responses to Discuss*, to see different strengths in contrasting approaches. This is more important than covering all of the material.

Whilst students are analyzing the *Sample Responses to Discuss*, support their thinking as before.

> *What approach is Gerry taking?*
>
> *How might Heather use that data to complete a formula?*
>
> *How does Judi's diagram connect with Gerry's?*

The three modeling strategies shown in the *Sample Responses to Discuss* are described in detail on the next page. You may want to use some of this information to support the whole-class discussion at the end of the lesson.

Heather uses the computer calculator to generate data. She generates the data by fixing two variables, *W* and *N*, and varying *S* to see the impact on *R*. She analyses some of her data to find patterns and uses the patterns to define aspects of the formula for *R*. She interprets some of the formula and data in the context of the problem. The way Heather sets out her data on the page is important. She aligns values so that it is possible to see how systematic changes in *N* and *W* result in patterned change in *R*. These are strengths in her solution.



Heather might usefully continue with her systematic exploration of values of *N* and *W* by looking at *N=2, W=3*, *N=3*, *W=4*, and so on. She might then be able to see patterns in the resulting values of *R*. Graphing values of *S*, *R* for sets of values of *N* with fixed *W*, or *W* with fixed *N* might be useful

However, Heather could complete the formula using her data on *N*=0.

She claims that $R = \dfrac{S \times ?}{W - N}$.

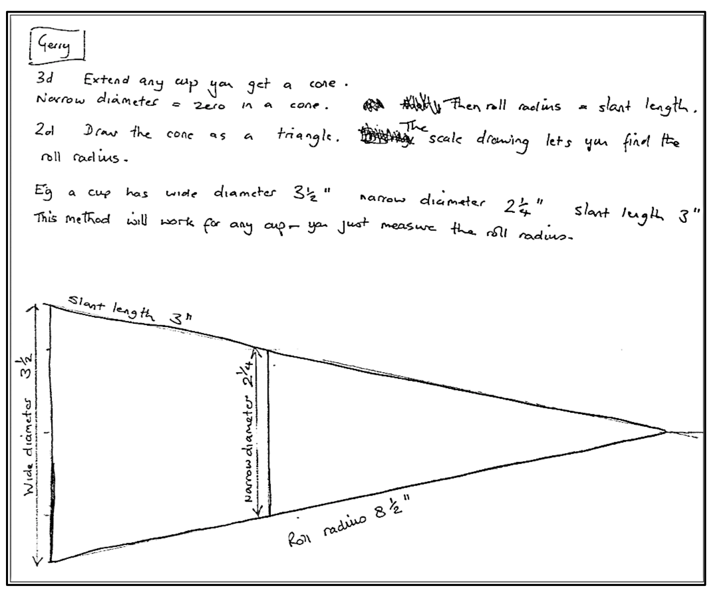
She knows that when *N*=0, the cup is a cone and it does not matter what *W* is. In that case, the formula must make $R = \dfrac{S \times ?}{W - 0} = S$ gives $R = \dfrac{SW}{W - 0}$. The formula is thus $R = \dfrac{SW}{W - N}$.

Some of Heather's argument is strong: she links the real cups she has seen with empirical data, patterns in the data, and the formula. However, her presentation is a bit disorganized and she does not bring all the pieces of her argument together.

The disorganized nature of Heather's work may be a useful stimulus for discussion with students on the differences between the notes they produce in working towards a solution and the reorganization of materials required to present and communicate a solution to others.

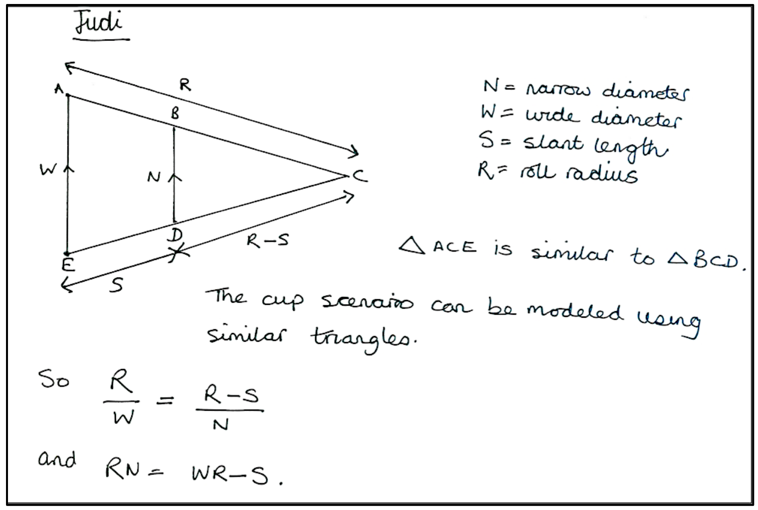
Gerry uses a simple, accessible strategy that gives a partial solution to the problem. Given any cup, Gerry could draw a scale diagram that would allow him to make an approximate measure of the roll radius, depending on the accuracy of the scale drawing. However, Gerry's approach must be used each time; it is not a general solution.

Gerry provides a little explanation of how to move from a cup to a cone, and from a cone to a triangle. He does not explain how he knows that his triangle is isosceles.



Asking students to use Gerry's method enables them to engage with his construction problem: how, given these measures, do you construct an isosceles triangle with an embedded isosceles trapezium. The value of *R* found given the values of *N, W*, and *S* should be about 4½".

The advantage of Gerry's method is that it is easy to understand. Some difficulties with Gerry's method are that the construction method needs to be used accurately to produce a reliable measure for the roll radius and the method does not produce a general solution: a scale drawing must be produced for each cup. As a result, his solution is not easily reversible, e.g. it would be difficult constructing the triangle, knowing the roll radius to find specific values of *N, W* and *S*.



Judi draws a diagram showing the cup in two dimensions as a pair of similar triangles. She says that the cup scenario can be modeled this way, but gives no explanation of how to move from a cup to the diagram.

Throughout, Judi uses capital letters to represent variables. While not incorrect, this may be confusing as the labels for vertices are also capitals. It is more conventional to use lower case for variables.

Judi claims that Triangle ACE is similar to Triangle BCD but does not explain how she knows this. Overall, although she has chosen quite sophisticated mathematics to use, she provides almost no explanation of her work.

The argument for similar triangles could be made as follows:

Angle BCD is common to both triangles.

Slicing the cone perpendicular to the ground, the face is a triangle. The wide diameter and narrow diameter included in that triangle are parallel because they are in the same plane (slice) and because the top of the cup is (assumed) to be parallel to the base. So *W* is parallel to *N*.

Then Angle EAB = Angle DBC.

Since the sum of the angles in a triangle is always 180º, the third angles must also be congruent.

So as the **three angles are equal**, the triangles must be similar.

Since the triangles are similar, corresponding side lengths are in the same ratio.

Hence $\frac{R}{W} = \frac{R-S}{N}$

Judi makes a standard error in the manipulation of this formula:

$RN = WR - S$

Judi's working can be corrected and her method used to find a formula for *R*.

$RN = W(R-S)$ and $RN = WR - WS$.

Thus $R(W - N) = SW$ and $R = \frac{SW}{(W - N)}$.

A full solution would include making sense of the formula in the context, by, for example, showing what happens for values of W and N that produce the extreme cases of the cone and cylinder.

Check that your students distinguish squashing a cup flat from slicing the cup perpendicular to the ground!

Whole-class discussion: comparing different approaches (15 minutes)

Ask students to compare the work they have seen on this problem and to identify the different strengths of the various methods. Slides P-3, P-4, and P-5 of the projector resource contain the three *Sample Responses to Discuss* and can be used to support this discussion.

Describe one strategy for solving the problem. Did any of you solve the problem in that way?

What are the strengths of that method? Were there any difficulties with that method?

Describe another strategy for solving the problem.

Which method did you prefer? Why?

Ask students to review their work on the modeling problem across the lesson:

What was your initial strategy for solving this problem?

Has your strategy changed during the lesson? How? Why?

What advice would you give to someone just starting on this problem?

Follow-up lesson: individual review (20 minutes)

Give each student a copy of the *How Did You Work?* questionnaire. This is intended to provoke students to reflect on and make comparisons between the methods used in this lesson.

Think carefully about your work on this task. On your own, answer the review questions as carefully as you can.

Some teachers give this as a homework task.

SOLUTIONS

1. This question is designed to see whether students can make deductions from the data by controlling variables.

They can deduce that:

If the wide diameter is increased and other variables held constant, then the roll radius will decrease.
This may be deduced from the data for cups A and G.

If the narrow diameter is increased (and other variables held constant), then the roll radius will increase.
This may be deduced from the data for cups A and F.

Modeling Rolling Cups

Cup	Dimensions in inches			
	Wide diameter	Narrow diameter	Slant length	Roll radius
A	3½	3	3¾	26¼
B	3	2	3½	10½
C	2½	2	5¾	28¾
D	3	3	4¼	Infinite!
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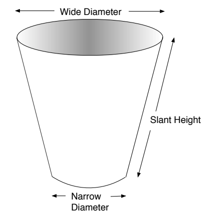


Here is a reminder of the data you saw in the video with a few extra cups added.

1. Describe how each of the three lengths on the picture affect the roll radius. Show how you used the data to explain your ideas.

2. Show how you can use math to predict the radius of the circle rolled by *any* size of cup. Show all your reasoning, including any diagrams and calculations.

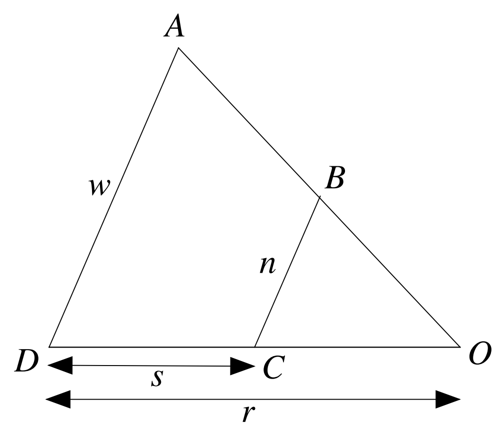
If the slant height is increased (and other variables held constant), then the roll radius will increase. This may be deduced from the data for cups B and E.

If the narrow diameter is zero, the roll radius is equal to the slant height of the cup. This can be deduced from cup H.

2. This question is best answered using a geometrical construction. Your students may, however, devise other, equally valuable approaches.

The diagram shows a cup ABCD on its side. It rolls in a circle radius r about a point O.

The wide diameter = w
The narrow diameter = n
The slant height = s
The roll radius = r



Triangles AOD and BOC are similar, since <DAO = <CBO (AD is parallel to BC) and <AOD is common.

Therefore:

$$\frac{r}{w} = \frac{r-s}{n}$$

$$\Leftrightarrow rn = rw - sw$$

$$\Leftrightarrow sw = r(w-n)$$

$$\Leftrightarrow r = \frac{sw}{w-n}$$

This result shows that that the roll radius is proportional to the slant height. It also shows that when n and w are nearly equal then the roll radius becomes very large, as is shown by the tin can!

Modeling Rolling Cups

Cup	Dimensions in inches			
	Wide diameter	Narrow diameter	Slant length	Roll radius
A	3½	3	3¾	26¼
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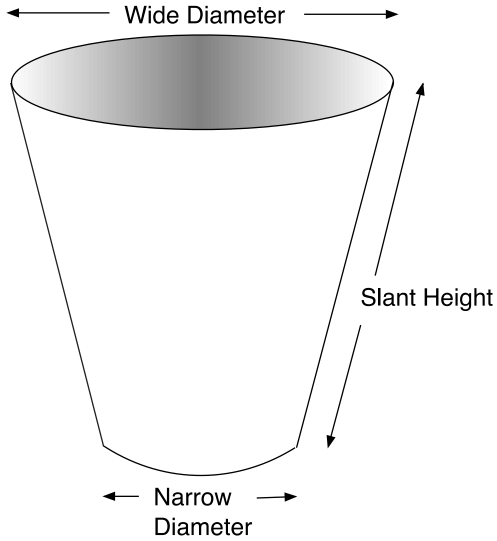

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   Show all your reasoning, including any diagrams and calculations.

Sample Responses to Discuss: Heather

N narrow diameter
W wide diameter
S slant height
R roll radius

When N = 0 the cup is a cone & R = S. It doesn't matter what W is.

Fix N=0 and W=4

S	R
1	1
2	2
3	3
4	4
5	5

Fix N=0 and W=5

S	R
1	1
2	2
3	3
4	4
5	5

$R = 1.5 \times S$

Fix N=1 and W=3

S	R
1	1.5
2	3
3	4.5
4	6
5	7.5

N=1 and W=4

S	R
1	1.25
2	2.75
3	4
4	5.25
5	6.75

N=1 and W=5

S	R
1	1.25
2	2.5
3	3.75
4	5
5	6.25

N=1 and W=6

S	R
1	1.25
2	2.5
3	3.5
4	4.75
5	6

Some of the values are rounded so it's hard to see patterns.

$R = 1.25 \times S$

$$R = \frac{? \times S}{W - N}$$

Fix N=2 and W=2

S	R
1	inf
2	inf
3	inf
4	inf
5	inf

Fix N=2 and W=3

S	R
1	3
2	6
3	9
4	12
5	15

Fix N=2 and W=4

S	R
1	2
2	4
3	6
4	8
5	10

When N = W R is infinite ie you have a cylinder that rolls in a straight line.
Formula is something like $R = \frac{?}{W - N}$

Fix N=4 and W=4

S	R
1	inf
2	inf
3	inf
4	inf
5	inf

HEATHER

What values for the narrow diameter and wide diameter do you think Heather should choose next? Justify your answer.

Heather has described some patterns in the data.

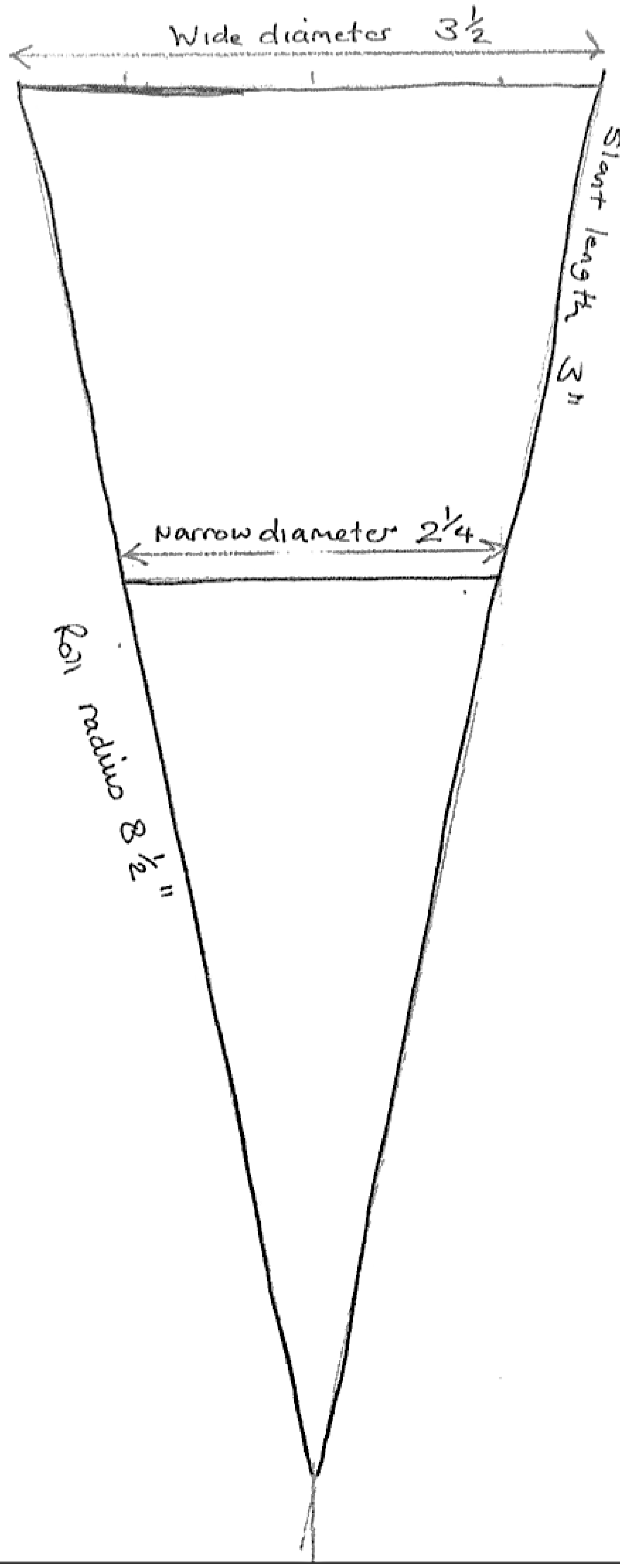
Find and describe a new pattern in the data.

What are the strengths of Heather's solution?

What would you do to improve Heather's solution?

Explain your answer.

Sample Responses to Discuss: Gerry



Gerry

3d Extend any cup you get a cone.
Narrow diameter = zero in a cone. Then roll radius = slant length.

2d Draw the cone as a triangle. The scale drawing lets you find the roll radius.

Eg a cup has wide diameter 3½" narrow diameter 2¼" slant length 3"
This method will work for any cup – you just measure the roll radius.

Use Gerry’s method to find the roll radius of this cup:

Wide diameter = 3"　　　Narrow diameter = 1"　　　Slant length = 3"

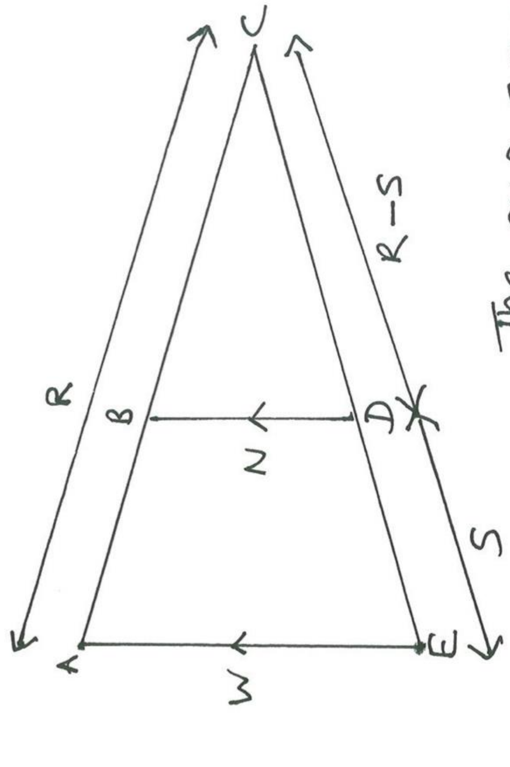
Gerry says he can use his method to find the roll radius of any cup.
Is this correct?
Explain your answer.

What are the strengths of Gerry’s approach?

What could Gerry do to improve his solution?

Sample Responses to Discuss: Judi

Judi



N = narrow diameter
W = wide diameter
S = slant length
R = roll radius

$\triangle ACE$ is similar to $\triangle BCD$.

The cup scenario can be modeled using similar triangles.

So $\dfrac{R}{W} = \dfrac{R-S}{N}$

and $RN = WR - S$.

Judi says Triangle ACE and Triangle BCD are similar.

Is Judi correct?

Explain your answer.

Judi draws a triangle diagram.

Explain how this represents a cup.

Judi has made a mistake in her formula.

Find and correct the mistake.

Now find a formula for *R*, the roll radius.

How Did You Work?

1. My own method was similar to one of the sample responses ☐ OR My own method was different from **all** of the sample responses ☐

   ☐ *(add name of sample response)*

   Because:

2. Our group's method was similar to sample response: ☐ OR Our group's method was different from **all** of the sample responses ☐

   ☐ *(add name of sample response)*

   Because:

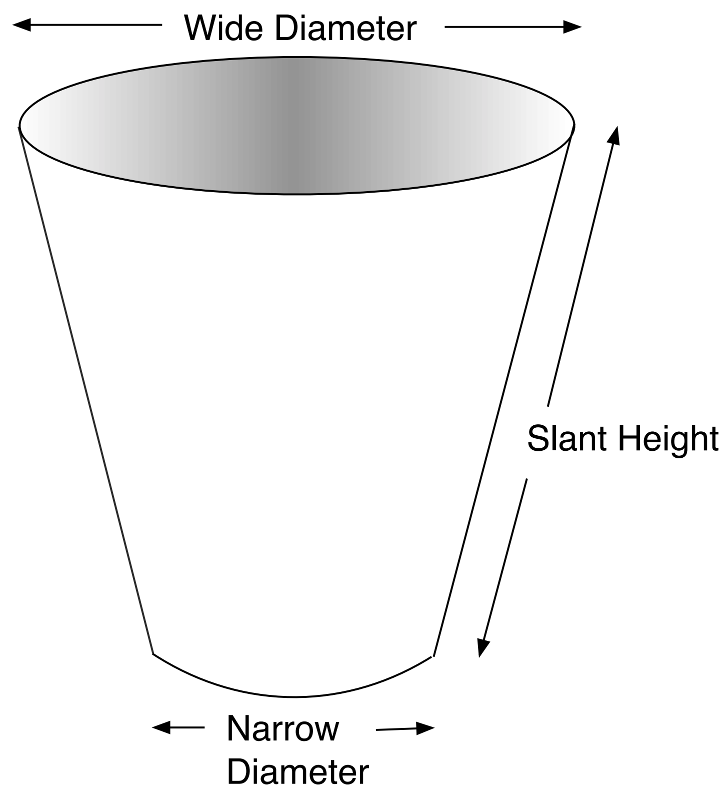
3. Explain giving reasons, whether or not you think the method(s) you used was more efficient/clearer than the sample response methods.

4. Explain how the method used by Heather is different to the method used by Gerry.

5. Write down the advantages and disadvantages of each of the methods shown in the sample responses.

Modeling Rolling Cups

Cup	Dimensions in inches			
	Wide diameter	Narrow diameter	Slant length	Roll radius
A	3½	3	3¾	26¼
B	3	2	3½	10½
C	2½	2	5¾	28¾
D	3	3	4¼	Infinite!
E	3	2	6	18
F	3½	2	3¾	8¾
G	3¾	3	3¾	18¾
H	3½	0	3¾	3¾



Collaborative small-group work

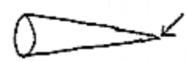
1. Take turns to explain your work.
   Ask questions if you don't understand.

2. Agree on a strategy for producing a joint solution that is better than your individual responses.

3. Work together on implementing your strategy.
   Think carefully about any additional data you want to collect.

Sample Responses to Discuss: Heather

N narrow diameter
W wide diameter
S slant height
R roll radius



N = 0

When N=0 the cup is a cone & R = S. It doesn't matter what W is.

Fix N=0 and W=4

S	R
1	1
2	2
3	3
4	4
5	5

Fix N=0 and W=5

S	R
1	1
2	2
3	3
4	4
5	5

$R = 1.5 \times S$

Fix N=1 and W=3

S	R
1	1.5
2	3
3	4.5
4	6
5	7.5

N=1 and W=4

S	R
1	1.25
2	2.75
3	4
4	5.25
5	6.75

N=1 and W=5

S	R
1	1.25
2	2.5
3	3.75
4	5
5	6.25

N=1 and W=6

S	R
1	1.25
2	2.5
3	3.5
4	4.75
5	6

Some of the values are rounded so it's hard to see patterns.

Fix N=2 and W=2

S	R
1	inf
2	inf
3	inf
4	inf
5	inf

Fix N=2 and W=3

S	R
1	3
2	6
3	9
4	12
5	15

Fix N=2 and W=4

S	R
1	2
2	4
3	6
4	8
5	10

$R = 1.25 \times S$

$$R = \frac{? \times S}{W - N}$$

When N = W R is infinite ie you have a cylinder that rolls in a straight line.
Formula is something like $R = \dfrac{?}{W - N}$

Fix N=4 and W=4

S	R
1	inf
2	inf
3	inf
4	inf
5	inf

HEATHER

Sample Responses to Discuss: Gerry

Gerry

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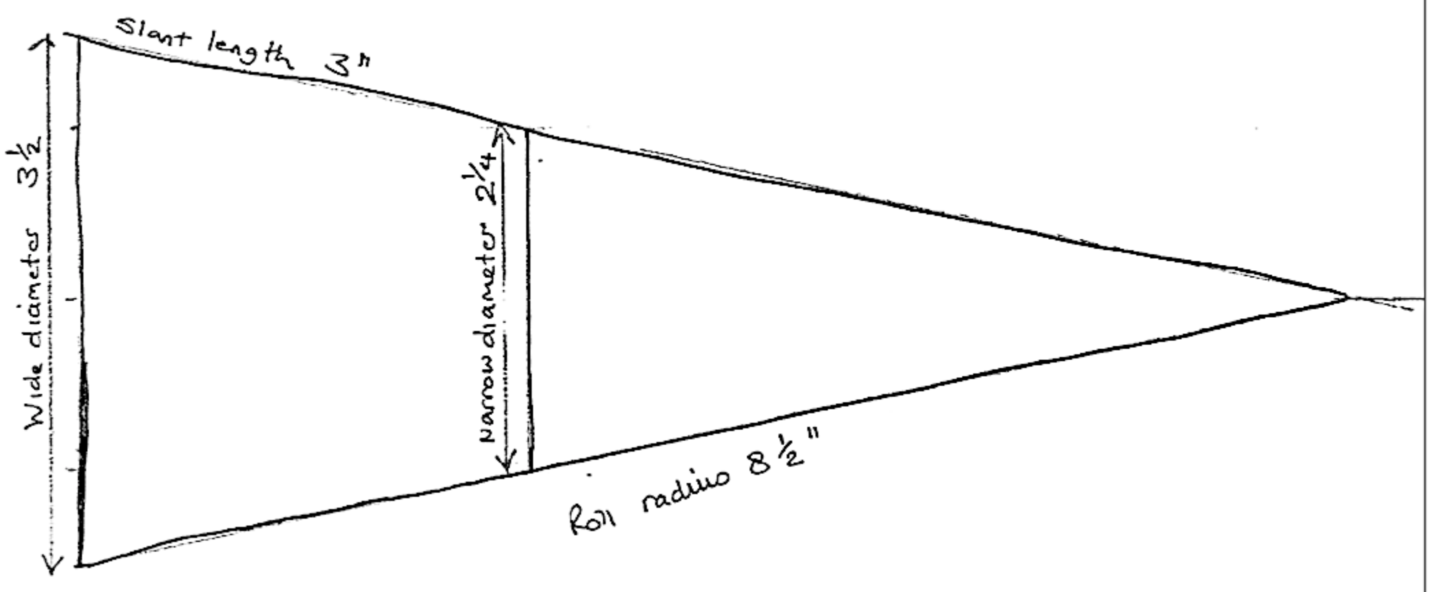
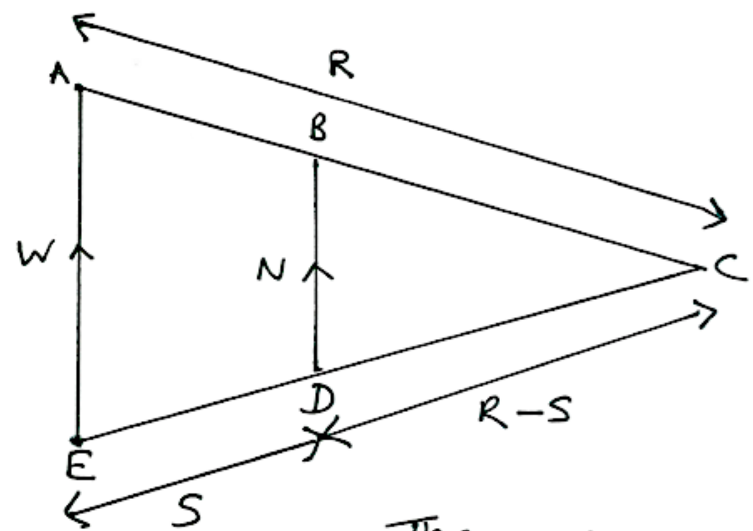


Sample Responses to Discuss: Judi

Judi



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Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
University of Nottingham, England:

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Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
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Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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http://map.mathshell.org