CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Defining Lines by Points, Slopes and Equations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Defining Lines by Points, Slopes and Equations

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Find the slopes and equations of linear graphs defined by pairs of coordinates.
- Calculate the slope and y-intercept of a straight line.
- Use the slope and y-intercept of a straight line to derive its equation.

Students may use the properties of similar triangles to achieve these goals.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.EE: Understand the connections between proportional relationships, lines, and linear equations.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2 and 7.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding. You then review their responses and create questions for students to consider when improving their work.
- After a whole-class introduction, students work in small groups on a collaborative discussion task, matching cards that describe the same line. Throughout their work, students justify and explain their thinking and reasoning.
- Students review their work by comparing their matches with those of their peers.
- In a whole-class discussion, students discuss what they have learned.
- In a follow-up lesson, students revisit their initial work on the assessment task and work alone on a similar task to the introductory task.

MATERIALS REQUIRED

- Each student will need a mini-whiteboard, pen, and wipe, and a copy of the assessment tasks *Lines, Slopes and Equations* and *Lines, Slopes and Equations (revisited)*.
- Each small group of students will need a cut-up copy of the *Card Set: Lines*, a pencil, a marker, a large sheet of poster paper and a glue stick.

TIME NEEDED

15 minutes before the lesson, an 90-minute lesson (or split into two shorter ones), and 15 minutes in a follow-up lesson (or for homework). These timings are not exact. Exact timings will depend on the needs of your students.

BEFORE THE LESSON

Assessment task: *Lines, Slopes and Equations* (15 minutes)

Ask students to complete this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

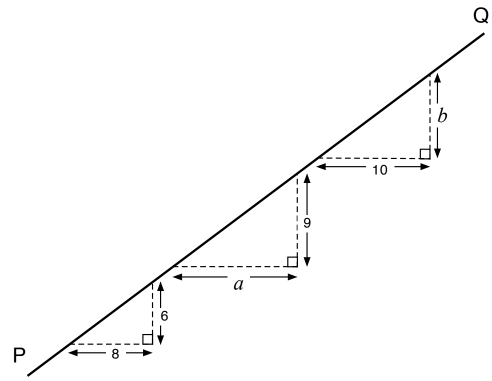
Give each student a copy of the assessment task *Lines, Slopes and Equations* and briefly introduce the task:

> *In this task the drawings are not to scale. The idea is to see if you can work out how to answer the questions by calculation.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. Students should not worry too much if they cannot understand nor do everything because in the next lesson they will work on a similar task that should help them. Explain to students that by the end of the next lesson they should be able to answer questions such as these more confidently. This is their goal.

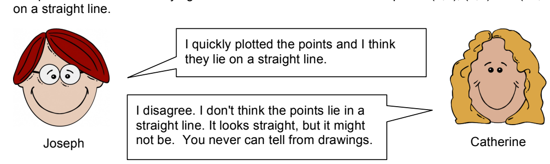
Lines, Slopes and Equations

1. Line PQ is a straight line.
   Three similar right-angled triangles have been drawn beneath this line.
   They have not been drawn to scale.

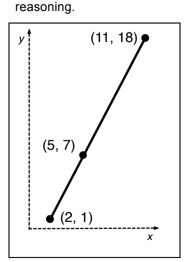


Calculate the lengths marked a and b.
Show all your reasoning in the space provided.

2. Joseph and Catherine are trying to decide whether or not the three points (2,1), (5,7) and (11,18) lie on a straight line.



Without drawing, decide whether or not the three points lie on a straight line. Show all your reasoning.



3. If the straight line that passes through (2,1) and (5, 7) is drawn accurately, it passes through the point (20, y). What number does y stand for?

The point (x, 21) lies on the same line. What number does x stand for?

What is the equation of the straight line that passes through (2,1) and (5, 7)?

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different ways of reasoning. The purpose of doing this is to forewarn you of issues that may arise during the lesson itself, so that you can prepare carefully. We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on the next page. We suggest that you make a list of your own questions, based on your students' work, using the ideas in the table. We recommend you write one or two questions on each student's work. If you do not have time to do this, you could:

- Select a few questions that will be of help to the majority of students, and write these questions on the board when you return the work to the students.
- Or give each student a printed version of your list of questions and highlight the questions for each individual student.

Common issues:	Suggested questions and prompts:
Student cannot get started	• What do you know? What do you need to find out? What does 'similar' mean?
Uses an additive rather than multiplicative strategy For example: The student writes a = 11 (6 + 3 = 9, so 8 + 3 = 11) and b = 8 (8 + 2 = 10, so 6 + 2 = 8) (Q1). Or: The student states that a = 9 (because the horizontal sides increase by 1) and b = 12 (because the vertical sides increase by 3) (Q1).	• Look at the triangle with sides 6 and 8. If you added 100 to each side, would the hypotenuse still have the same slope? If you subtracted 5 from each side, would it have the same slope then? • Use your dimensions to accurately construct your right-angled triangles on squared paper. Compare the angles and slopes of their hypotenuses. Are these equal?
Confuses operations of multiplication and division For example: The student multiplies instead of dividing by the correct scale factor. Finds a = 12, then calculates b = 9 x 12/10 = 10.8 (Q1).	• What do you notice about the base lengths of the triangles compared with their heights?
Applies an incorrect procedure for calculating the slope For example: The student calculates 18 ÷ 7 and 11 ÷ 5 and then subtracts (Q2).	• What does this value you have calculated represent? • What determines the slope of a straight line?
Correctly answers questions without calculating slopes For example: The student correctly adds multiples of 3 to 5 and multiples of 6 to 7 to obtain other correct points on the line: (8, 13); (11, 19); (14, 25); (17, 31); (20, 37) (Q3).	• The point (x, 21) doesn't appear in your list. How could you calculate the value of x? • Can you calculate the slope of the line joining (2,1) and (5,7)? • How could you use the slope to find other points on the line?
Assumes proportionality instead of only linearity For example: The student believes that if a line passes through (2,1) it must pass through (20,10), (multiplying both coordinates by 10) (Q3). Or: The student believes that if a line passes through (5,7) it must pass through (15, 21), (multiplying both coordinates by 3) (Q3).	• If you multiply both of the coordinates by the same number, will the points always lie on this line? What happens when you multiply both coordinates by 7? • If you extend the line passing through (2,1) and (5,7) will it go through (0,0)? How do you know?
Student provides little or no explanation For example: The student's work is limited to calculations only.	• Can you write some instructions that show someone new to the problem what you have done?
Student successfully completes the task	• Find the equation of the line passing through (5,7) and (11,18). • Write down some other points on this line.

SUGGESTED LESSON OUTLINE

Whole-class introduction (20 minutes)

The aim of the introduction is for students to recognize that they cannot rely on appearances to determine whether or not points lie on a straight line. Instead they must use a mathematical strategy. Use the student responses to the pre-assessment task to adapt the introduction appropriately.

Give each student a mini-whiteboard, pen, and wipe.

Display Slide P-1 of the projector resource:

Are the lines the same?

Do these two cards describe the same straight line?
How can you tell?

Line A passes through (0, 2) and (6, 10)	Line B passes through (6, 10) and (10, 16)

Students may suggest plotting points to check, but this is not enough as looks can be deceptive.

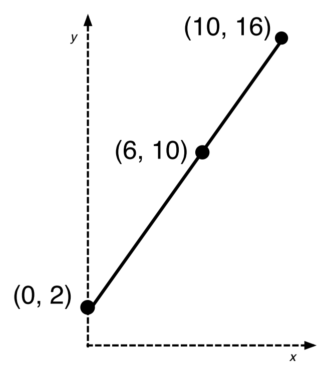
Display Slide P-2, which shows a diagram drawn to scale:

Let's plot them!

If we plot the points they look like they might be the same straight line ….

But looks can be deceptive.

How can we be sure?

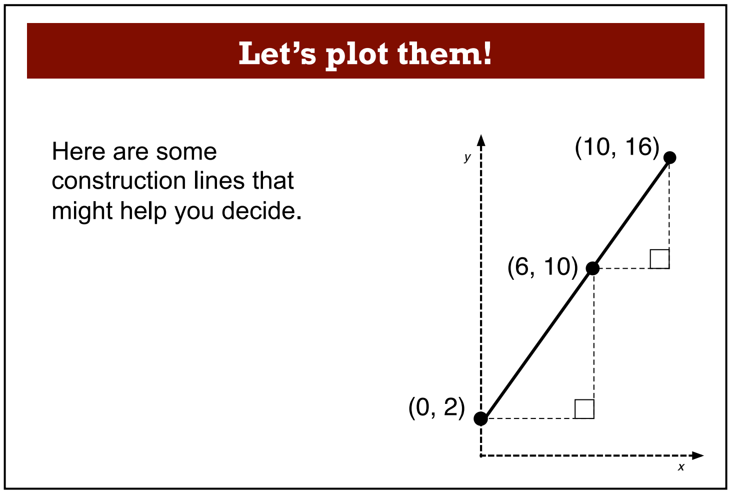


The points certainly *look* as though they are on a straight line, but are they *exactly* on a straight line?

> *How can we tell whether they lie on a straight line without drawing?*
> *Spend a few minutes discussing your ideas.*
>
> *Think about what calculations you could do.*
> *Use your mini-whiteboard for any calculations you think might help.*

Some students may suggest looking at whether or not the two line segments have the same slope, or whether they have the same equation.

Slide P-3 shows some possible construction lines:

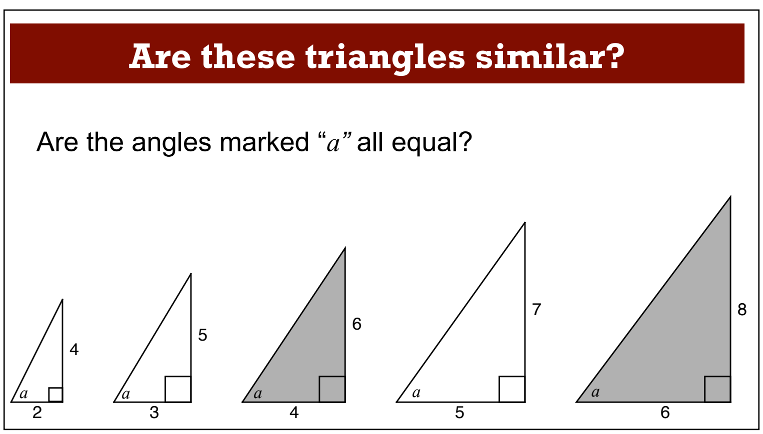


Can these two right-angled triangles help you decide?

What are the lengths of the sides of the triangles? [Lower: 6 horizontally and 8 vertically. Upper: 4 horizontally and 6 vertically.]

How could you use this information to see if the points lie on a straight line?

[Optional interlude] Some students view an enlargement as an additive transformation rather than a multiplicative one and so may assume the points lie on a straight line because the measures of the sides of the lower triangle are two units more than the corresponding measures of the sides of the upper triangle ($4 + 2 = 6$; $6 + 2 = 8$). If this issue arises, one approach is to consider the two constructed triangles as members of a sequence of right-angled triangles whose sides all differ by constant amounts, as shown on Slide P-4:



The two triangles in question are shaded. You may like to ask students to consider adding additional triangles to this sequence:

If I add a new triangle to the left of the sequence, what will it look like? (… and another?)

What will the 100th triangle in the sequence look like?

Will the angles marked "a" all be equal? Why/Why not?

Continue the discussion of slide P-3 by directing students' attention to the slope of the lines:

What is the slope of the hypotenuse of the lower triangle? [(10 - 2) ÷ (6 - 0) = 8 ÷ 6 = 1⅓]

What is the slope of the hypotenuse of the upper triangle? [(16 - 10) ÷ (10 - 6) = 6 ÷ 4 = 1½]

These are close but are not equal.

If we drew a straight line from (0, 2) to (10, 16) what would its gradient be? [14 ÷ 10 = 1⅖]

This lies between the other two slopes.

What does this tell us about lines A and B? [They are not the same straight line. If the points were all in a straight line these slopes would all be equal.]

You may like to probe further to seek equations.

What is the equation of the hypotenuse in the lower triangle? [y = 4x/3 + 2]

What about the hypotenuse of the upper triangle? [y = 3x/2 + something]

Will the hypotenuse of the upper triangle go through the origin? How can we find out?

Students may suggest substituting (6, 10) or (10, 16) into the equation and solving. You may want to show students one point, which they can then check with the other:

$y = \frac{3}{2}x + c$

When $x = 6, y = 10$

$\Rightarrow 10 = \frac{3}{2} \bullet 6 + c$

$\Rightarrow c = 1$

So the equation is: $y = \frac{3}{2}x + 1$

or

$y = \frac{3}{2}x + c$

When $x = 10, y = 16$

$\Rightarrow 16 = \frac{3}{2} \bullet 10 + c$

$\Rightarrow c = 1$

So the equation is: $y = \frac{3}{2}x + 1$

Summarize the purpose of the introduction and introduce the collaborative activity:

In today's lesson you will not be able to rely on appearances. You will be given a collection of cards and your task will be to find pairs of cards that describe the same line.

Collaborative small group work (30 minutes)

Ask students to work in groups of two or three. Give each group a cut-up copy of *Card Set: Lines*, a glue stick, a pencil, a marker and a large sheet of poster paper.

Display Slide P-5 and explain how students are to work together:

> **Working Together**
>
> 1. Look for pairs of cards that describe the same line.
> 2. Some of the cards have information missing. You will need to work out the missing information once you have matched the card.
> 3. Before writing anything down, describe your reasoning to your partner. Your partner must challenge your explanation if they disagree, or describe it in their own words if they agree.
> 4. Once agreed, glue the pair of cards onto the poster and write your reasoning **in pencil** next to it.
>
> Work together until all the cards are placed.

The purpose of this structured work is to encourage students to engage with their partner's explanations and take responsibility for each other's understanding. Students should use their mini-whiteboards for calculations and to explain their thinking to each other. While students are working, you have two tasks: to notice their approaches to the task and to support student reasoning.

Notice different student approaches to the task

Listen to and watch students carefully. Notice how students make a start on the task, where they get stuck, and how they overcome any difficulties.

Do students begin by first sketching a diagram? Do they attempt to calculate slopes and equations? Do they assume that because slopes are equal then the lines are the same? Are students checking their work? If so, what method are they using?

Note also any common mistakes. For example, do students use the orientation of the triangle correctly when calculating the slope? Do they make any incorrect assumptions? You may want to use the questions in the *Common issues* table to help address any misconceptions that arise.

Support student problem solving

Help students to work constructively together, referring them to Slide P-5. Check that one student listens to another by asking the listener to repeat the speaker's reasoning in different words. Check that students are recording their discussions as rough notes on their mini-whiteboards.

Try not to do the reasoning for them. Instead, ask strategic questions to suggest ways of moving forward.

What do you know? What additional information can you add?
In what ways does this help you to determine whether or not the two lines are the same?

What do you need to find out?
What math do you know that connects to that?

What's the same about these two triangles? What is different?

In what ways is this line similar to one that you have already matched? How does this help?

Is it possible to match the cards without working out the equation first?

Support students in developing written explanations. Suggest that they draft explanations on their mini-whiteboards and then refine these explanations for their poster.

If students are struggling, suggest that they start with the lines where the y intercept is already known (Lines A, D, E and F).

If students succeed with the task, ask them to deduce the shape created by the six lines from the cards (without plotting accurately). Where do the lines intersect? Which are parallel? [The six lines describe a parallelogram with its two diagonals.]

Extending the lesson over two days

If you are taking two days to complete the unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of their previous work before moving on to students sharing their work.

Sharing work (15 minutes)

As students finish their posters, ask them to critique each other's work by asking one student from each group to visit another group.

It may be helpful for the students visiting another group to first jot down a list of their matches with equations/slopes (e.g. Lines D & J, slope 1, $y = x + 3$) on their mini-whiteboard.

Slide P-6 of the projector resource, *Sharing Work*, summarizes the instructions.

Sharing Work

1. One person in your group jot down your matches/equations/ slopes on your mini-whiteboard and then visit another group's desk and compare your work with theirs.
2. If there are differences in your matches, ask for an explanation. If you still don't agree, explain your own thinking.
3. Check that you understand each other's methods and explanations.
4. If you are staying at your desk, be ready to explain the reasons for your group's decisions.

Poster review (10 minutes)

After students have had a chance to share their work and discuss their matches and reasoning with their peers, give them a few minutes to review their posters.

Now that you have discussed your work with someone else, you need to consider as a group whether to make any changes to your own work.

When you are confident with your decisions, go over your work in pen (or make amends in pen if you have changed your mind).

Whole-class discussion (15 minutes)

Organize a whole-class discussion about what has been learned and explore the different methods used when matching the lines.

For two lines to be the same, what did you need to show?
[That they have the same slope and a point in common; or that they have the same equation.]

You may want to first select a pair of lines that most groups matched correctly as this may encourage good explanations.

Can you explain your method to us clearly?

Can anyone improve this explanation?

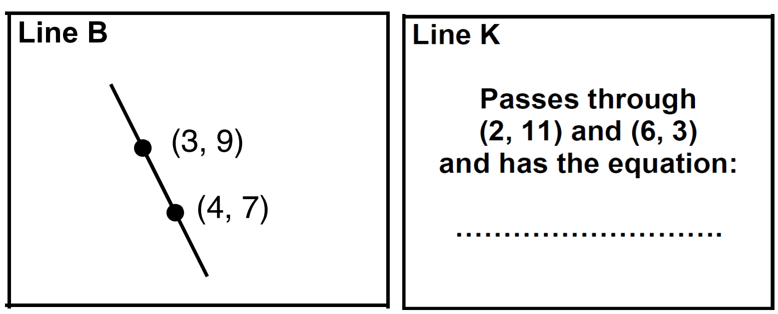
Does anyone have a different explanation?

Which explanation do you prefer? Why?

To help students explain their work, there are slides in the projector resource showing the correct matched pairs from the lesson task (Slides P-7 to P-12). Try to include a discussion of lines that have both positive and negative slopes. After one group has justified their choice for a particular match, ask other students to contribute ideas of alternative approaches and their views on which reasoning was easier to follow. You might like to ask if and how students have calculated the slope and/or equation of the line(s).

For example:

Consider Lines B and K:



What is the slope of line B? [-2] How did you decide this?

What is the slope of line K? [-2] Again, how did you decide?

Does this mean that the lines are the same? [No, they may just be parallel.]

How did you find out whether or not they are the same line?

Different methods are possible. One method might involve finding the equations of both lines and showing that they are equal. An alternative method would be to consider one point from Line B (e.g. (3,9)) and one point from line K (e.g. (6,3) and show that the slope between them is also -2.

Cards F and L are matched by default. There is insufficient information on card L to define a line. This reverses the task; given that they do match, what information must we have on card L?

Line F	Line L
Passes through (0, 3) and (1, 1)	**Passes through (4, …..) has a slope of -2 and has the equation: ……………………………**

To conclude, ask students what they learned by looking at other students' work and whether or not this helped them with cards they had found difficult to match or were unsure about:

Which lines did you find the most difficult to match? Why do you think this was?

Did seeing the work of another group help? If so, can you give an example?

In what ways did having another group critique your poster help? Can you give an example?

During the discussion, draw out any issues you noticed as students worked on the activity, making specific reference to the misconceptions you noticed. You may want to use the questions in the *Common issues* table to support your discussion.

Follow-up lesson: reviewing the assessment task (15 minutes)

Give each student a copy of the review task, *Lines, Slopes and Equations (revisited)*, and their original scripts from the assessment task, *Lines, Slopes and Equations*. If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Look at your original responses and the questions [on the board/written on your script.]

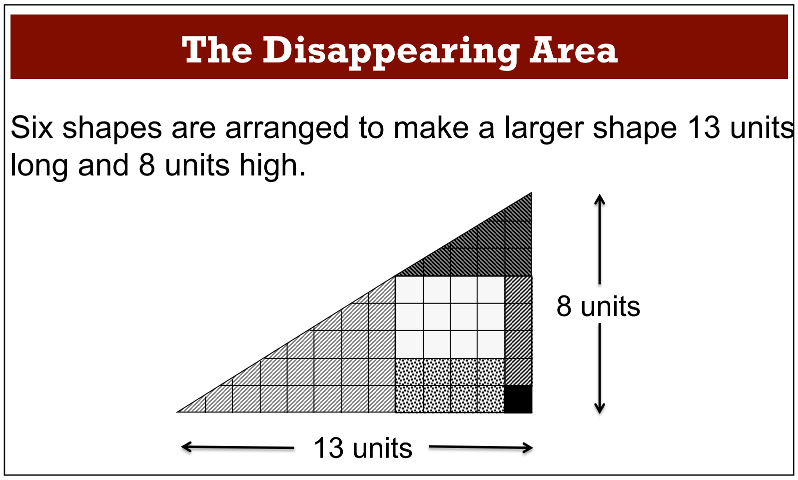
Use what you have learnt to answer the questions.

Now look at the new task sheet, Lines, Slopes and Equations (revisited). Can you use what you have learned to answer these questions?

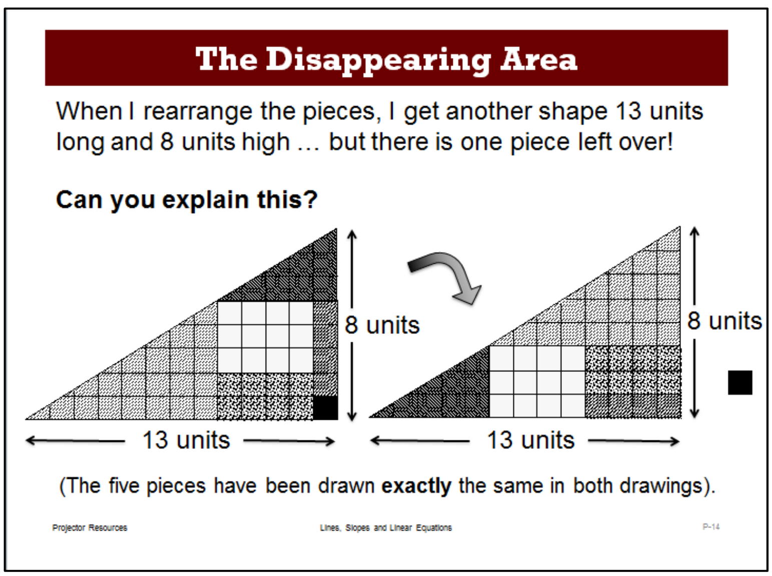
Some teachers give this as homework.

POSSIBLE EXTENSION

There is an interesting extension to these problems that students may enjoy, called the disappearing area. This problem is presented on Slides P-13 and P-14:



Here, six shapes (2 triangles, 3 rectangles and a square) have been arranged to make a larger shape 13 units long and 8 units high.



The ‘triangles’ look the same, but they aren’t. They are not in fact triangles at all because the hypotenuses are not straight. The ‘hypotenuse’ on the left hand ‘triangle’ bends in towards the right angle and the ‘hypotenuse’ on the right hand ‘triangle’ bends away from the right angle.

This activity reinforces that we need to rely on mathematical properties and reasoning, rather than drawings.

SOLUTIONS

Assessment Task: *Lines, Slopes and Equations*

1. The lengths marked a and b are 12 and 7.5 respectively.
   One approach is to calculate the slope as 6/8 = 3/4 from the lower triangle.
   This means that $9/a = 3/4 = 9/12$, and hence $a = 12$ and that $b/10 = 3/4 = (7.5)/10$ and so b = 7.5.

   A second approach is to use the similarity of the triangles.
   Corresponding sides are in proportion, so: $9/6 = a/8$; $b/6 = 10/8$.

2. The three points do not lie on a straight line.
   This can be seen by calculating the slopes of each line segment:

   (2,1) to (5,7) has slope of 6/3 = 2
   (5,7) to (11,18) has slope of 11/6 < 2, so the slopes are different.

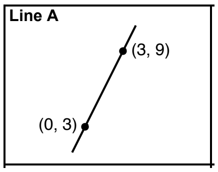
3. The line that passes through (2,1) and (5, 7) also passes through (20, 37) and (12, 21).
   This may be seen by again finding slopes. As the slope of the line is 2, then:

   $$\frac{y-1}{20-2} = 2 \qquad\qquad \frac{21-1}{x-2} = 2$$
   $$\Rightarrow y = 37 \qquad\qquad \Rightarrow x = 12$$

   The equation of the straight line is $y = 2x - 3$.
   This may be obtained by knowing the slope (2) and thus substituting either point in to $y = 2x + c$ to obtain $c = -3$.

Lesson task

The cards are matched below:

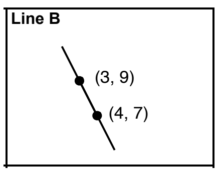


Line G

Passes through (10, 23) has a slope of 2 and has the equation:

....

These two cards both have the equation $y = 2x + 3$



Line K

Passes through (2, 11) and (6, 3) and has the equation:

....

These two cards both have the equation $y = -2x + 15$

Line C

Passes through (1, 1) and (4, 7)

Line H

Passes through (20, 39) has a slope of 2 and has the equation:

....

These two cards both have the equation $y = 2x -1$

Line D

Passes through (0, 3) and (4, 7)

Line J

Passes through (36, 39) has a slope of .... and has the equation:

....

These two cards both have the equation $y = x + 3$

Line E

Passes through (0, -3) and (3, 9)

Line I

Passes through (10, 37) has a slope of ...... and has the equation:

....

These two cards both have the equation $y = 4x – 3$

Line F

Passes through (0, 3) and (1, 1)

Line L

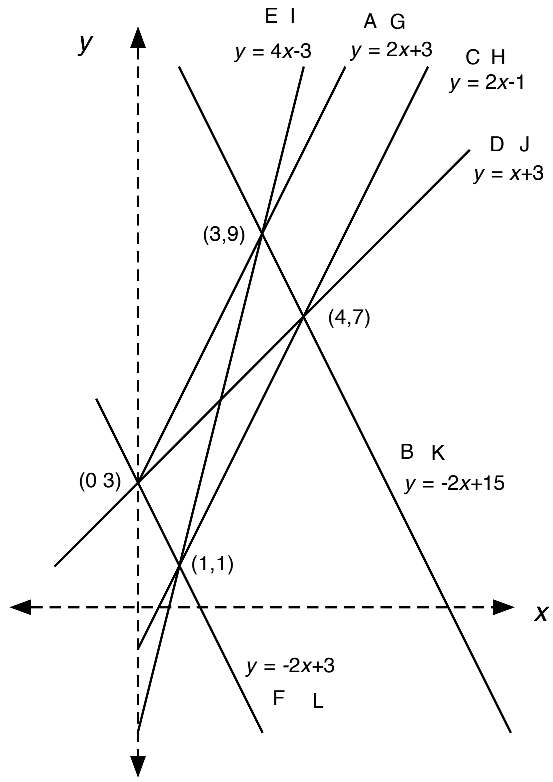
Passes through (4, ...) has a slope of -2 and has the equation:

....

Line L must pass through (4, -5) for the lines to be the same.

These two cards both have the equation $y = -2x + 3$

If a graph were drawn, then the lines would appear as follows:



Thus they form a parallelogram along with its diagonals.

Assessment Task: Lines, Slopes and Equations (revisited)

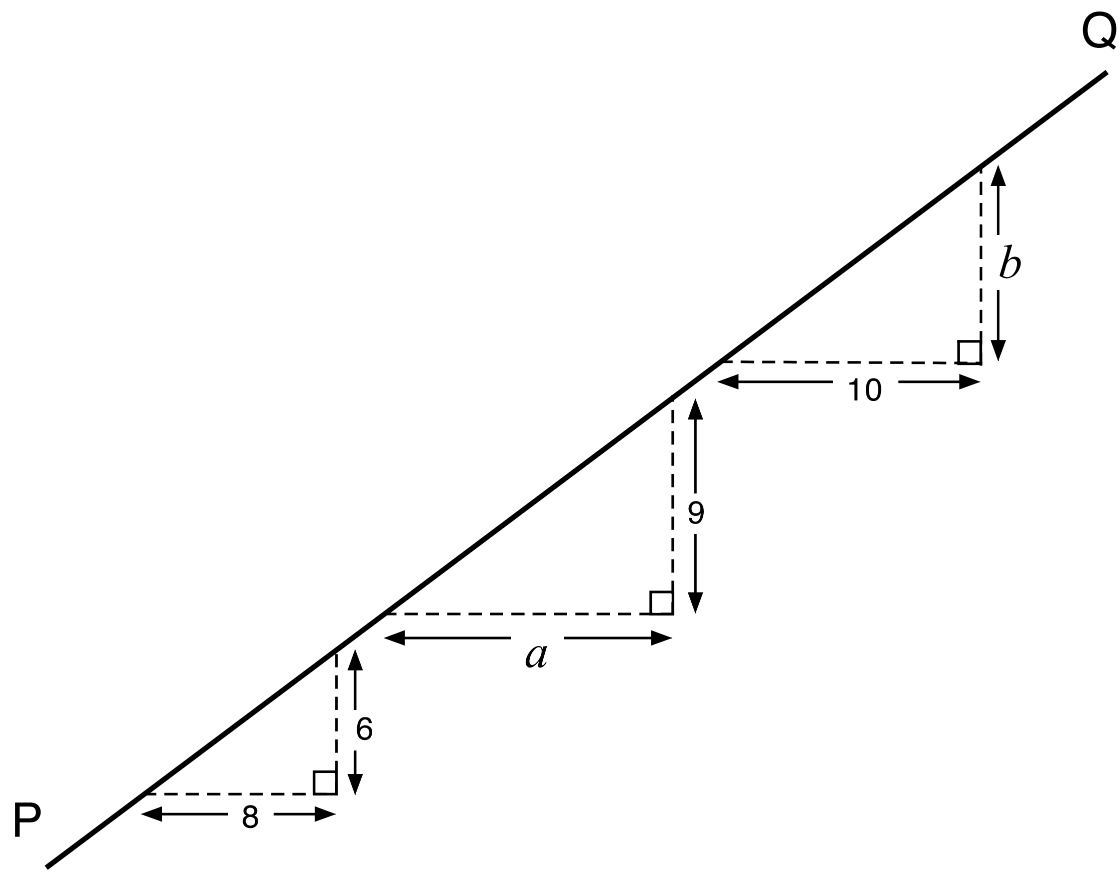
The slopes of the lines are:

Line A: 3
Line B: 2
Line C: 2
Line D: 1.5

1. Line A is steepest.
2. Lines B and C are parallel
3. The point (10, 30) lies on Line A.
4. The point (11, 21) lies on Line B.
5. The equation of line C is $y = 2x - 3$.

Lines, Slopes and Equations

1. Line PQ is a straight line.
   Three similar right-angled triangles have been drawn beneath this line.
   They have not been drawn to scale.



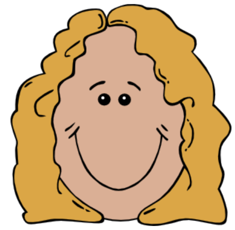
Calculate the lengths marked a and b.
Show all your reasoning in the space provided.

2. Joseph and Catherine are trying to decide whether or not the three points (2,1), (5,7) and (11,18) lie on a straight line.

Joseph

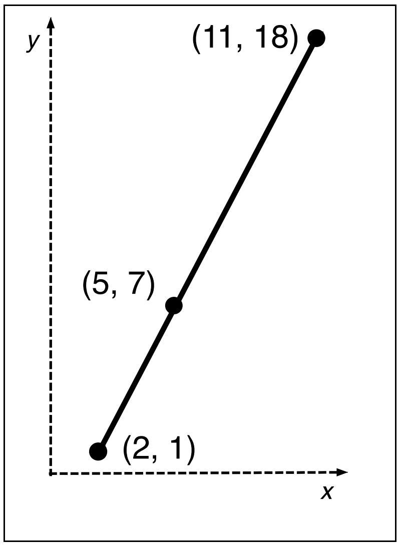
I quickly plotted the points and I think they lie on a straight line.

I disagree. I don't think the points lie in a straight line. It looks straight, but it might not be. You never can tell from drawings.



Catherine

Without drawing, decide whether or not the three points lie on a straight line. Show all your reasoning.



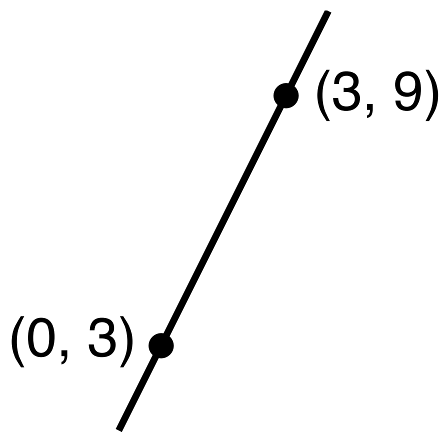
3. If the straight line that passes through (2,1) and (5, 7) is drawn accurately, it passes through the point (20, y). What number does y stand for?

The point (x, 21) lies on the same line. What number does x stand for?

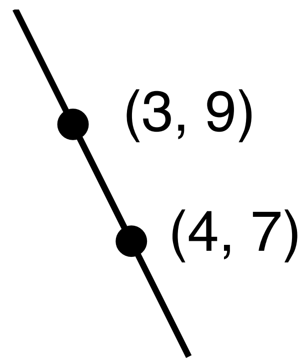
What is the equation of the straight line that passes through (2,1) and (5, 7)?

Card Set: Lines

Line A



Line B



Line C

Passes through (1, 1) and (4, 7)

Line D

Passes through (0, 3) and (4, 7)

Line E

Passes through (0, -3) and (3, 9)

Line F

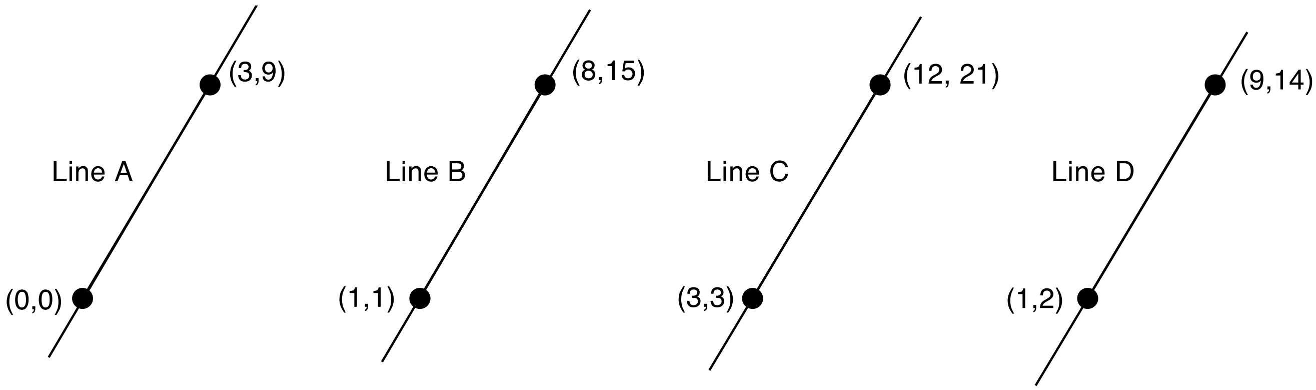
Passes through (0, 3) and (1, 1)

Card Set: Lines (continued)

Line G	Line H
Passes through (10, 23) has a slope of 2 and has the equation: **…………………………**	**Passes through (20, 39) has a slope of 2 and has the equation:** **…………………………**
Line I **Passes through (10, 37) has a slope of …… and has the equation:** **…………………………**	**Line J** **Passes through (36, 39) has a slope of ……. and has the equation:** **…………………………**
Line K **Passes through (2, 11) and (6, 3) and has the equation:** **…………………………**	**Line L** **Passes through (4, …..) has a slope of -2 and has the equation:** **…………………………**

Lines, Slopes and Equations (revisited)

The four straight lines below are not drawn to the same scale.



1. Which line has the steepest slope? Explain how you can tell.

2. Which two lines are parallel? Explain how you can tell.

3. The point (10, y) also lies on **line A**. What number does y stand for?

4. The point (x, 21) also lies on **line B**. What number does x stand for?

5. Write down the equation of **line C**.

Are the lines the same?

Do these two cards describe the same straight line?
How can you tell?

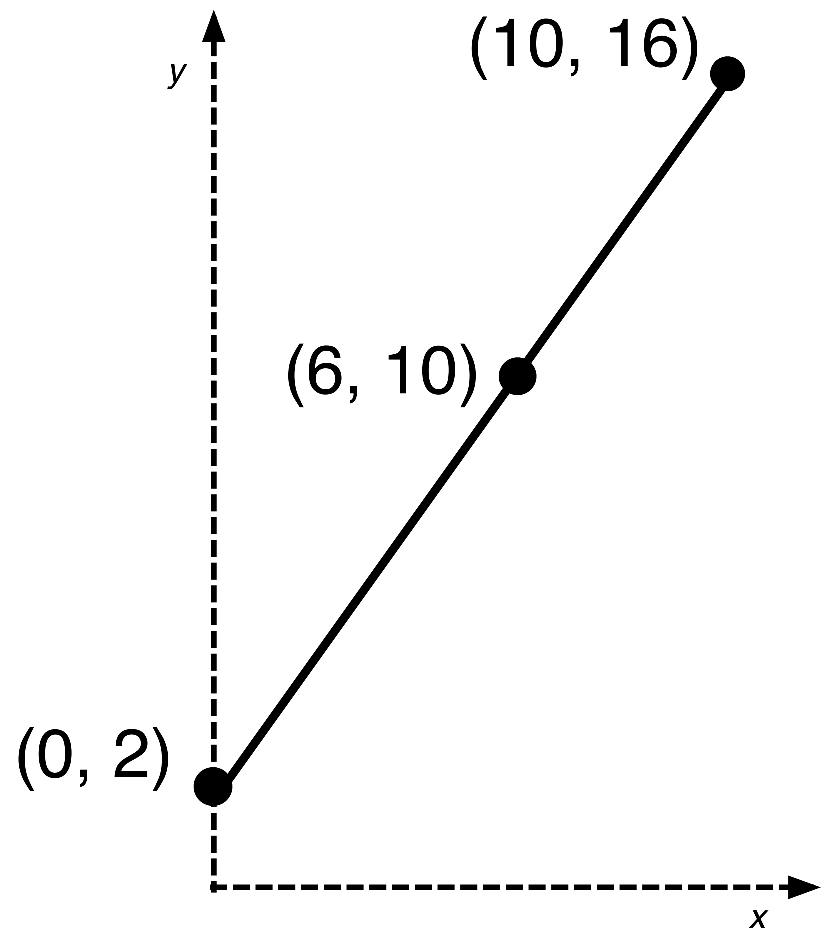
Line A passes through (0, 2) and (6, 10)	Line B passes through (6, 10) and (10, 16)

Let's plot them!

If we plot the points they look like they might be the same straight line ….

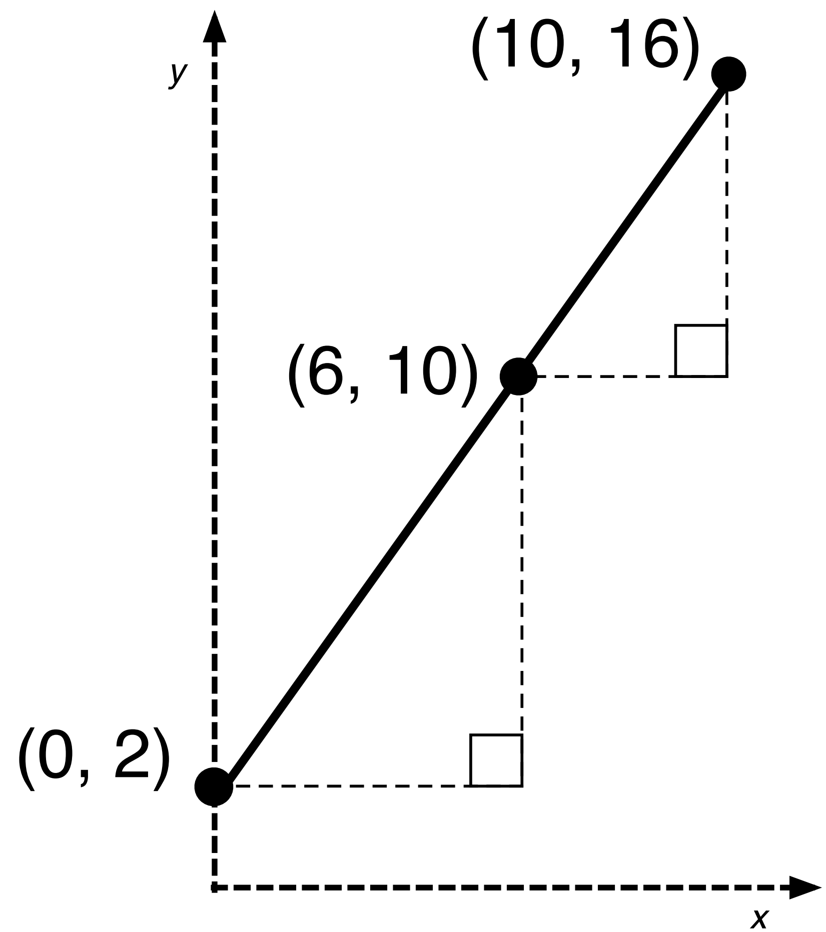
But looks can be deceptive.

How can we be sure?



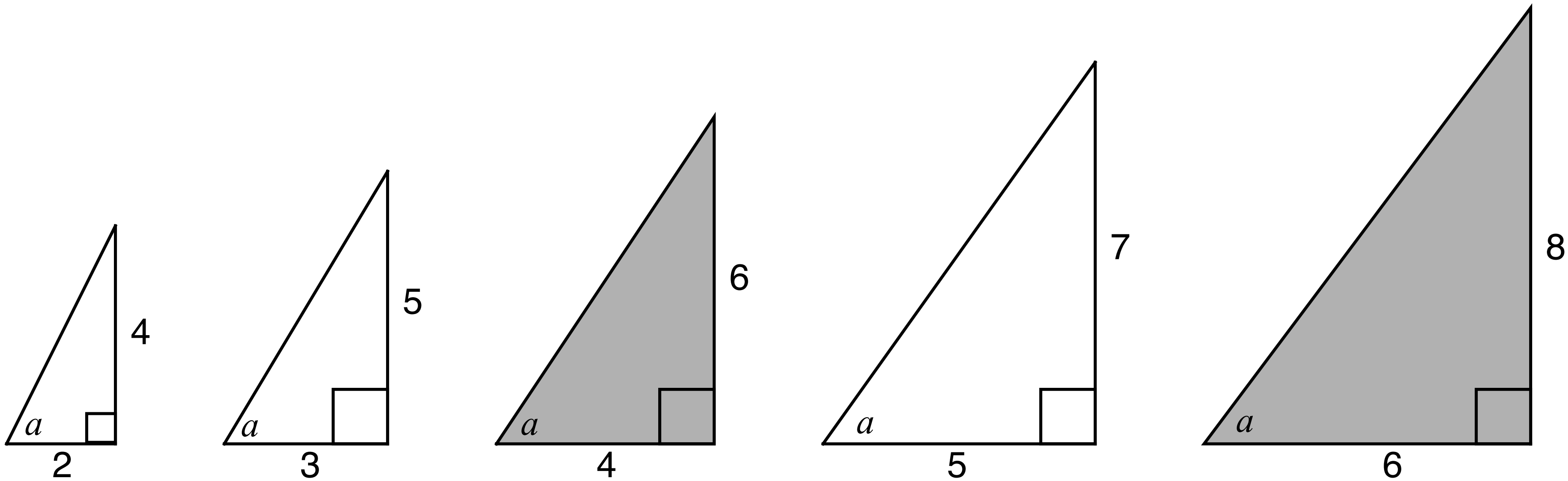
Let's plot them!

Here are some construction lines that might help you decide.



Are these triangles similar?

Are the angles marked "*a*" all equal?



Working Together

1. Look for pairs of cards that describe the same line.
2. Some of the cards have information missing. You will need to work out the missing information once you have matched the card.
3. Before writing anything down, describe your reasoning to your partner. Your partner must challenge your explanation if they disagree, or describe it in their own words if they agree.
4. Once agreed, glue the pair of cards onto the poster and write your reasoning **in pencil** next to it.

Work together until all the cards are placed.

Sharing Work

1. One person in your group jot down your matches/equations/ slopes on your mini-whiteboard and then visit another group's desk and compare your work with theirs.

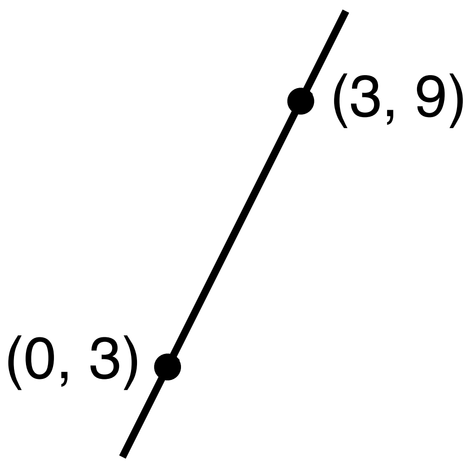
2. If there are differences in your matches, ask for an explanation. If you still don't agree, explain your own thinking.

3. Check that you understand each other's methods and explanations.

4. If you are staying at your desk, be ready to explain the reasons for your group's decisions.

Cards A and G

Line A



Line G

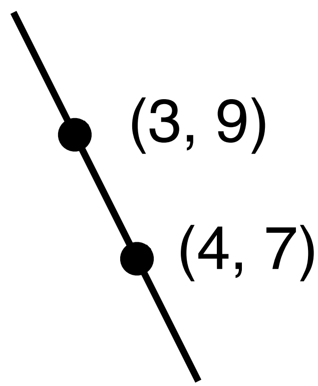
Passes through (10, 23) has a slope of 2 and has the equation:

……………………………

How can we tell that these are the same line?

Cards B and K

Line B



Line K

Passes through
(2, 11) and (6, 3)
and has the equation:

....

How can we tell that these are the same line?

Cards C and H

Line C

Passes through (1, 1) and (4, 7)

Line H

Passes through (20, 39) has a slope of 2 and has the equation:

....

How can we tell that these are the same line?

Cards D and J

Line D

Passes through (0, 3) and (4, 7)

Line J

Passes through (36, 39) has a slope of ……. and has the equation:

…………………………

How can we tell that these are the same line?

Cards E and I

Line E

Passes through (0, -3) and (3, 9)

Line I

Passes through (10, 37) has a slope of …… and has the equation:

……………………………

How can we tell that these are the same line?

Cards F and L

Line F

Passes through (0, 3) and (1, 1)

Line L

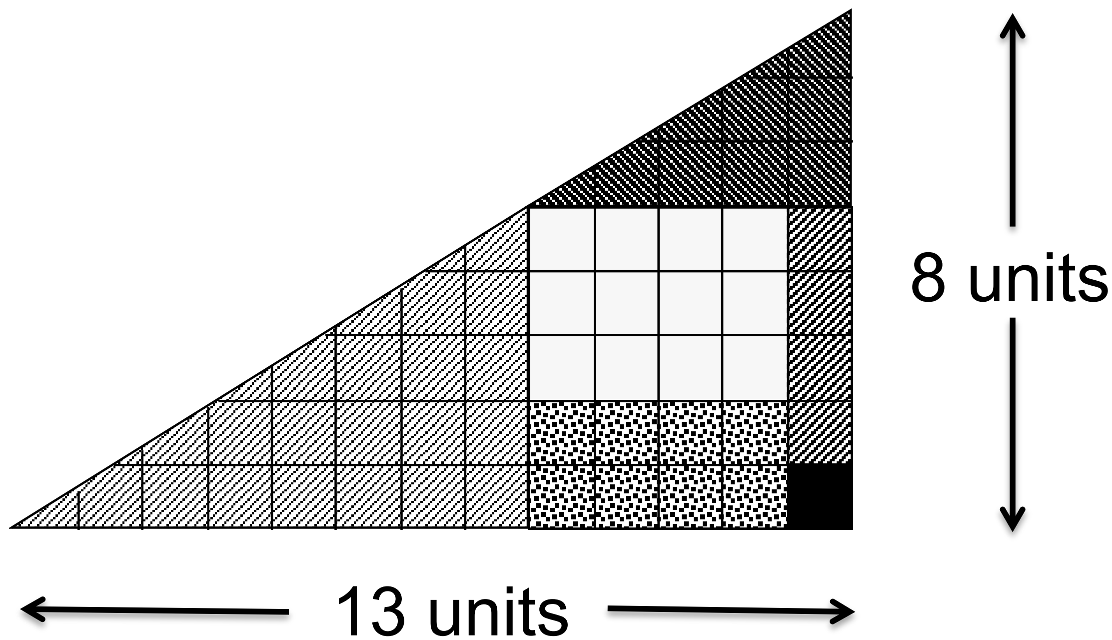
Passes through (4, …..) has a slope of -2 and has the equation:

…………………………

L doesn't have enough information to define a line.
If they are the same line, what must this information be?

The Disappearing Area

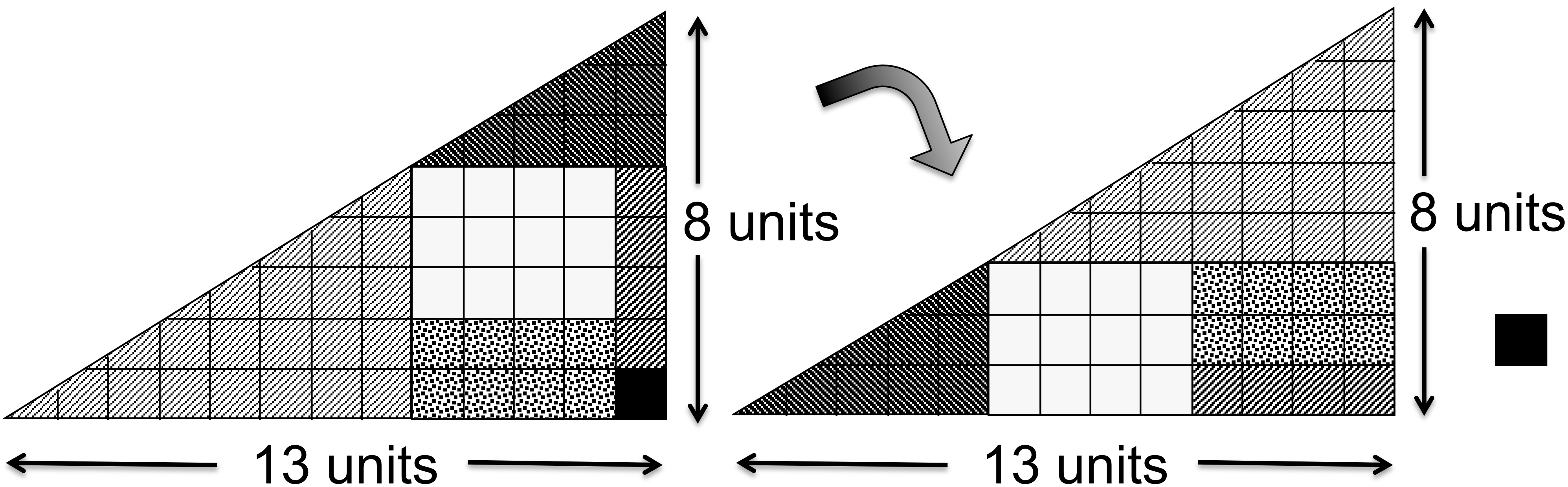
Six shapes are arranged to make a larger shape 13 units long and 8 units high.



The Disappearing Area

When I rearrange the pieces, I get another shape 13 units long and 8 units high … but there is one piece left over!

Can you explain this?



(The five pieces have been drawn **exactly** the same in both drawings).

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
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Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org