CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Describing and Defining Triangles

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Describing and Defining Triangles

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how students reason about geometry, and in particular, how well they are able to:

- Recall and apply triangle properties.
- Sketch and construct triangles with given conditions.
- Determine whether a set of given conditions for the measures of angle and/or sides of a triangle describe a unique triangle, more than one possible triangle or do not describe a possible triangle.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.G: Draw, construct, and describe geometrical figures and describe the relationships between them.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2, 3, 5, 6, and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding and ability to reason using geometrical properties. You then review their responses and create questions for students to consider when improving their work.
- After a whole-class introduction, students work in pairs or threes on a collaborative task, determining whether sets of conditions describe possible triangles (unique or otherwise) or whether it is impossible to draw a triangle with these conditions. Throughout their work, students justify and explain their thinking and reasoning. They then compare their categorizations with their peers before reviewing their work and what they have learned in a whole-class discussion.
- In a follow-up lesson, students review their initial work on the assessment task and work alone on a similar task to the introductory task.

MATERIALS REQUIRED

- Each student will need a copy of the tasks *Triangles or Not?* and *Triangles or Not? (revisited),* some plain paper to work on, a mini-whiteboard, pen, and eraser.
- Each small group of students will need the *Card Set: Possible Triangles?* (cut-up), a pencil, a marker, some poster paper, and a glue stick. Rules, protractors and compasses should be made available.

TIME NEEDED

15 minutes before the lesson, a 90-minute lesson (or two 50-minute lessons), and 20 minutes in a follow-up lesson. Exact timings will depend on the needs of your students.

BEFORE THE LESSON

Assessment task: *Triangles or Not?* (15 minutes)

Ask students to complete this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student some plain paper to work on and a copy of the assessment task *Triangles or Not?*

Briefly introduce the task:

In this task you are asked to decide whether, from the information given: only one possible triangle can be drawn; more than one triangle can be drawn; or it is not possible to draw a triangle.

If more than one triangle can be drawn then try to say how many!

There is some plain paper for you to use when completing question one. Make sure you explain your answers clearly. Your explanations may include drawings and words.

It is important that, as far as possible, students are allowed to answer the questions without assistance.

Students should not worry too much if they cannot understand or do everything because in the next lesson they will work on a similar task that should help them. Explain to students that by the end of the next lesson they should be able to answer questions such as these confidently. This is their goal.

Triangles or Not?

1. Decide from the information given whether:
   - It is possible to construct a unique triangle ABC.
   - It is possible to construct more than one triangle ABC.
   - It is not possible to construct a triangle ABC with these properties.

   Give reasons for your answers (You are not required to perform any accurate constructions).

Information	Triangle possible? Check (☑) the correct answer		Reason
a. Angle B = 50°, AC = 3 cm, BC = 5 cm.	Unique triangle		
	More than one triangle		
	Not possible		
b. AB = 10 cm, BC = 11 cm, AC = 9 cm.	Unique triangle		
	More than one triangle		
	Not possible		
c. Angle A = 40°, Angle B = 60°, Angle C = 80°.	Unique triangle		
	More than one triangle		
	Not possible		
d. AB = 4 cm, BC = 3 cm, Angle B = 30°.	Unique triangle		
	More than one triangle		
	Not possible		

2. Triangle ABC is isosceles with AB = 5 cm and Angle B = 48°.
   Triangle DEF is isosceles with DE = 5 cm and Angle E = 48°.
   Explain why the two triangles may not be identical.

....

....

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different ways of reasoning. The purpose of doing this is to forewarn you of issues that may arise during the lesson itself, so that you can prepare carefully.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the

Common issues table on the next page. These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight appropriate questions for each student.

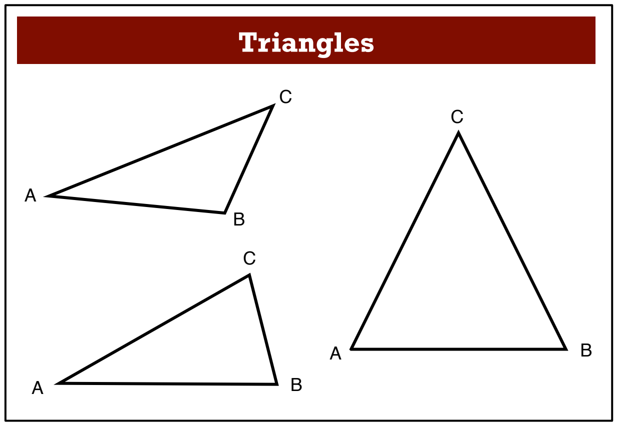
If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students in the follow-up lesson.

Common issues:	**Suggested questions and prompts:**
Has difficulty getting started	• Can you sketch a triangle with the information given? Would it be possible to construct this triangle? How do you know? • Try to construct a triangle with the information given. What do you notice? What decisions (if any) do you have to make?
Relies solely on a sketch without considering viability of dimensions For example: The student produces a labeled sketch without checking that the dimensions will viably produce a triangle (Q1a).	• Your sketch looks like a possible triangle. Can you check that a triangle may *actually* be drawn with these measurements?
Makes incorrect assumptions For example: The student assumes that when given the lengths of three sides, multiple triangles can be drawn, as the angles can be anything you choose (Q1b). Or: The student assumes that when three angles are given, only one triangle can be drawn, as a different triangle would have to have different angles (Q1c).	• How would you draw this triangle accurately? • Is it possible to draw a different triangle with the same three sides/angles?
Does not provide reasons for assertions For example: The student correctly determines whether a unique triangle, multiple triangles or no triangles can be drawn with no justification (Q1).	• Suppose someone does not believe your answers. How can you convince them that you are correct?
Provides incorrect reasons for assertions For example: The student states that the triangle is not possible, as we are not given information on *all* three angles/sides lengths (Q1a and/or Q1d).	• What is the smallest amount of information needed to construct a triangle?
Relies on just one form of reasoning For example: The student states that more than one triangle will never be possible as a different triangle would have different angles/side measurements (Q1).	• Are there any decisions to make when drawing any of these triangles? Would someone else necessarily make the same decisions?
Fails to apply properties of triangles For example: The student sketches two triangles each containing a 5 cm side and 48° angle that are not isosceles (Q2).	• What properties does an isosceles triangle have? • Are the triangles that you have sketched isosceles? • Can you sketch two different *isosceles* triangles with a side of 5 cm and an angle of 48°?

SUGGESTED LESSON OUTLINE

Whole-class introduction (20 minutes)

Give each student a mini-whiteboard, pen, and eraser and display Slide P-1 of the projector resource:



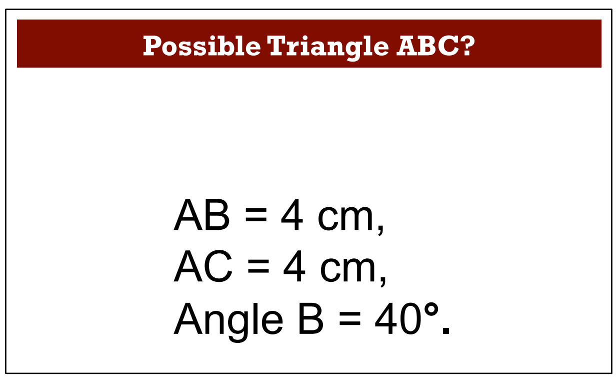
What facts do you know about triangles?

What names do you know for different types of triangle?

How do we label sides and angles if they are equal in magnitude?

Encourage the class to give as much information as they can about the properties of a triangle: sum of interior angles is 180°; triangles can be equilateral, isosceles, scalene, right-angled etc. Check also that they know how sides and angles may be labeled as equal in magnitude.

Now display Slide P-2:

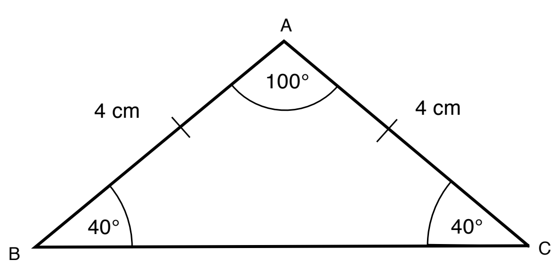


We know the magnitudes of two sides and one angle.
Is it possible to construct a triangle with these properties?

Using your mini-whiteboard, try to answer this question. Do this on your own.

Try to figure this out without constructing the triangle accurately.

Emphasize that students do not need to try to construct the triangle; a sketch is sufficient at this stage.



However, they should think carefully about the measurements in their sketch and determine their implications for drawing the triangle. For example, the triangle is isosceles, so angle C is also 40°.

After a few minutes ask the class whether or not they think the triangle is possible. If students think it is not possible, ask them to explain why. If they think it is possible, ask them to sketch the triangle and explain how they know. If more than one student thinks it is possible, compare their sketches.

> *These sketches look the same – how can we check whether or not more than one triangle is possible?*
>
> *These sketches look different, does that mean that more than one triangle is possible? How can we check?*

Students need to recognize that orienting the triangle differently does not make it a different triangle. They also need to recognize that, while a sketch gives them a way of visualizing the given features, they need to think mathematically about the *properties* of the triangle in order to determine whether or not it may be constructed and whether there is more than one possibility. (For example, thinking about the possible positions of side AC in relation to side BC is an important factor when establishing that only one unique triangle can be constructed in this case).

The purpose of this whole-class introduction is to reinforce the need to think mathematically about sketches, rather than simply adopting a trial and error approach to finding a solution. The collaborative activity will give students the opportunity to explore this further:

> *In today's lesson you are going to be using mathematical reasoning to determine whether or not the information given describes a possible triangle.*
>
> *You will need to think carefully about any sketches you draw and what these can tell you about a possible triangle construction.*

Collaborative small group work (30 minutes)

Ask students to work in groups of two or three.

Give each group a cut-up copy of *Card Set: Possible Triangles?*, a pencil, a marker, a glue stick and a large sheet of poster paper.

> *Divide your poster paper into three columns and label them 'Unique', 'More than one' and 'Not possible'.*
>
> *For each of the cards you need to decide whether from the information given; a unique triangle can be drawn, more than one triangle can be drawn, it is not possible to draw a triangle.*

Display Slide P-3 and explain how students are to work together:

Working Together

1. Take turns to select any card.
2. Work individually: can a triangle be produced?
3. Decide together how many triangles are possible (one, more than one, none).
   - If a triangle is not possible, agree on your reasoning.
   - If you disagree, challenge each other's explanations and work together to resolve your disagreements.
4. Once agreed, glue the card in the appropriate column and write an explanation **in pencil** on your poster, before moving on to another card.

The purpose of this structured work is to encourage each student to engage with their partner's explanations and to take responsibility for their partner's understanding. Students should use their mini-whiteboards for sketches and to explain their thinking to each other.

It is not the intention that students try to *construct* each triangle as a means of determining whether or not it is possible, but this may provide a way in for students that are having trouble getting started. They may find it helpful to use a rule, protractor and/or compasses to try to construct the triangle. However, encourage them rather to sketch each triangle and to think about the *process* of constructing each triangle.

It does not matter if students do not manage to categorize all ten cards. It is more important that everyone in each group understands the categorization of each card and can explain it in their own words.

While students are working, you have two tasks: to notice their approaches to the task and to support student problem solving.

Notice different student approaches to the task

Listen to and watch students carefully. Notice which cards students choose to tackle first. Do they have a strategy or do they choose cards at random? Do they group the cards in any way before they start? Notice too whether students are addressing the difficulties they experienced in the assessment task. Are students engaging with mathematical properties or are they relying on perceptual reasoning about surface features of a sketch? Do they notice or make connections between cards? Do they recognize types of triangle, such as right-angled, isosceles, equilateral and articulate these properties in their discussions?

Support student problem solving

Help students to work constructively together. Remind them to look at the Slide P-3 for instructions on how to work.

Check that one student listens to another by asking the listener to paraphrase the speaker's statements. Check that students are recording their thinking on their mini-whiteboards.

Try not to solve students' problems or do the reasoning for them. Instead, you might ask strategic questions to suggest ways of moving forward:

How do the given side lengths relate to each other?

Is there any information you have not been given that you need/can work out?
What does this tell you about the number of possible triangles?

Why can you draw more than one triangle for this card but not this card?

Support students in developing written explanations. Suggest that they write rough notes on their mini-whiteboards as they make decisions. Suggest that they refer to these notes as they write a fuller, clearer version of the explanation on their poster.

Extending the lesson over two days

If you are taking two days to complete the unit then you may want to end the first lesson here, ensuring that students have glued their categorized cards onto their poster. Then, at the start of the second day, give students time to familiarize themselves with their own work before comparing posters with another group.

Sharing work (10 minutes)

As students finish their posters, get them to critique each other's work by asking one student from each group to visit another group. It may be helpful for the students visiting another group to first jot down a list of the categorized diagrams (i.e. Unique: B, D, G etc.) on their mini-whiteboard.

Display Slide P-4 of the projector resource and explain how students are to share their work:

Sharing Work

1. One person in your group jot down your card categorizations on your mini-whiteboard and then go to another group's desk and check your work against their categorizations.
2. If there are differences, explain your thinking to each other.
3. If you have categorized cards in the same columns, compare your methods and check that you understand each other's explanations.
4. If you are staying at your desk, be ready to explain the reasons for your group's decisions.

Poster review (10 minutes)

Once students have had chance to share their work and discuss their categorizations and reasoning with their peers, give them a few minutes to review their posters.

Now that you have discussed your work with someone else, you need to consider as a group whether to make any changes to your own work.

If you think a card is in the wrong column, draw an arrow on your poster to show which column it should move to. If you are confident with your decisions, go over your work in pen (or make amends in pen if you have changed your mind.)

Whole-class discussion (20 minutes)

Organize a whole-class discussion about what has been learned and explore the different methods of justification used when categorizing cards.

You may want to first select a card that most groups categorized correctly, as this may encourage good explanations.

Which column did you put this card in? Can you explain your decision?

Can anyone improve this explanation?

Does anyone have a different explanation?

Which explanation do you prefer? Why?

Try to include a discussion of at least one card from each of the three columns.

Give me a card that describes a unique triangle. Why is it unique?

Give me a card that describes more than one possible triangle. How many triangles can be drawn for this card?

Give me a card that does not describe a possible triangle. Why is it not possible?

Did anyone put this card in a different column?

Once one group has justified their choice for a particular card, ask other students to contribute ideas of alternative approaches and their views on which reasoning was easier to follow.

To help students explain their work, there are slides in the projector resource showing each of the cards A to J from the lesson task (Slides P-5 to P-14).

Ask students what they learned by looking at other students' work and whether or not this helped them with cards they had found difficult to categorize or were unsure about:

Which card did you find the most difficult to categorize? Why do you think this was?

Did seeing where another group had placed this card help? If so, in what way did it help?

In what ways did having another group critique your poster help?

Did looking at another group's poster help you with your own work? Can you give an example?

During the discussion, draw out any issues you noticed as students worked on the activity, making specific reference to the misconceptions you noticed. You may want to use the questions in the *Common issues* table to support your discussion.

If your students seem to be confident, finish up by showing Slides P-15 and P-16 which aim to push the discussion towards noticing patterns and making generalizations:

Choose Some Measures	Choose Some Measures
Angle A = ….. , Angle B = ….. , Angle C = 50°.	BC = 4 cm , AB = ….. , AC = ….. .

Begin by discussing Slide P-15:

Can you suggest measures for angle A and angle B to make ABC a possible triangle?
How many different triangles ABC could you make with these measures?

Can you suggest measures for angle A and angle B that will make triangle ABC unique?

Can you suggest measures for angle A and angle B so that triangle ABC is not possible?

Now go on to Slide P-16:

Can you suggest measures AB and AC to make ABC a possible triangle?
How many different triangles ABC could you make with these measures?

Can you suggest measures for AB and AC so that triangle ABC is not possible?

Follow-up lesson: reviewing the assessment task (20 minutes)

Give each student a copy of the review task, *Triangles or Not? (revisited)* and their original responses to the assessment task, *Triangles or Not?*.

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Look at your original responses and the questions [on the board/written on your script.] Answer these questions and revise your response.

Now look at the new task sheet, Triangles or Not? (revisited). Can you use what you have learned to answer these questions?

Note that Question 2 on *Triangles or Not (revisited)* assumes knowledge of the term ‘isosceles’ triangle.

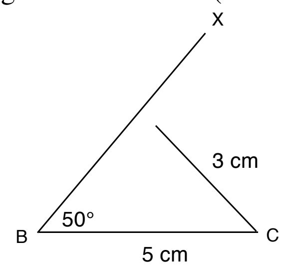
Some teachers give this as homework.

SOLUTIONS

Assessment task: *Triangles or Not?*

1.

a) It is impossible to draw a triangle with Angle B = 50°, AC = 3 cm and BC = 5 cm. If one draws BX at 50° to BC, then A must lie on BX. However, BX never gets within 3 cm of C (see diagram below). The smallest possible length for AC is when angle A = 90° and this occurs when AC is greater than 3 cm (3.83 cm, to 2 d.p.)

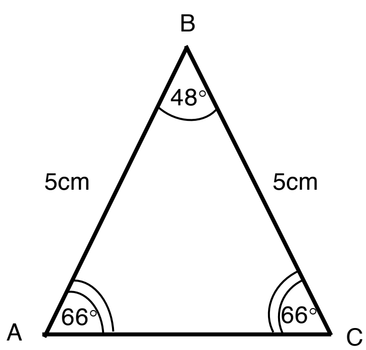
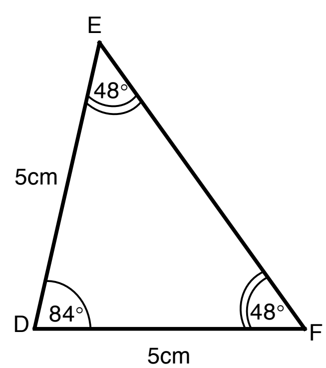


b) A unique triangle can be drawn with sides AB = 10 cm, BC = 11 cm and AC = 9 cm (with Angle A = 71°, Angle B = 50° and Angle C = 59°). In general, a unique triangle may always be drawn if three side lengths are given and the sum of any two is greater than the third.

c) More than one triangle can be drawn with Angle A = 40°, Angle B = 60° and Angle C = 80°.
The angles sum to 180°, so at least one triangle may be drawn.
However, since no side lengths are given, an infinite number of similar triangles may be drawn.

d) A unique triangle can be drawn with sides AB = 4 cm, BC = 3 cm and Angle B = 30°.
When two sides and an included angle are defined, then a unique triangle may always be drawn.

2.

	
A possible isosceles triangle with side AB = 5 cm and Angle B = 48°.	A possible isosceles triangle with side DE = 5 cm and Angle E = 48°.

These are not identical because there can be one 48° angle (as in triangle ABC) or two 48° angles (as in triangle DEF) in the isosceles triangle.

Collaborative activity: *Possible Triangles?*

The quality of the written reasoning with references to geometry and triangle properties is the focus of this task.

We provide, however, a list of categorizations for your convenience:

Unique triangle	More than one triangle	Not possible to draw a triangle
B	C (infinite number)	A
D	E (two possible)	F
G	H (infinite number)	
I		
J		

Students may also notice that Card C describes isosceles triangles, Cards H and I describe equilateral triangles and Card J describes a right-angled triangle.

Assessment task: *Triangles or Not? (revisited)*

1.

a) It is impossible to draw a triangle with sides AB = 3 cm, BC = 3 cm and AC = 8 cm as side AC is greater than the sum of the other two sides.

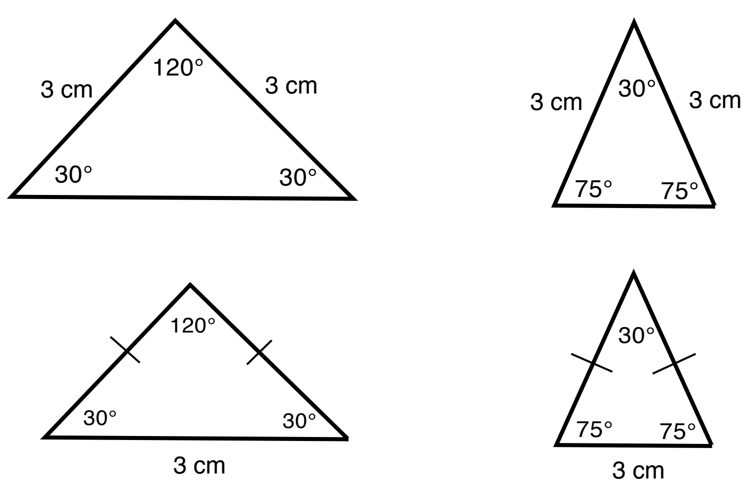
b) A unique triangle can be drawn with Angle A = 30°, Angle B = 45° and side AC = 6 cm. If two angles are given, the third may be determined. The angles thus determine the shape but not the size of the triangle. The additional side length then determines the size of the triangle.

c) An infinite number of triangles can be drawn with Angle A = 90°, Angle B = 30° and Angle C = 60°. They are all similar triangles.

d) Two possible triangles can be drawn with sides AB = 6 cm, BC = 4 cm and Angle A = 40°. This depends on whether Angle C is acute or obtuse.

2.

The possible isosceles triangles that have at least one 30° angle and at least one side that is 3 cm long are as follows (not drawn to scale):



Triangles or Not?

1. Decide from the information given whether:

   - It is possible to construct a unique triangle ABC.
   - It is possible to construct more than one triangle ABC.
   - It is not possible to construct a triangle ABC with these properties.

   Give reasons for your answers (You are not required to perform any accurate constructions).

Information	Triangle possible? Check (☑) the correct answer		Reason
a. Angle B = 50°, AC = 3 cm, BC = 5 cm.	Unique triangle		
	More than one triangle		
	Not possible		
b. AB = 10 cm, BC = 11 cm, AC = 9 cm.	Unique triangle		
	More than one triangle		
	Not possible		
c. Angle A = 40°, Angle B = 60°, Angle C = 80°.	Unique triangle		
	More than one triangle		
	Not possible		
d. AB = 4 cm, BC = 3 cm, Angle B = 30°.	Unique triangle		
	More than one triangle		
	Not possible		

2. Triangle ABC is isosceles with AB = 5 cm and Angle B = 48°.
Triangle DEF is isosceles with DE = 5 cm and Angle E = 48°.
Explain why the two triangles may not be identical.

Card Set: Possible Triangles?

A AB = 3 cm, BC = 7 cm, AC = 4 cm.	B BC = 6 cm, AC = 10 cm, Angle B = 90°.
C Angle A = 50°, Angle B = 50°, Angle C = 80°.	D Angle A = 40°, Angle B = 60°, AB = 5 cm.
E AC = 5 cm, AB = 7 cm, Angle B = 30°.	F BC = 7 cm, AC = 4 cm, Angle B = 45°.
G BC = 8 cm, AC = 6 cm, Angle C = 50°.	H Angle A = 60°, Angle B = 60°, Angle C = 60°.
I AB = 6 cm, BC = 6 cm, AC = 6 cm.	J BC = 4 cm, Angle B = 30°, Angle C = 60°.

Triangles or Not? (revisited)

1. Decide from the information given whether:

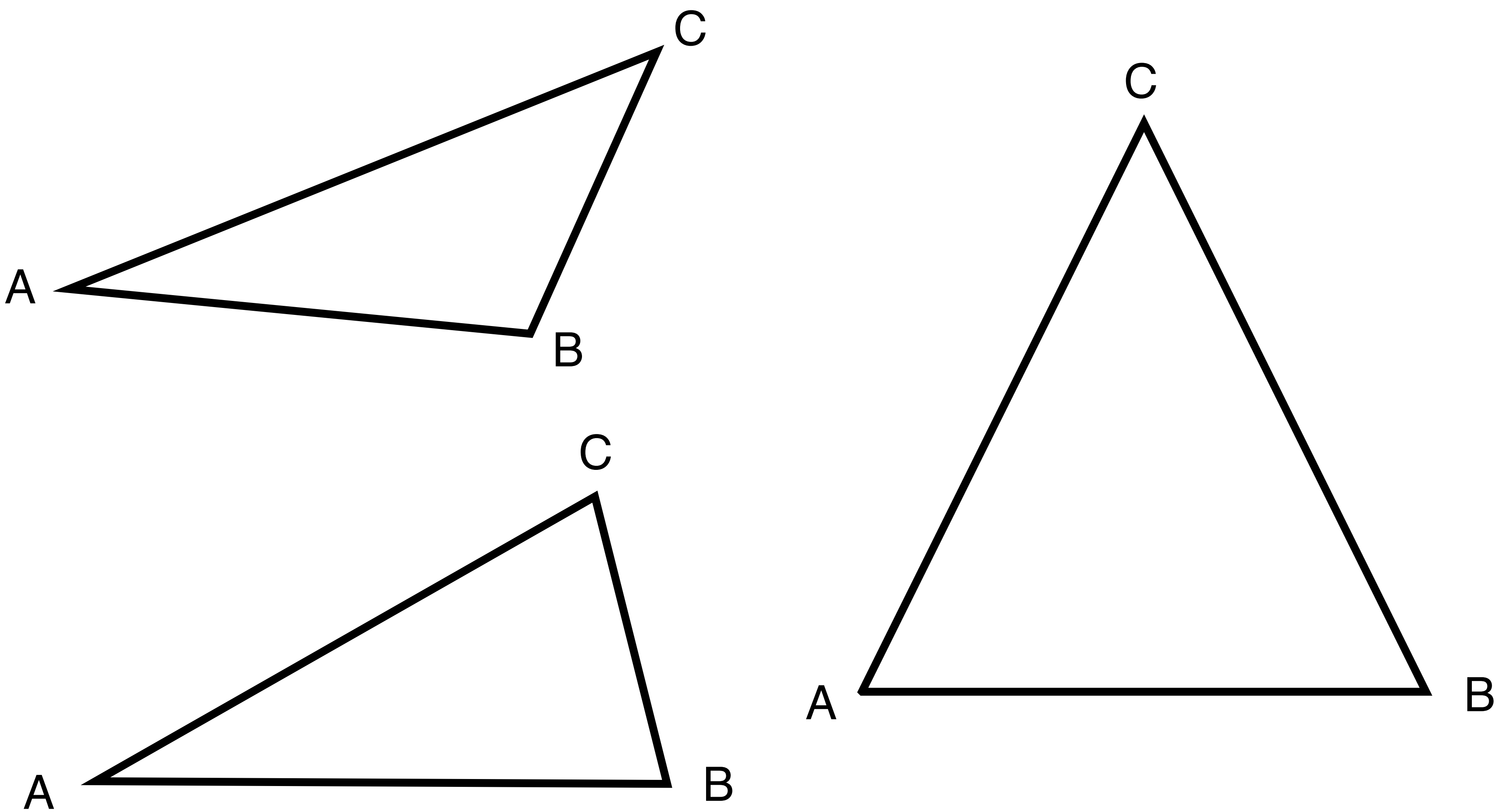
- It is possible to construct a unique triangle ABC.
- It is possible to construct more than one triangle ABC.
- It is not possible to construct a triangle ABC with these properties.

Give reasons for your answers (You are not required to perform any accurate constructions).

Information	Triangle possible? Check (☑) the correct answer		Reason
a. AB = 3 cm, BC = 3 cm, AC = 8 cm.	Unique triangle		
	More than one triangle		
	Not possible		
b. Angle A = 30°, Angle B = 45°, AC = 6 cm.	Unique triangle		
	More than one triangle		
	Not possible		
c. Angle A = 90˚, Angle B = 30˚, Angle C = 60˚.	Unique triangle		
	More than one triangle		
	Not possible		
d. AB = 6 cm, BC = 4 cm, Angle A = 40°.	Unique triangle		
	More than one triangle		
	Not possible		

2. An isosceles triangle has at least one 30° angle and at least one side that is 3 cm long.
   Sketch **all** different possible triangles with the features described.
   Label all the sides and angles clearly.
   Explain why each one is a possible triangle.

Triangles



Possible Triangle ABC?

AB = 4 cm,

AC = 4 cm,

Angle B = 40°.

Working Together

1. Take turns to select any card.

2. Work individually: can a triangle be produced?

3. Decide together how many triangles are possible (one, more than one, none).
    - If a triangle is not possible, agree on your reasoning.
    - If you disagree, challenge each other's explanations and work together to resolve your disagreements.

4. Once agreed, glue the card in the appropriate column and write an explanation **in pencil** on your poster, before moving on to another card.

Sharing Work

1. One person in your group jot down your card categorizations on your mini-whiteboard and then go to another group's desk and check your work against their categorizations.

2. If there are differences, explain your thinking to each other.

3. If you have categorized cards in the same columns, compare your methods and check that you understand each other's explanations.

4. If you are staying at your desk, be ready to explain the reasons for your group's decisions.

Card A

AB = 3 cm,

BC = 7 cm,

AC = 4 cm.

Card B

BC = 6 cm,

AC = 10 cm,

Angle B = 90°.

Card C

Angle A = 50°,
Angle B = 50°,
Angle C = 80°.

Card D

Angle A = 40°,
Angle B = 60°,
AB = 5 cm.

Card E

AC = 5 cm,
AB = 7 cm,
Angle B = 30°.

Card F

BC = 7 cm,

AC = 4 cm,

Angle B = 45°.

Card G

BC = 8 cm,
AC = 6 cm,
Angle C = 50°.

Card H

Angle A = 60°,
Angle B = 60°,
Angle C = 60°.

Card I

AB = 6 cm,

BC = 6 cm,

AC = 6 cm.

Card J

BC = 4 cm,

Angle B = 30°,

Angle C = 60°.

Choose Some Measures

Angle A = ….. ,
Angle B = ….. ,
Angle C = 50°.

Choose Some Measures

BC = 4 cm ,

AB = ... ,

AC = ...

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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University of Nottingham, England:

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Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and **Daniel Pead**

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Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

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Carina Wong, Melissa Chabran, and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org