CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Describing and Defining Quadrilaterals

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Describing and Defining Quadrilaterals

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Name and classify quadrilaterals according to their properties.
- Identify the minimal information required to define a quadrilateral.
- Sketch quadrilaterals with given conditions.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.G: Draw, construct, and describe geometrical figures and describe the relationships between them.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 3, 6, and 7:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their solutions and create questions for students to consider to help them improve their work.
- After a whole-class introduction, students work first individually, then in small groups, on a collaborative task, sketching quadrilaterals from a set of properties and identifying the minimal information required to complete the sketch.
- A whole-class discussion is held to explore the different combinations of property cards used by students when sketching the quadrilaterals.
- Finally, students work individually either on a new assessment task, or return to the original task and try to improve their responses.

MATERIALS REQUIRED

- Each individual student will need a copy of the assessment task *Classifying Quadrilaterals*, a copy of the assessment task *Classifying Quadrilaterals (revisited)*, a mini-whiteboard, a pen, and an eraser.
- Each pair of students will need a maximum of six copies of *Sketching Quadrilaterals*, a copy of the *Card Set: Properties* (cut the six property sets into strips), a pair of scissors, and a glue stick.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson, an 80-minute lesson (or two shorter lessons), and 15 minutes in a follow-up lesson. Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Classifying Quadrilaterals* (15 minutes)

Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Introduce the task briefly and help the class to understand what they are being asked to do.

This task is all about quadrilaterals. What are we referring to when we talk about a quadrilateral?

Different quadrilaterals have different properties and we can use these to help us to identify and classify a shape.

What do we mean by mathematical 'properties'? [Features of the shape.]

Before giving each student a copy of *Classifying Quadrilaterals*, you may want to display Slide P-1 for students to refer to when working on the assessment.

Note: Although there may be other definitions for some shapes, for this lesson, the definitions on the slide will be used.

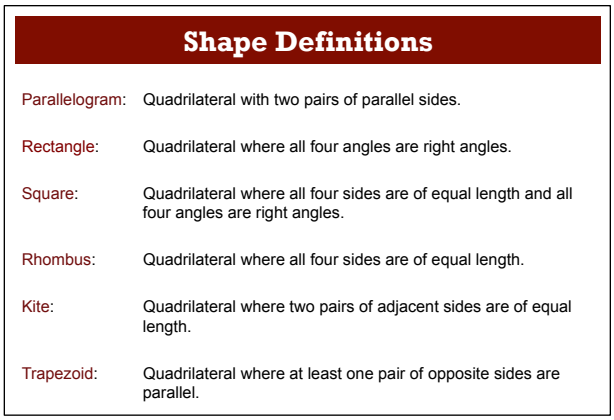
You may also want to check that your students understand the terms 'bisect' and 'diagonal'.

Once students have a copy of the task:

Read through the questions and try to answer them as carefully as you can. Give reasons and explain your answers fully.

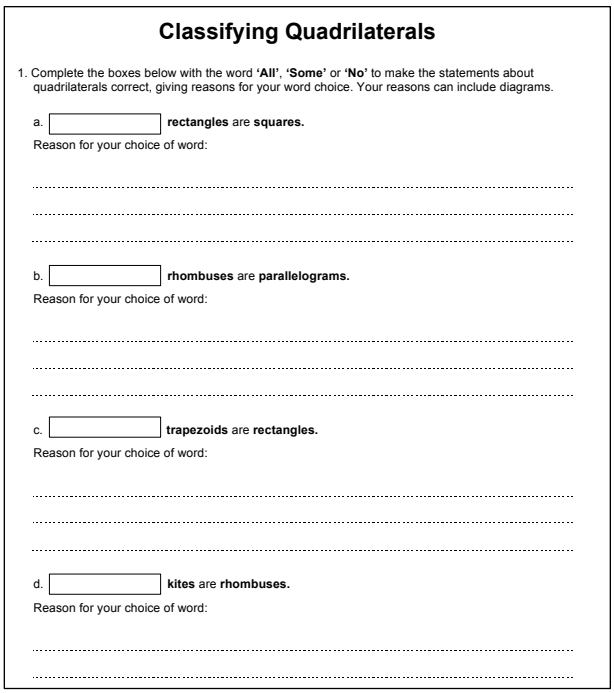
It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions like these confidently. This is their goal.



Shape Definitions

Parallelogram:	Quadrilateral with two pairs of parallel sides.
Rectangle:	Quadrilateral where all four angles are right angles.
Square:	Quadrilateral where all four sides are of equal length and all four angles are right angles.
Rhombus:	Quadrilateral where all four sides are of equal length.
Kite:	Quadrilateral where two pairs of adjacent sides are of equal length.
Trapezoid:	Quadrilateral where at least one pair of opposite sides are parallel.



Classifying Quadrilaterals

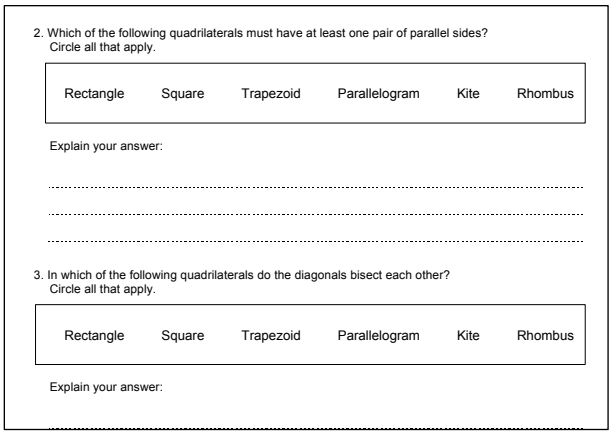
1. Complete the boxes below with the word **'All'**, **'Some'** or **'No'** to make the statements about quadrilaterals correct, giving reasons for your word choice. Your reasons can include diagrams.

a. [ ] **rectangles** are **squares.**
Reason for your choice of word:

b. [ ] **rhombuses** are **parallelograms.**
Reason for your choice of word:

c. [ ] **trapezoids** are **rectangles.**
Reason for your choice of word:

d. [ ] **kites** are **rhombuses.**
Reason for your choice of word:



2. Which of the following quadrilaterals must have at least one pair of parallel sides?
Circle all that apply.

Rectangle Square Trapezoid Parallelogram Kite Rhombus

Explain your answer:

3. In which of the following quadrilaterals do the diagonals bisect each other?
Circle all that apply.

Rectangle Square Trapezoid Parallelogram Kite Rhombus

Explain your answer:

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students in the follow-up lesson.

Common issues:	Suggested questions and prompts:
Understands different types of quadrilaterals as being distinct shapes rather than some quadrilaterals being subsets of others For example: The student states that 'no' rectangles are squares. (Q1a)	• What properties does a rectangle/square have? • Does a rectangle/square have all the properties of a square/rectangle? • Is it possible that one type of quadrilateral could be a special kind of a different quadrilateral? How could you tell from the properties if this was the case?
Assumes that the opposite sides of a rhombus are not parallel For example: The student states that 'no' rhombuses are parallelograms. (Q1b) Or: The student states that 'some' kites are rhombuses. (Q1d) Or: Fails to circle 'rhombus' as having at least one pair of parallel sides. (Q2)	• What do you know about the angles in a rhombus?
Assumes that a kite contains parallel sides For example: The student circles 'kite' as having at least one pair of parallel sides. (Q2)	• Does a kite have congruent sides? • Which sides in a kite are congruent?
Assumes diagonals that bisect must do so at 90° For example: The student circles just the square. (Q3)	• What does it mean for diagonals to bisect each other?
Assumes that the diagonals in an isosceles trapezoid bisect each other For example: The student provides an explanation that the diagonals of isosceles trapezoids bisect each other whereas non-isosceles trapezoids contain non-bisecting diagonals. (Q3)	• In what way is an isosceles trapezoid different to a non-isosceles trapezoid? • Draw in the diagonals of an isosceles trapezoid. What properties would the two triangles that are formed have if the diagonals were bisecting?
Provides little or no explanation For example: The student gives no reason for their choice of word (Q1) and/or fails to explain their answers. (Q2 & Q3)	• Which properties of (rectangles) do (trapezoids) not satisfy? • Can you convince me that a (rhombus) satisfies all the properties of a (parallelogram)? • What additional properties does a (square) have?

SUGGESTED LESSON OUTLINE

Whole-class interactive introduction (20 minutes)

Give each student a mini-whiteboard, pen, and eraser.

Remind the class of the assessment task they have already attempted.

Recall what we were working on previously. What was the task about?

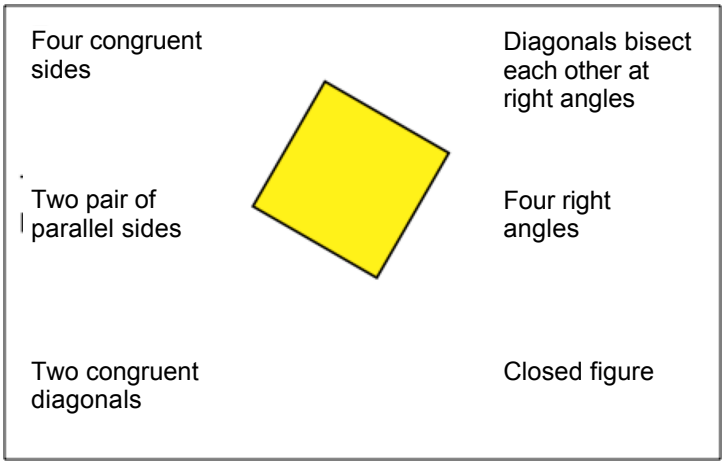
What do we mean by the 'properties' of a quadrilateral? [The mathematical features that the shape possesses.]

Let's now think about a specific quadrilateral.

Display Slide P-2 of the projector resource showing a square.

Spend a few minutes, on your own, writing on your whiteboard as many properties of a square as you can. Try to be as detailed as possible.

Once students have had a chance to identify a list of properties, list the students' ideas on the board. As you do this, encourage students to express the properties using correct mathematical language:



If students do not mention all of the features shown above, draw their attention to them and to the language needed to describe them, as they will need to understand this vocabulary for the rest of the lesson.

What do we mean by the word 'congruent'?

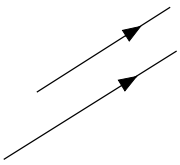
What do we mean by the word 'parallel'?

What is a 'diagonal'?

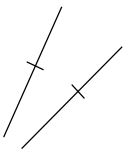
What does 'bisect' mean?

What does 'bisect at right angles' mean?

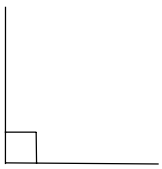
It may be helpful when collating ideas about the properties of a square to discuss ways of showing some of these properties on the diagram, for example:



These lines are parallel



These lines are congruent



This angle is 90°

When a range of properties have been identified, ask the following:

Remember that for this lesson we are talking about quadrilaterals only.

Does this property [e.g. two equal diagonals] by itself define a square?
If not, what other quadrilaterals have this property? [E.g. rectangle.]

Can you identify two properties that together define a square?
Can you find another pair?
What else do you need to know in order to draw the square?
[E.g. four right angles and four congruent sides.]

Can you identify a pair of properties that won't necessarily define a square?
What other quadrilaterals could these properties be defining?
[E.g. 'diagonals meet at 90°' and 'four congruent sides' could be describing a rhombus.]

It may be appropriate to extend this questioning further to include, for example, more than two properties. However, being able to identify properties that define a square will depend on the original list generated by the class.

Individual work, then collaborative work: *Sketching Quadrilaterals* (40 minutes)

Organize students into pairs and give each group of students the six sets of *Properties* cards, cut into strips. Ask students to work individually to start with. Introduce the activity by showing and explaining to students Slide P-3 of the projector resource:

Working Individually

1. Each strip of 5 properties describes a quadrilateral.
   Each person should select just one set.
2. For this set, draw the quadrilateral described by the 5 properties on your mini-whiteboard.
   Name the quadrilateral you have drawn.
   Label the sides and angles.
3. Now select the **smallest** number of cards you need in order to define the shape and size of the quadrilateral.
4. Be prepared to explain to your partner how you know that the shape you have sketched is correct and why you only need these cards to define it.

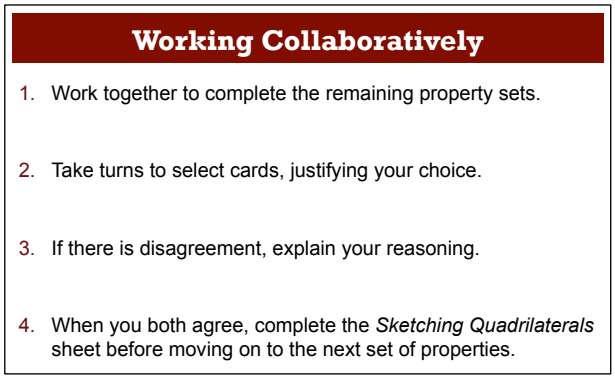
When most students have at least one card set completed, ask students to work in pairs. Give each pair some scissors, a glue stick and six copies of *Sketching Quadrilaterals*. Explain Slide P-4 of the projector resource:

Sharing Work

1. Take turns to share your drawing and explanation with your partner. Ask questions if you do not understand an explanation.
2. Make sure you both agree and can explain:
   - why your chosen cards define the shape and size of your quadrilateral,
   - why this is the smallest number of cards needed.
3. Complete the *Sketching Quadrilaterals* sheet, gluing down the cards in the agreed order.

Once students have agreed upon and completed the cards they worked on individually, they need to work collaboratively on the remaining *Properties* cards.

Display Slide P-5 of the projector resource and explain how students are to work together:



You have two tasks during the group work: to make a note of student approaches to the task and to support students working as a group.

Make a note of student approaches to the task

Listen and watch students carefully. In particular, notice how students make a start on the task, where they get stuck and how they overcome any difficulties.

Do students sort the set of property cards in any way before they start to sketch the quadrilateral? If so, how? What do they focus on first? Are there any cards that they consider to be irrelevant or do they use the information on these cards to check that the quadrilateral they have drawn is correct? What do they do when a property card that they haven't referred to when drawing their sketch contradicts what they have drawn? To make the minimal set of property cards needed to define the shape, do they eliminate cards from the original set or do they build up the minimal set?

Support students working as a group

As students work on the task support them in working together. Encourage them to take turns and if you notice that one partner is doing all the sketching or that they are not working collaboratively on the task, ask students in the group to explain a sketch drawn by someone else in the group.

Encourage students to clearly explain their choice of cards. Some shapes can be defined using more than one combination of cards. If this is the case, encourage students to make a note of the other possible card combination(s) somewhere on their sheet.

Try to avoid identifying the information students need to complete a sketch. If students are struggling to get started, encourage them to think about what quadrilaterals they know and their properties. This may help them recognize which properties these quadrilaterals share and which make them distinct shapes.

Check that students have completed each sheet before moving on to the next set of properties.

> *How did you figure out the minimal set of property cards to define the shape?*
>
> *Is there a different set of property cards that could also define the shape?*
>
> *If I removed this property card from your minimal set of shapes, what shapes can now be defined?*
>
> *Is it possible to figure out all the angles and lengths for the quadrilateral? [Not for Shape E and F. Students would need to draw the shapes accurately or use trigonometry!]*

It is not essential that students work on all six property sets, but rather that they are able to develop good explanations.

If students do successfully complete all six sketches, encourage them to produce an accurate drawing of each of the six quadrilaterals using a ruler and protractor and/or compasses.

Extending the lesson over two days

If your lessons are shorter, you may wish to stop students part-way through the collaborative work and continue with this in the next lesson. If this is the case, at the end of the first lesson, make sure that all of the *Properties* cards that have been agreed upon have been stuck down. Then, at the start of the second lesson, give students time to familiarize themselves with their work and complete any sets of properties they have yet to work on. It is not essential that all students complete all six property sets, but most groups should have at least two or three sets completed before moving on to discuss them as a whole-class.

Whole-class discussion (20 minutes)

The aim of this discussion is to explore the different combinations of property cards used by students when completing their sketches. There may not be time to discuss all six quadrilaterals but aim to discuss at least two or three. Use your knowledge of the students' group work to call on a wide range of students for contributions.

Charlie, what quadrilateral did your group draw for property card set C?

Did any group sketch a different quadrilateral?

Charlie come and sketch the shape your group drew for property card set C on the board.

If students have sketched a different quadrilateral for a particular property card set or labeled the sketch differently, ask them to re-produce their sketch on the board as well so that the sketches can be compared. Alternatively a document reader may be used, if available, to enable the class to compare sketches.

Charlie, which property cards did your group use to define this quadrilateral?

Has Charlie's group used the least possible number of cards?

Let's test his answer.
If we remove this property card, what else could the shape be?
Now let's remove this one instead…

Did any group use a different minimal set of cards to define the quadrilateral?

Once the completion of sketches for a few of the quadrilaterals has been discussed, explore further the different strategies used when completing the sketches.

Which quadrilaterals were the easiest to sketch? Why was this?

Did you look for a particular type of property when starting to sketch the quadrilateral or did it vary from shape to shape?

Were the property cards that didn't get selected for the minimal set used to check the sketch and/or quadrilateral type?

Is it possible to draw any of these shapes without knowing all the measurements? [Yes, Shapes E and F. Trigonometry is needed to figure out the missing angles and lengths!]

You may want to draw on the questions in the *Common issues* table to support your own questioning. Slides P-6 to P-11 (printed on transparency film if preferred) may be used to support this discussion.

Follow-up lesson: reviewing the assessment task (15 minutes)

Give each student a copy of the assessment task *Classifying Quadrilaterals (revisited)* and their original solutions to the assessment task *Classifying Quadrilaterals*.

Read through your papers from Classifying Quadrilaterals and the questions [on the board/written on your paper.] Answer these questions and revise your response.

Now look at the new task sheet, Classifying Quadrilaterals (revisited). Can you use what you have learned to answer these questions?

If students struggled with the original assessment task, you may feel it more appropriate for them to revisit *Classifying Quadrilaterals* rather than attempting *Classifying Quadrilaterals (revisited)*. If this is the case, give them another copy of the original assessment task instead.

SOLUTIONS

Definitions:

In the solutions below we use the following definitions.

Parallelogram: quadrilateral with two pairs of parallel sides.

Rectangle: quadrilateral where all four angles are right angles.

Square: quadrilateral where all four sides are of equal length, and all four angles are right angles.

Rhombus: quadrilateral where all four sides are of equal length.

Kite: quadrilateral where two pairs of adjacent sides are of equal length.

Trapezoid: quadrilateral where at least one pair of opposite sides are parallel.

Assessment task: *Classifying Quadrilaterals*

1a. **SOME** rectangles are squares. A square has all the properties of a rectangle with the additional property of four congruent sides.

1b. **ALL** rhombuses are parallelograms. Parallelograms have congruent and parallel opposite sides, opposite angles are equal and diagonals bisect each other but are not congruent. A rhombus has all of these properties with the additional properties that all sides are congruent and the diagonals bisect each other at right angles.

1c. **SOME** trapezoids are rectangles. All rectangles are trapezoids, but not all trapezoids are rectangles.

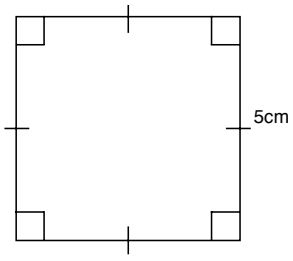
1d. **SOME** kites are rhombuses. A kite has two pairs of adjacent congruent sides and if all four sides are congruent then the kite is a rhombus.

2. A **kite** is the only quadrilateral in the list that does not have to have at least one pair of parallel sides.

3. The diagonals in a **rectangle**, **square**, **parallelogram** and **rhombus** must bisect each other. The diagonals in **trapezoids** and **kites** do not necessarily bisect each other.

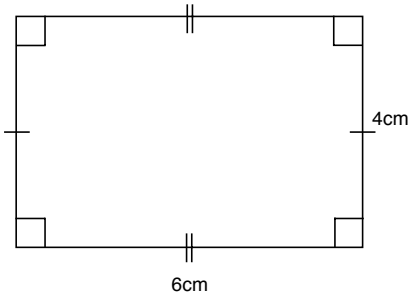
Collaborative task:

Shape A is a square:



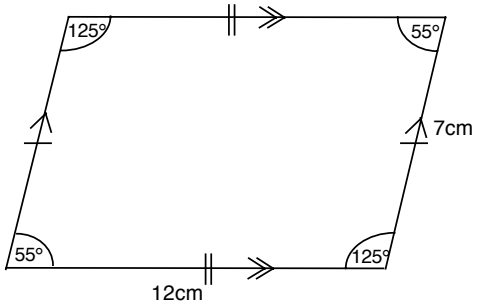
The minimal set of properties contains three cards, for example A2, A3, & A4 define the square.

Shape B is a rectangle:



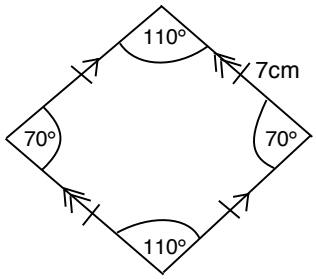
The minimal set of properties contains three cards, for example B1, B3, & B5 define the rectangle.

Shape C is a parallelogram:



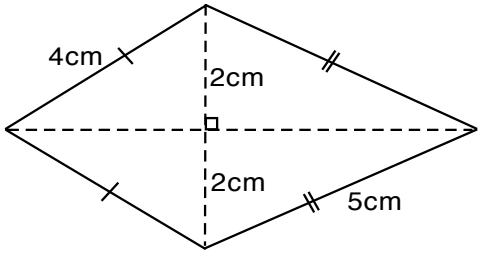
The minimal set of properties contains four cards, for example C2, C3, C4, & C5 define the parallelogram.

Shape D is a rhombus:



The minimal set of properties contains three cards, for example D2, D3, & D5 define the rhombus.

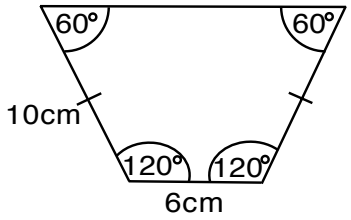
Shape E is a kite:



The minimal set of properties contains four cards, for example E1, E2, E4, & E5 define the kite.

No angles are given for **Shape E** so when students are sketching the kite they will not be able to label any angles on their sketch. However, it is possible to construct the kite from the information given.

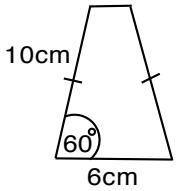
Shape F is an isosceles trapezoid:



The length of the longest side of the trapezoid is not given in the properties of **Shape F** so students will not be able to label the length of this side on their sketch. However, it is possible to construct the trapezoid from the information given.

All five cards are needed to define the trapezoid.

Note: Some students may sketch Shape F as shown below:



This is not possible to draw.

Assessment task: *Classifying Quadrilaterals (revisited)*

1a. **ALL** rectangles are parallelograms. A rectangle has all of the properties of a parallelogram with the additional properties of four congruent angles and congruent diagonals.

1b. **SOME** parallelograms are squares. Parallelograms have congruent and parallel opposite sides, opposite angles are equal and diagonals bisect each other. Squares have four congruent sides and four congruent angles and diagonals that bisect each other.

1c. **ALL** squares are rhombuses. A square is a rhombus with four congruent angles so all squares are rhombuses.

1d. **SOME** trapezoids are kites. A trapezoid with two pairs of adjacent sides equal (i.e. it is a rhombus) is also a kite.

2. A **rectangle**, a **square**, a **parallelogram**, a **kite**, and a **rhombus** all have at least one pair of congruent sides. A **trapezoid** is the only quadrilateral in the list that does not necessarily have at least one pair of congruent sides.

3. **Squares** and **rhombuses** are the only quadrilaterals in the list with diagonals that bisect each other at right angles.

Classifying Quadrilaterals

1. Complete the boxes below with the word **'All'**, **'Some'** or **'No'** to make the statements about quadrilaterals correct, giving reasons for your word choice. Your reasons can include diagrams.

   a. ☐ **rectangles** are **squares.**

   Reason for your choice of word:

   b. ☐ **rhombuses** are **parallelograms.**

   Reason for your choice of word:

   c. ☐ **trapezoids** are **rectangles.**

   Reason for your choice of word:

   d. ☐ **kites** are **rhombuses.**

   Reason for your choice of word:

2. Which of the following quadrilaterals must have at least one pair of parallel sides?
Circle all that apply.

Rectangle	Square	Trapezoid	Parallelogram	Kite	Rhombus

Explain your answer:

3. In which of the following quadrilaterals do the diagonals bisect each other?
Circle all that apply.

Rectangle	Square	Trapezoid	Parallelogram	Kite	Rhombus

Explain your answer:

Card Set: Properties

A1 The diagonals of the shape are congruent	A2 The shape has at least one side that is 5cm long	A3 The diagonals of the shape bisect each other at right angles	A4 The shape has 4 equal angles	A5 The shape has two pairs of parallel sides

B1 The shape has at least one side that is 4cm long	B2 The diagonals of the shape bisect each other	B3 The shape has 4 equal angles	B4 Opposite sides of the shape are congruent	B5 The shape has at least one side that is 6cm long

C1 The diagonals of the shape are not congruent	C2 The shape has at least one side that is 12cm long	C3 The shape has at least one side that is 7cm long	C4 The shape contains at least one 55° angle	C5 Opposite sides of the shape are parallel

D1 The diagonals of the shape bisect each other at right angles	D2 All four sides are congruent	D3 The shape contains at least one 70° angle	D4 Opposite sides of the shape are parallel	D5 The shape has at least one side that is 7cm long

E1 The shape has at least one side that is 5cm long	E2 One diagonal bisects the other diagonal into two 2cm segments	E3 The shape has two pairs of congruent sides	E4 The diagonals of the shape intersect each other at right angles	E5 The shape has at least one side that is 4cm long

F1 The shape contains exactly one pair of parallel sides	F2 The shape has more than one side that is 10cm long	F3 The shape contains at least one 60° angle	F4 The shape has a side that is 6cm long	F5 The shape contains a pair of opposite sides that are congruent

Sketching Quadrilaterals

Sketch the quadrilateral and label it appropriately:

What is the mathematical name of the quadrilateral?

Find the **smallest** number of property cards that you need to *define* the quadrilateral.

Cut out and stick them below:

Explain how you know that you need **all** of these cards to define the quadrilateral:

Classifying Quadrilaterals (revisited)

1. Complete the boxes below with the word **'All'**, **'Some'** or **'No'** to make the statements about quadrilaterals correct, giving reasons for your word choice. Your reasons can include diagrams.

a. [ ] **rectangles** are **parallelograms.**

Reason for your choice of word:

b. [ ] **parallelograms** are **squares.**

Reason for your choice of word:

c. [ ] **squares** are **rhombuses.**

Reason for your choice of word:

d. [ ] **trapezoids** are **kites.**

Reason for your choice of word:

2. Which of the following quadrilaterals must have at least one pair of congruent sides?
Circle all that apply.

Rectangle	Square	Trapezoid	Parallelogram	Kite	Rhombus

Explain your answer:

3. Which of the following quadrilaterals' diagonals must bisect each other at right angles?
Circle all that apply.

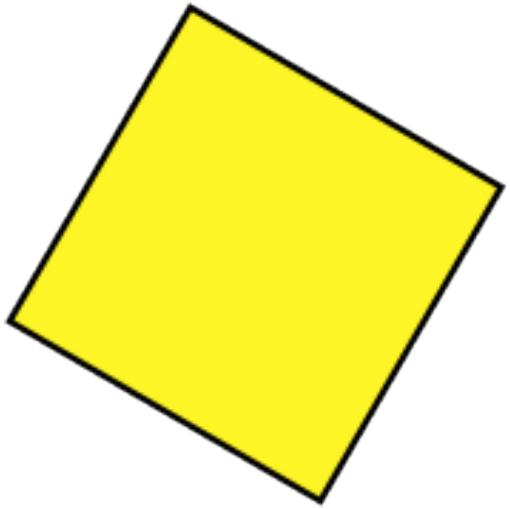
Rectangle	Square	Trapezoid	Parallelogram	Kite	Rhombus

Explain your answer:

Shape Definitions

Parallelogram:	Quadrilateral with two pairs of parallel sides.
Rectangle:	Quadrilateral where all four angles are right angles.
Square:	Quadrilateral where all four sides are of equal length and all four angles are right angles.
Rhombus:	Quadrilateral where all four sides are of equal length.
Kite:	Quadrilateral where two pairs of adjacent sides are of equal length.
Trapezoid:	Quadrilateral where at least one pair of opposite sides are parallel.

A Square



Working Individually

1. Each strip of 5 properties describes a quadrilateral.
   Each person should select just one set.

2. For this set, draw the quadrilateral described by the 5 properties on your mini-whiteboard.
   Name the quadrilateral you have drawn.
   Label the sides and angles.

3. Now select the **smallest** number of cards you need in order to define the shape and size of the quadrilateral.

4. Be prepared to explain to your partner how you know that the shape you have sketched is correct and why you only need these cards to define it.

Sharing Work

1. Take turns to share your drawing and explanation with your partner. Ask questions if you do not understand an explanation.
2. Make sure you both agree and can explain:
   - why your chosen cards define the shape and size of your quadrilateral,
   - why this is the smallest number of cards needed.
3. Complete the *Sketching Quadrilaterals* sheet, gluing down the cards in the agreed order.

Working Collaboratively

1. Work together to complete the remaining property sets.

2. Take turns to select cards, justifying your choice.

3. If there is disagreement, explain your reasoning.

4. When you both agree, complete the *Sketching Quadrilaterals* sheet before moving on to the next set of properties.

Property Card Set A

A1 The diagonals of the shape are congruent	A2 The shape has at least one side that is 5cm long	A3 The diagonals of the shape bisect each other at right angles	A4 The shape has 4 equal angles	A5 The shape has two pairs of parallel sides

Property Card Set B

B1 The shape has at least one side that is 4cm long	B2 The diagonals of the shape bisect each other	B3 The shape has 4 equal angles	B4 Opposite sides of the shape are congruent	B5 The shape has at least one side that is 6cm long

Property Card Set C

C1	C2	C3	C4	C5
The diagonals of the shape are not congruent	The shape has at least one side that is 12cm long	The shape has at least one side that is 7cm long	The shape contains at least one 55° angle	Opposite sides of the shape are parallel

Property Card Set D

D1 The diagonals of the shape bisect each other at right angles	D2 All four sides are congruent	D3 The shape contains at least one 70° angle	D4 Opposite sides of the shape are parallel	D5 The shape has at least one side that is 7cm long

Property Card Set E

E1 The shape has at least one side that is 5cm long	E2 One diagonal bisects the other diagonal into two 2cm segments	E3 The shape has two pairs of congruent sides	E4 The diagonals of the shape intersect each other at right angles	E5 The shape has at least one side that is 4cm long

Property Card Set F

F1	F2	F3	F4	F5
The shape contains exactly one pair of parallel sides	The shape has more than one side that is 10cm long	The shape contains at least one 60° angle	The shape has a side that is 6cm long	The shape contains a pair of opposite sides that are congruent

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
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Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org