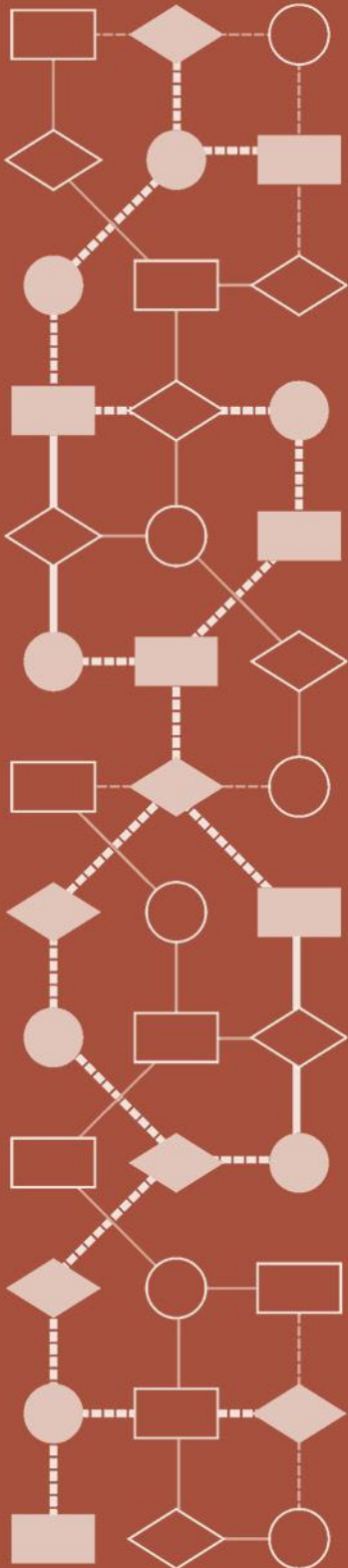


Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Drawing to Scale: *A Garden*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

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Drawing to Scale: *A Garden*

MATHEMATICAL GOALS

This lesson unit is intended to help assess how well students are able to interpret and use scale drawings to plan a garden layout. This involves using proportional reasoning and metric units.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 3, 4, 5, and 6:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

- 7.G: Draw, construct, and describe geometrical figures and describe the relationships between them.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
- 7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on a task designed to reveal their current levels of understanding. You review their responses and create questions to help them improve their work.
- At the start of the lesson, students reflect on their individual responses, before producing a collaborative improved solution to the task. Then, in the same small groups students analyze sample responses. They then discuss as a whole-class the methods they have seen and used.
- In a follow-up lesson, students reflect on their work. If time allows, an extension task is available.

MATERIALS REQUIRED

- Each student will need a copy of *Design a Garden* and *Garden Plan*, some blank paper, a mini-whiteboard, pen, and eraser, and the *How Did You Work?* questionnaire. The *Garden Plan* should be copied at exactly 100% scale so the measurements are accurate. If this is not possible, photocopy the rules on S-3, one rule per student, which should then match the *Garden Plan* measurements. It will be useful to have spare copies of the *Garden Plan*.
- Each small group of students will need a new copy of the *Garden Plan*, the *Assistants' Methods*, a glue stick, felt-tipped pen, and a sheet of poster paper. For the optional extension *Mandy's Second Email* will be needed. Provide short rules, meter rules, string, protractors, scissors, glue, card, plain paper, graph paper, and colored pencils for students who choose to use them.

TIME NEEDED

20 minutes before the lesson, a 110-minute lesson (or two 60-minute lessons), and 10 minutes in a follow-up lesson. Actual timings will depend on the needs of your students.

BEFORE THE LESSON

Assessment task: *Design a Garden* (20 minutes)

Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the lesson that follows.

Display Slide P-1 of the projector resource. Introduce the task briefly, helping the class to understand the problem and its context.

Here are some pictures of gardens that have been designed. They are drawn carefully to scale so that the customer can get a good idea of what the finished garden will look like.

In this task, you will be taking the role of garden designers.

Does anyone have a nice-looking garden? What do you have in it?

Has anyone ever used a garden designer?

This is an opportunity for students to talk briefly about any relevant experiences they may have had, in order to help them to get into the problem.

Give each student a copy of the assessment task *Design a Garden* and *Garden Plan*. They will also need some blank paper to work on.

Spend a few minutes reading the first sheet, Design a Garden.

If you cannot photocopy or print the sheets at precisely 100%, print also some copies of the paper rules that are provided. (This will distort the plan and the rules equally.) The paper rules work best if they are folded along the line next to the scale, to give a sharp, straight edge.

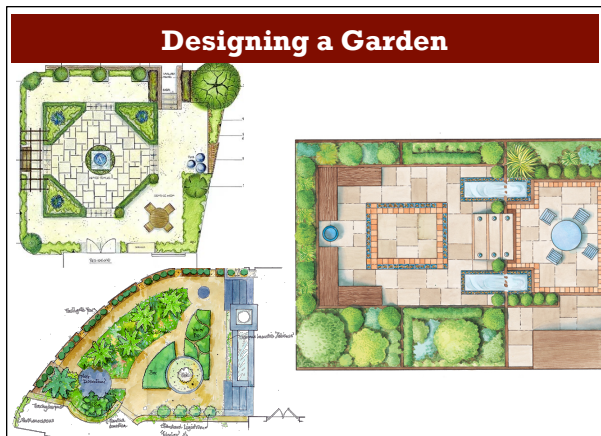
Now look at the second sheet, Garden Plan. What direction are you looking from in this drawing?

The plan shows that the garden is 10 meters long.

Read through the resources carefully and follow the instructions for designing the garden.

Show your calculations and reasoning on a blank piece of paper.

Students who sit together often produce similar answers, so when they come to compare their work, they have little to discuss. For this reason, we suggest that when students work individually on the assessment task, you ask them to move to different seats.



Design a Garden

Imagine you are a garden designer.
You receive this email from a customer.

Dear Garden Designer,

I have moved into a house with a small garden that needs a total redesign. Please design my garden for me. I have attached an accurate scale drawing of my garden to this email. I've listed below some features I want in the garden. I will email you later about some other things I also want.

To start, please could you draw these features accurately on the plan, showing where you think they should go in the garden. Send me your plan with an explanation of your thinking.

Best wishes,
Mandy

Shed I've ordered this shed. It is 2 meters wide, 3.25 meters long and 2.8 meters tall.	
Decking for barbeques I want some decking near the patio doors. It should be big enough to seat at least six people.	
Circular pond I would like a circular pond. I'd like its area to be about 7 m ² .	
Path and Borders I would like some flower borders. These should not be more than one meter wide as I find wider ones difficult to look after. I'd like a gravel path 1 meter wide to go from the shed to the house and from the garden gate to the house. I will cover the rest with grass.	

Use the sheet *Garden Plan* to draw the features from the email.
Record all your calculations and reasoning on a separate sheet.
Make sure to record the scale you use on the plan.

At the beginning of the formative assessment lesson allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

It is important that, as far as possible, students are allowed to answer the questions without assistance. If they are struggling to get started, ask questions to help students understand what is required, but make sure you do not do the task for them. The questions corresponding to the first issue in the *Common issues* table might be helpful.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

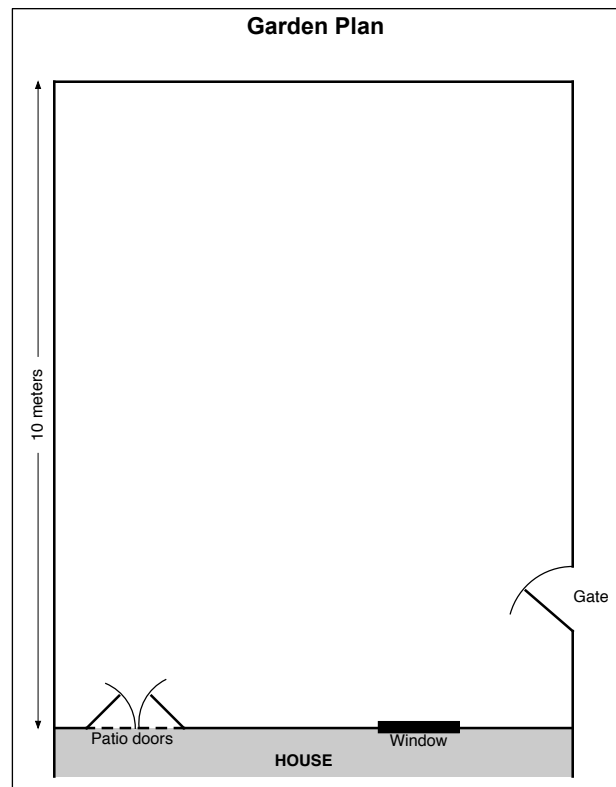
We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.



Common issues**Suggested questions and prompts**

<p>Has little or no written work</p>	<ul style="list-style-type: none"> • Which direction are you looking from when you make a plan view? • What useful information are you given? Underline this. What do you need to find out? How can you use the information you know to do this?
<p>Lacks precision in their work</p> <p>For example: The student writes the scale in words or as an equality '2 cm is/= 1 m'.</p>	<ul style="list-style-type: none"> • What do you mean by 'is' / 'equals'? Can you be more precise?
<p>Has difficulty in expressing a scale</p> <p>For example: The student does not calculate the scale ratio or has calculated it incorrectly or drawn objects with no apparent calculations of scale.</p> <p>For example: The student has the scale ratio as 1 : 200 or 1cm : 4m.</p> <p>Or: The student writes the figures in reverse order, for example 50 : 1 instead of 1 : 50.</p> <p>Or: The student uses fractions in the scale ratio, for example 1 : 0.5.</p> <p>Or: The student has not calculated a unitary scale ratio, for example 2 cm : 1 m.</p>	<ul style="list-style-type: none"> • How long is the real-life garden in centimeters? How long is the drawing in centimeters? Can you use this to figure out the scale? • If an object on the plan is 1cm long, how long is the real object? How do you know? • Usually a scale ratio is written in the order length on the plan : length in real life. What is your scale ratio when written this way? • Can you write the scale ratio more simply, without using fractions or decimals?
<p>Has difficulty calculating measures on the plan</p> <p>For example: The student has difficulty when the actual lengths are not whole numbers.</p>	<ul style="list-style-type: none"> • How do you convert 1m in real life to a measurement on the plan? What about 3m? Now apply the same method to figure out what the length 3.25 m is on the plan.
<p>Does not calculate an appropriate radius for the pond</p> <p>For example: The student uses guess and check unsuccessfully.</p> <p>Or: The student does not use consistent units in calculations.</p>	<ul style="list-style-type: none"> • Can you draw a square with area 7 m²? Could this help you to draw a circle with a similar area? • Can you pick a radius for the pond and work out what area that would give? • What is the formula for the area of a circle? Can you use this to find the pond's radius?
<p>Does not explain a method for determining the area needed for the decking</p>	<ul style="list-style-type: none"> • How much space do six people occupy when sitting down? How much space do you think you need to allow between them?
<p>Makes a technical error</p>	<ul style="list-style-type: none"> • How can you determine that your scale is/measures are reasonable? What's your evidence? • How can you verify that your design has met all of Mandy's requests?

SUGGESTED LESSON OUTLINE

Individual work (15 minutes)

Return the assessment task to the students along with some blank paper. If you did not add questions to individual pieces of work, write your list of questions on the board. Students can then select questions appropriate to their own work.

Begin the lesson by briefly reintroducing the problem:

Recall your work on the design task. What was the task about?

Today we are going to improve your work on this task.

I have looked over your papers and have some questions for you.

Work individually. Read my questions. Use the questions to figure out how to improve your work.

Write notes on the sheet, or on the blank paper, about what you think will improve your work.

While students are working, focus your attention on any students that struggled to begin the assessment task, to identify the scale factor or make sense of the plan view. Give students time to revise their work, using your written questions and notice changes they make. Use what you notice to organize groupings for the next task. After ten minutes if there is any student who continues to struggle beginning the task, drawing objects in plan view, or finding the scale factor, you might pair him or her with a student who has succeeded on that part of the work in the next task.

Collaborative small group work: improving the *Garden Plan* (25 minutes)

Give each small group a copy of the task sheet, *Garden Plan*, a large sheet of poster paper, a felt-tipped pen, and a glue stick.

Slide P-2 of the projector resource summarizes the process students should follow:

Collaborating With Your Partner

- Take turns explaining your *Garden Plan* to your partner. Explain how you would improve your solution. Listen to each other carefully.
- Ask 'clarifying questions' that will help you understand your partner's reasoning.
- When you have both taken a turn, decide how to design a new, better garden together.
- Draw your plan on the *Garden Plan* sheet and stick it in the middle of the poster paper.
- Use the space around the edge to write your reasoning, decisions, and calculations.

While students are working you have three tasks: to notice students' strengths and difficulties, to support their reasoning, and to ensure collaborative working.

Notice students' strengths and difficulties

Note different approaches to the task and the assumptions students make.

Are there any students who struggle to imagine what objects would look like in plan view?

Do any students continue to struggle with using the scale? Notice how students express the scale ratio, their use of units and whether they express it in conventional form.

Are there any students struggling with calculating measures on the plan or translating measures on the plan to real-life measures? They may be having difficulty working with scaling relationships. Students might not recognize that there is a multiplicative relationship between the two measures and attempt to use additive methods. Other students might use informal methods and they may then find it difficult to perform calculations on non-integer values.

Do students struggle with the relationships between units? How do they handle moving to relationships between units when area and volume are introduced?

Notice how students carry out their work. Do they plan, developing a strategy? Do students work systematically? What do students do when they become stuck?

Do students check their work? Have students made practical mistakes such as placing the shed right next to the window? Do students check whether answers are sensible by thinking about the context?

You may want to use the questions in the *Common issues* table to support your own questioning. If the whole class is struggling on the same issue, you could write one or two relevant questions on the board and hold a brief whole-class discussion.

Support students' reasoning

Rather than resolve students' difficulties for them, ask questions to support them in making progress. You might ask strategic questions, to help them direct and organize their work:

What have you done so far?

Can you explain your strategy?

What math do you know about [working with scale ratios/the area of a circle]? How can you use that to solve this problem?

If students have made errors, or are struggling to make progress with a particular part of the design, ask more focused questions to help them make sense of the mathematics.

Think of a right rectangular prism. If you were drawing a plan view, what would it look like?

Tell me the information given about measures on the Garden Plan sheet.

If students continue to struggle, move the lesson on to the *Collaborative Analysis of Assistants' Methods*. These show a variety of approaches to drawing the pond feature. When students have analyzed the sample work they are to return to improving their own garden design.

Develop collaborative working

Encourage students to take turns in talking and to listen carefully to each other's explanations. Ask the students to be responsible for making sure that they all understand.

If one student dominates the written activities, encourage turn taking there too. You can check that there are samples of each student's handwriting on the poster.

Make sure that all students in the group are able to explain what is written on the poster. If you see one student writing an explanation, ask another student to explain to you how they figured out their solution.

Poster gallery (20 minutes)

Once students have had sufficient time producing an improved collaborative garden design organize them so that one student stays with the group's poster and the other visits another group's poster.

Give each student two sticky notes and explain how they are to share their work.

Slide P-3 of the projector resource summarizes the process students should follow:

Poster Gallery
<ul style="list-style-type: none">• One person from each group get up and visit another group's poster.• If you are the visitor, read the poster. If there is math you do not understand, ask clarifying questions.• If you are staying with your poster, explain the math to the visitor.• If you find things you could do to improve your poster, write them on your sticky notes and attach to your poster.

After five minutes, ask students to swap roles.

Extending the lesson over two days

If you are planning to teach the unit over two class periods, this is a good place to break. Begin the next lesson with a quick review of work so far and then move on to the next activity.

Collaborative analysis of Assistants' Methods (30 minutes)

Organize students into pairs or threes. Distribute copies of each of the *Assistants' Methods* to each group.

Imagine you have three assistants who have worked on designing this garden. Their work is incomplete. They all use different methods in their work.

Your job is to try to understand what they have done and to improve their work.

Slide P-4 of the projector resource summarizes the process students should follow:

Analysis of Assistants' Methods
<ul style="list-style-type: none">• Choose one assistant's work and read it carefully.• Answer the questions underneath.• Try to understand what they have done and think about how the work could be improved.• Take turns explaining your thinking to your partner.• Listen carefully and ask clarifying questions.• When your group has reached its conclusions, write your ideas below the assistant's work.• Now check out another assistant's work in the same way.

This task gives students an opportunity to evaluate different approaches to the task. The methods used by the assistants are incomplete and contain some errors. They illustrate different strategies for figuring out measures for the length of objects on the scale drawing and obtaining real-life measures from the lengths drawn on the plan.

While students work, notice their strengths and difficulties, support their reasoning, and develop collaborative working as before. Listen to students' discussion of the explanations. Is there one particular method that many students struggle to understand? Do any students reason strongly about a particular method? Note this and call on their expertise in later discussions.

Encourage students to focus on evaluating the math contained in the assistants' work, rather than surface features like neatness. Ask questions to help them articulate detailed and accurate reasoning before they record their answers on the worksheets. For example, if students are struggling to find ways to improve the work, suggest they try the methods out:

Use Bill's rule to calculate the length of the shed.

Use Hina's result for the radius of the pond on your drawing.

What do you think the horizontal and vertical lines on Lisa's graph are about?

Use Lisa's method to find the width of the shed.

Do you agree with their answers?

If one student writes an answer to a question, ask the other student to explain that answer. If one student's reasoning is incomplete, ask the other to help articulate the reasoning more fully.

Below, we discuss math issues arising from each of the *Assistants' Methods*.

Bill has provided a 'rule' to use for drawing the features in the plan. His rule is a double number line, specifying the relationship between one set of numbers and another. The scale of Bill's rule is correct but he does not make it explicit.

To measure more accurately Bill could use millimeter-squared paper.

Bill uses his rule to accurately draw five squares, representing 5 m^2 . He has also drawn four right triangles, representing a further 2 m^2 . Bill has used this framework to sketch the outline of a pond that has a surface area slightly bigger than 7 m^2 . Bill has not found the radius on the plan for his radius of the garden pond in real-life.

I've created my own rule for this garden.

I will cut this rule out, stick it on card, and use when drawing all the garden features.

Pond $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$ square
 $= 2 \text{ cm} \times 2 \text{ cm}$ on plan

Hina uses guess and check to figure out the radius of the pond. This method does provide a correct answer. If an exact measurement were required, then an algebraic method could be more appropriate (and more efficient).

Hina leaves her ratios in different units of measure but does not record the units, giving the ratio $1 : 2$. This is incorrect: the ratio is $1 \text{ cm} : 0.5 \text{ m}$ or $1 : 50$. The mathematical convention is, also, to write the plan measure before the real-life length. In effect, Hina works with the ratio $1 \text{ m} : 2 \text{ cm}$.

Area of pond = $7 \text{ m}^2 = \pi r^2$.

$r = 3 \text{ m}$ Area = $\pi \times 3 \times 3$ - too big
 $r = 2 \text{ m}$ Area = $\pi \times 2 \times 2 = 12$ too big
 $r = 1 \text{ m}$ Area = $\pi = 3.1$ too small
 $r = 1.5 \text{ m}$ Area = $\pi \times 1.5 \times 1.5 = 7 \text{ m}^2$. Perfect!

Scale $10 \text{ m} = 20 \text{ cm}$
 $= 1 : 2$
 $= 1.5 : 3$
 Radius is 3 cm on plan.

Lisa uses an algebraic method to figure out the radius of the pond.

She rounds π down to 3 (and does not explain why he has done this) and

rounds the square root of $\frac{7}{3}$ down to

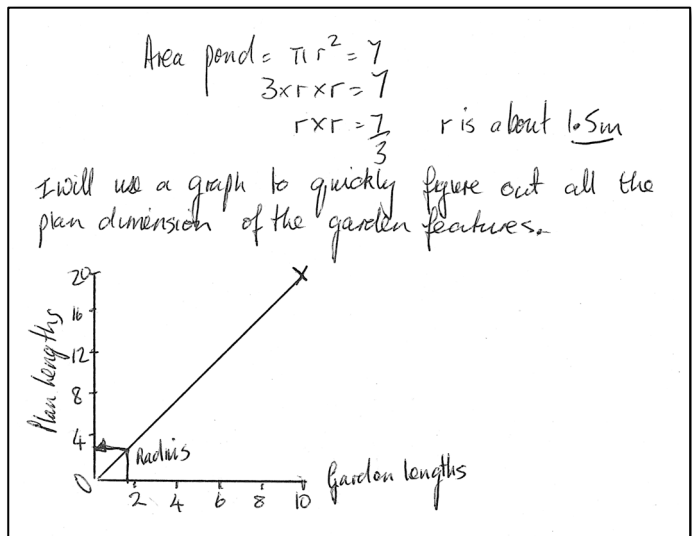
$r = 1.5$ m.

Calculating the surface area of the pond from these values:

$$A = \pi r^2 = 3 \times 1.5 \times 1.5 = 6.75m^2$$

Lisa has used a graph to figure out the radius of the pond on the plan. She can read from the graph the measurement of any garden length on the plan and likewise read any plan length from a given garden length. Lisa's graphical method gives her a way of calculating the measures in Mandy's first email. It would be good to state the units for the variables on each axis.

However, the graph is not a good method for finer degrees of accuracy. For this Lisa could use millimeter graph paper. Drawing a graph for this plan is not efficient for such a simple scale.



Whole-class discussion: comparing different approaches (20 minutes)

Hold a whole-class discussion to consider the different approaches used. Slides P-5 to P-7 may be helpful with this discussion; they show the assistants' methods.

Ask the students to compare and evaluate the methods:

Which do you think is the best method? Why? What are the advantages of that method?

Which of the assistants' methods did you find most difficult to understand?

What was difficult about that method?

Which method might you use yourself? Why?

If the scale was more complicated, for example 1 : 25, which method would you prefer?

Follow-up lesson: review work (10 minutes)

Remind students of their work on *Designing a Garden* and the different methods they have seen and used. Ask students to sit in the small groups in which they worked during the lesson. Return individual and group scripts and give students a few minutes to read their work.

Distribute the sheet *How Did You Work?* and ask students to complete this questionnaire.

The questionnaire should help students review their progress. Some teachers give this task for homework.

Optional follow-up lesson: extension work (30 minutes)

Hand out *Mandy's Second Email* sheet:

Mandy wants some costings for her garden. Can you estimate these for her?

Students could continue to work in the same groups or in different ones. If there is time, you could hold a whole-class discussion in which students share their solutions.

SOLUTIONS

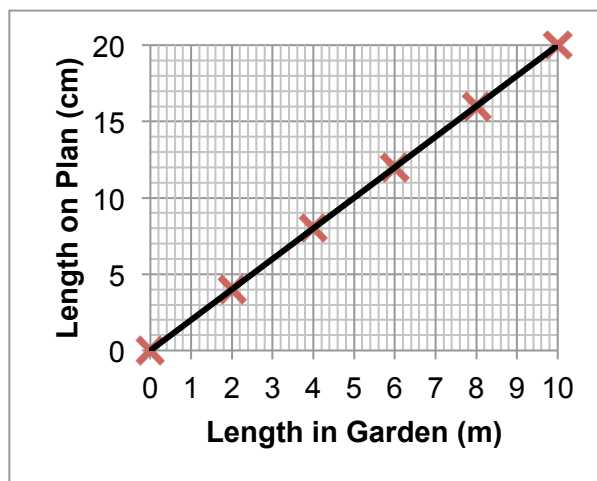
Assessment task: *Design a Garden*

The scale is calculated using the length of the garden on the plan, 20cm, and the given real-life measure of 10 m. 1 cm on the plan is equivalent to 0.5m in the garden.

The scale is 1 : 50 or 1cm : 0.5m.

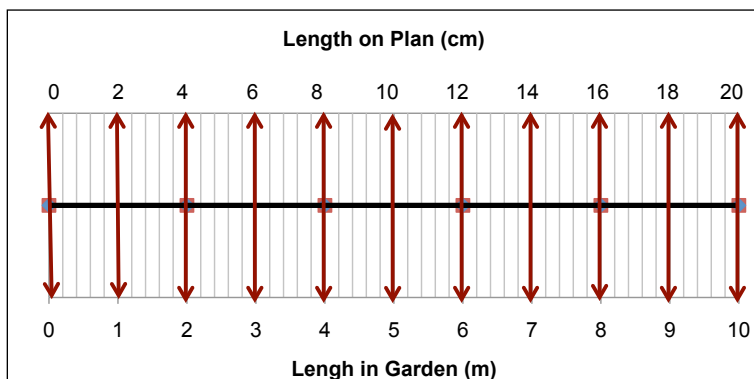
There are many ways students could draw the design. Below are some examples of how students could efficiently figure out measurements on the plan of actual measurements in the garden.

A graphical method



More generally if 1cm on the plan represents a m, 1 m on the ground represents $\frac{1}{a}$ cm on the plan.

A scale rule or double number line method



Numerical methods

The ratio is 1 : 50 or 1 cm : 0.5 m.

Given a length on the plan, the real-life length might be calculated by multiplying by 50 to find its length in centimeters, or multiplying by 0.5 to find its length in meters.

Given a real-life length in meters, the plan length in centimeters might be found by dividing the measure by 0.5 or multiplying by $\frac{1}{0.5} = 2$.

Given a real-life length in centimeters, the plan length might be found by dividing the measure by 50, or multiplying by $\frac{1}{50} = 0.02$.

Some students may attempt to use the method of setting up a proportion and cross-multiplying:

$$1 : 50 = x : 160$$

$$\text{So } \frac{1}{50} = \frac{x}{160}$$

$$\frac{1 \times 160}{50} = x$$

$$\frac{16}{5} = x = 3.2$$

This is a powerful method, but some students may have learned the procedure without linking it to which variables stand in which multiplicative relationships and which units are involved.

Garden features

Students may use their own method, or one of the above methods to figure out the dimensions on the plan of the garden features:

Feature	Dimensions on Plan
<p>Shed I've ordered this shed. It is 2 meters wide, 3.25 meters long and 2.8 meters tall.</p>	4 cm × 6.5 cm. Students do not need to calculate the height because that is not represented on the plan.
<p>Decking for barbeques I'd like this near the patio doors. It should be big enough to seat at least six people.</p>	<p>If the decking is rectangular then the dimensions would be at least 3 m × 3 m. This would be 6 cm × 6 cm on the plan.</p>
<p>Circular pond I would like a circular pond. I'd like the surface area to be about 7m².</p>	<p>$\pi r^2 = 7 \text{ m}^2$ so $r = 1.5 \text{ m}$. The plan radius should be about 3 cm. Counting squares to find a circle with area about 7 m² is an adequate method.</p>
<p>Path and Borders I would like some flower borders. I find borders wider than a meter difficult to garden. I'd like a 1 meter wide path to go from the shed to the house and from the garden gate to the house. <i>Extension:</i> Please tell me the approximate cost of the gravel path you have drawn. The gravel should be about 8cm deep. Gravel costs \$40 per cubic meter.</p>	<p>The path and borders should be shown 2 cm wide and in appropriate places.</p> <p><i>Extension:</i> Volume of path = path length in meters × 1m × 0.08m. Cost of path = volume of path × 40 (\$ per cubic meter) (= Path length (m) × 3.2 (\$ per m))</p>
<p>Grass Please tell me the approximate cost of the grass area. Grass costs \$3 per square meter.</p>	<p>Students' answers will vary. Some will try to estimate the remaining area directly by dividing the garden into pieces and estimating the area of each piece. Others may find the combined areas of the shed, patio, pond, path and borders and subtract this total from the area of the garden.</p>

Design a Garden



Imagine you are a garden designer.
You receive this email from a customer:

Dear Garden Designer,

I have moved into a house with a small garden that needs a total redesign. Please design my garden for me. I have attached an accurate scale drawing of my garden to this email. I've listed below some features I want in the garden. I will email you later about some other things I also want.

To start, please could you draw these features accurately on the plan, showing where you think they should go in the garden. Send me your plan with an explanation of your thinking.

Best wishes,

Mandy

Shed

I've ordered this shed. It is 2 meters wide, 3.25 meters long and 2.8 meters tall.



Decking for barbeques

I want some decking near the patio doors.
It should be big enough to seat at least six people.



Circular pond

I would like a circular pond.
I'd like its area to be about 7 m^2 .

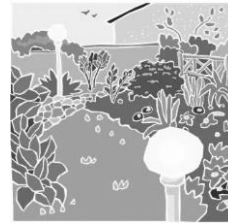


Path and Borders

I would like some flower borders. These should not be more than one meter wide as I find wider ones difficult to look after.

I'd like a gravel path 1 meter wide to go from the shed to the house and from the garden gate to the house.

I will cover the rest with grass.

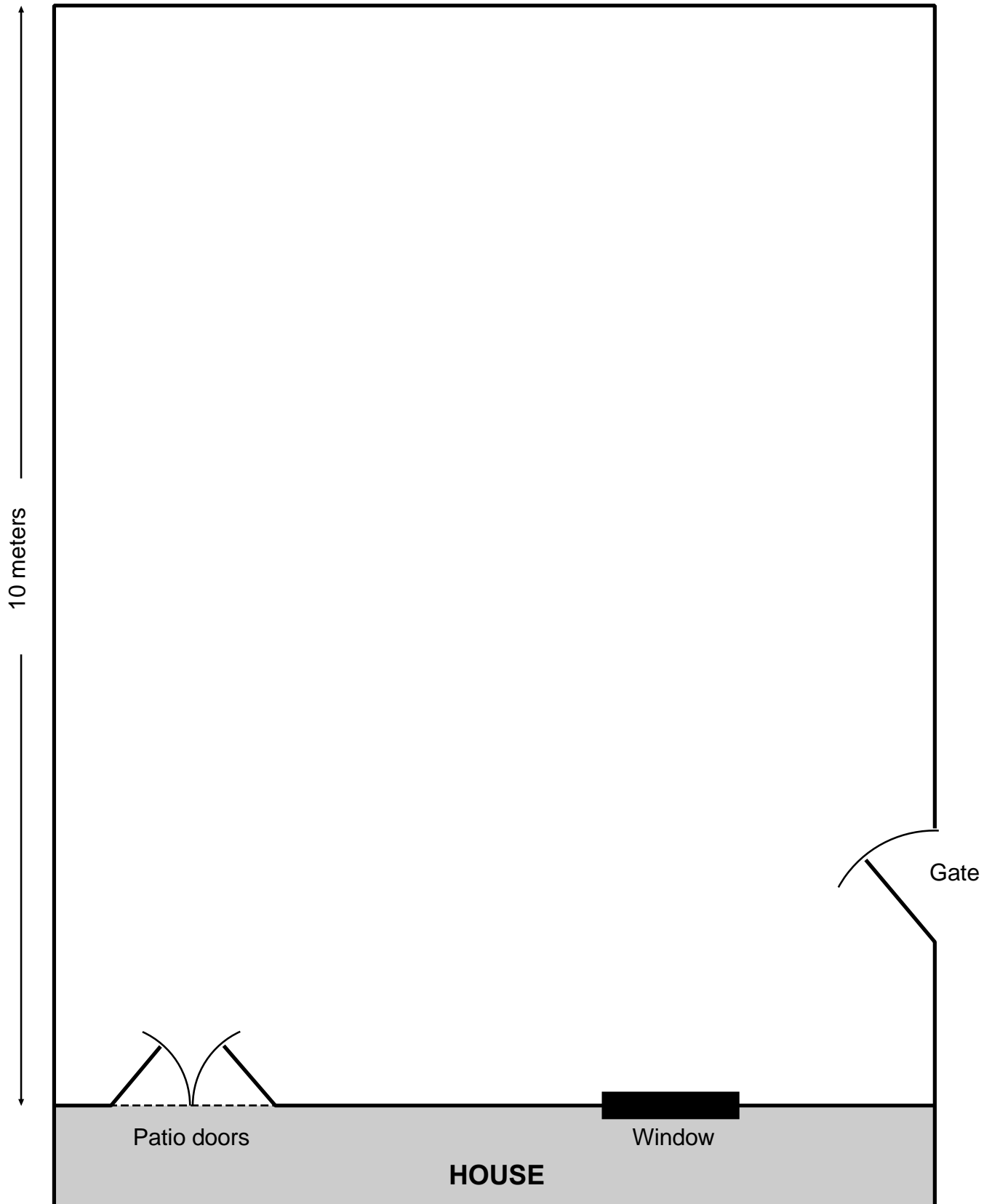


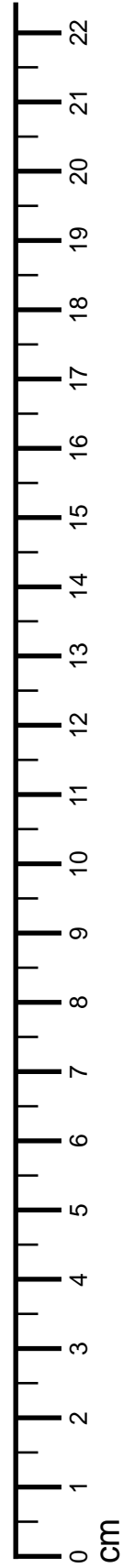
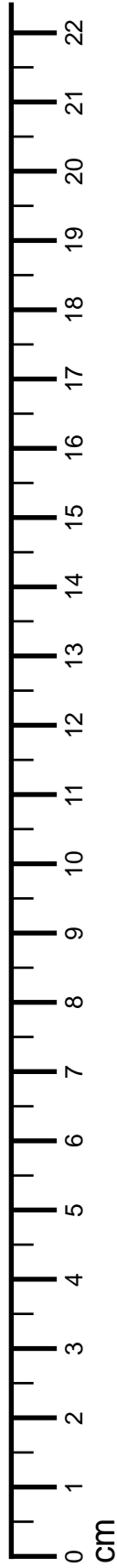
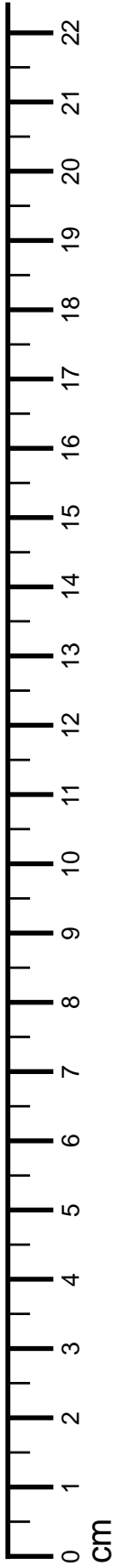
Use the sheet *Garden Plan* to draw the features from the email.

Record all your calculations and reasoning on a separate sheet.

Make sure to record the scale you use on the plan.

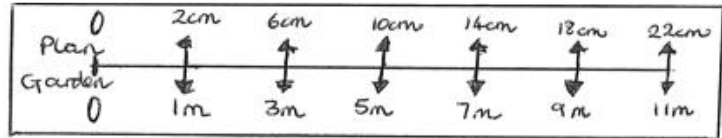
Garden Plan





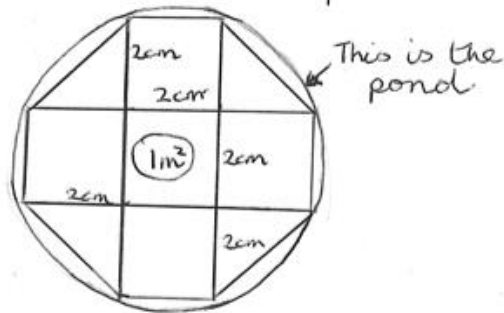
Assistant Bill's Method

I've created my own rule for this garden.



I will cut this rule out, stick it on card, and use when drawing all the garden features.

Pond $1m^2 = 1m \times 1m$ square
 $= 2cm \times 2cm$ on plan



What is Bill's method for calculating lengths on the plan?

Are there any problems with Bill's method? Explain how Bill could overcome these problems.

Explain Bill's method for drawing the pond on the plan.

Assistant Hina's Method

Area of pond = $7m^2 = \pi r^2$.

$r = 3m$ Area = $\pi \times 3 \times 3$ - too big

$r = 2m$ Area = $\pi \times 2 \times 2 = 12$ too big

$r = 1m$ Area = $\pi = 3.1$ too small

$r = 1.5m$ Area = $\pi \times 1.5 \times 1.5 = 7m^2$. Perfect!

Scale $10m = 20cm$

= 1:2

= 1.5:3

Radius is 3cm on plan.

What is Hina's method for calculating the radius of the pond on the plan?

How could Hina's method be improved?

Assistant Lisa's Method

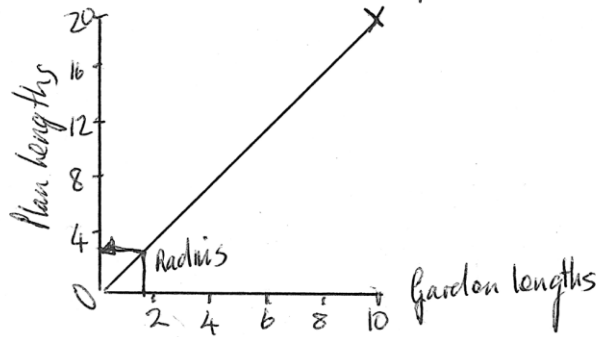
$$\text{Area pond} = \pi r^2 = 7$$

$$3 \times r \times r = 7$$

$$r \times r = \frac{7}{3}$$

r is about 1.5m

I will use a graph to quickly figure out all the plan dimension of the garden features.



What is Lisa's method to find the radius of the pond on the plan?

Are there any problems with Lisa's method? Explain how Lisa could overcome these problems.

Mandy's Second Email

Hello again,

I need to know some costs of the things on your plan.

Please can you work these out for me?

Best wishes,

Mandy

Gravel path

Please tell me the approximate cost of the gravel path you have drawn.

The gravel should be about 8 cm deep.

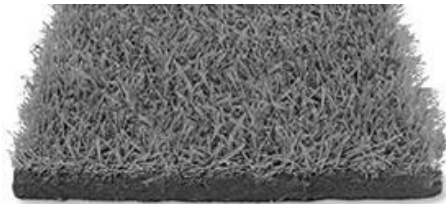
Gravel costs \$40 per cubic meter.



Grass

Please tell me the approximate cost of the grass area.

Grass costs \$3 per square meter.



How Did You Work?

1. Please tick one sentence about your individual work:

Our group's method **was better** than my own work.

Our group's method was **not better** than my own work.

Explain your response:

2. Underline the part of the sentence that applies to your group's work:

Our group method was similar to:

Bill's Method

Hina's Method

Lisa's Method

None of the assistants' methods.

If your work was similar to one of the assistant's methods, answer the question below:

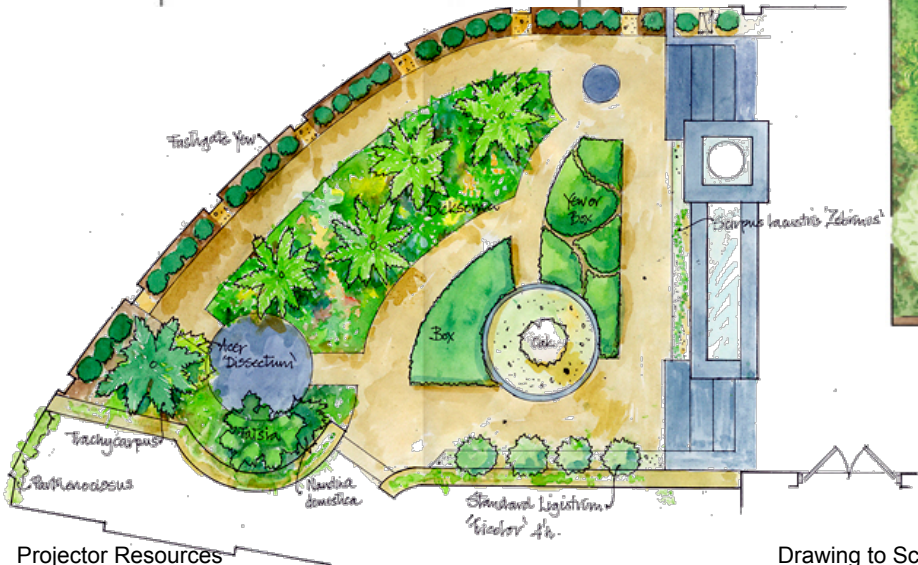
I prefer our work/the assistant's method because

If your work was similar to none of the assistants' methods, answer the question below:

Our work was different from every assistant's method because

3. What advice would you give to a student about to start this task?

Designing a Garden



Collaborating With Your Partner

- Take turns explaining your *Garden Plan* to your partner. Explain how you would improve your solution. Listen to each other carefully.
- Ask ‘clarifying questions’ that will help you understand your partner’s reasoning.
- When you have both taken a turn, decide how to design a new, better garden together.
- Draw your plan on the *Garden Plan* sheet and stick it in the middle of the poster paper.
- Use the space around the edge to write your reasoning, decisions, and calculations.

Poster Gallery

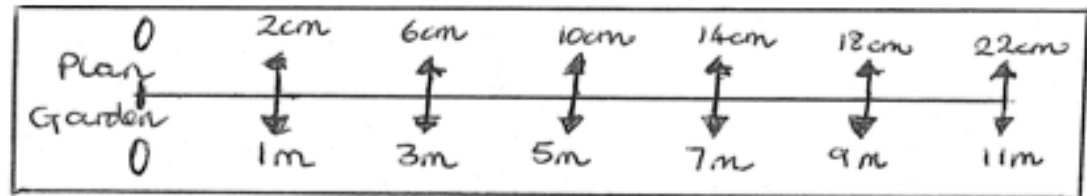
- One person from each group get up and visit another group's poster.
- If you are the visitor, read the poster. If there is math you do not understand, ask clarifying questions.
- If you are staying with your poster, explain the math to the visitor.
- If you find things you could do to improve your poster, write them on your sticky notes and attach to your poster.

Analysis of Assistants' Methods

- Choose one assistant's work and read it carefully.
- Answer the questions underneath.
- Try to understand what they have done and think about how the work could be improved.
- Take turns explaining your thinking to your partner.
- Listen carefully and ask clarifying questions.
- When your group has reached its conclusions, write your ideas below the assistant's work.
- Now check out another assistant's work in the same way.

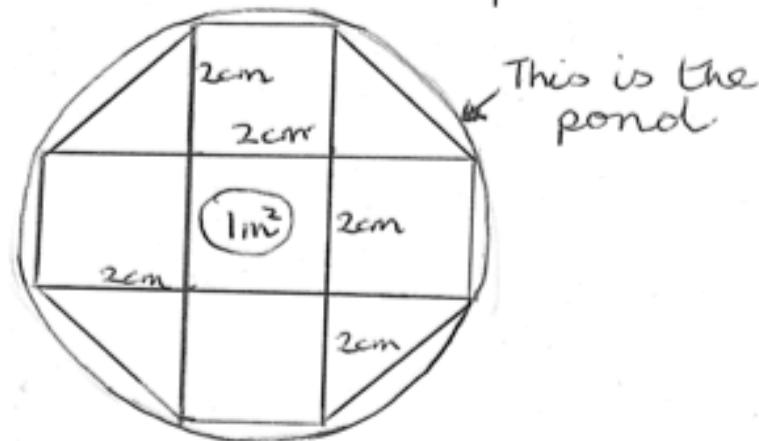
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I will cut this rule out, stick it on card, and use when drawing all the garden features.

Pond $1m^2 = 1m \times 1m$ square
 $= 2cm \times 2cm$ on plan



Assistant Hina's Method

$$\text{Area of pond} = 7\text{m}^2 = \pi r^2.$$

$$r = 3\text{m} \quad \text{Area} = \pi \times 3 \times 3 - \text{too big}$$

$$r = 2\text{m} \quad \text{Area} = \pi \times 2 \times 2 = 12 \text{ too big}$$

$$r = 1\text{m} \quad \text{Area} = \pi = 3.1 \text{ too small}$$

$$r = 1.5\text{m} \quad \text{Area} = \pi \times 1.5 \times 1.5 = 7\text{m}^2. \text{ Perfect!}$$

$$\text{Scale } 10\text{m} = 20\text{cm}$$

$$= 1:2$$

$$= 1.5:3$$

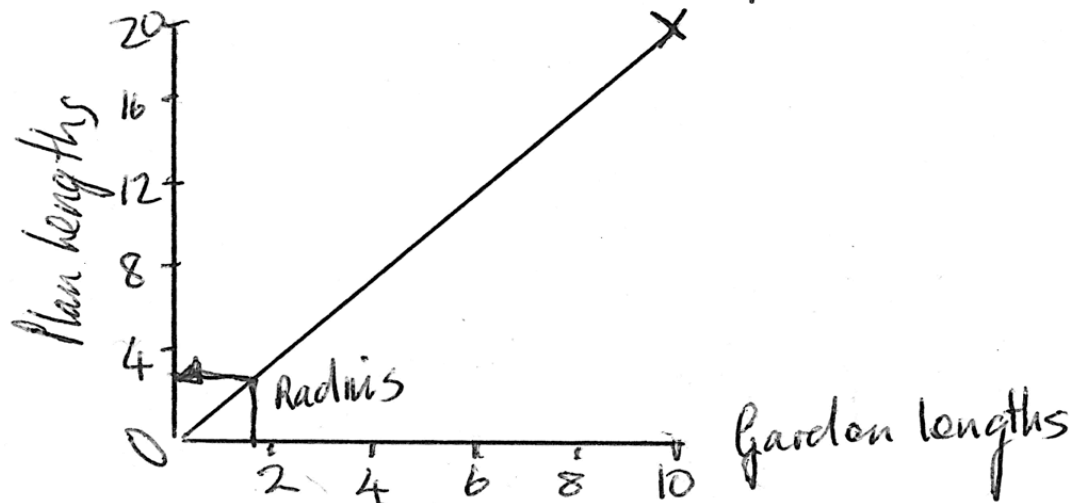
Radius is 3cm on plan.

Assistant Lisa's Method

$$\begin{aligned} \text{Area pond} &= \pi r^2 = 7 \\ 3 \times r \times r &= 7 \\ r \times r &= \frac{7}{3} \end{aligned}$$

r is about 1.5m

I will use a graph to quickly figure out all the plan dimension of the garden features.



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
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We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and Diane Schaefer

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and Malcolm Swan at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and Jamie McKee

The full collection of Mathematics Assessment Project materials is available from

<http://map.mathshell.org>