CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Using Positive and Negative Numbers in Context

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Using Positive and Negative Numbers in Context

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to understand and use directed numbers in context. It is intended to help identify and aid students who have difficulties in ordering, comparing, adding, and subtracting positive and negative integers. Particular attention is paid to the use of negative numbers on number lines to explore the structures:

Starting temperature + Change in temperature = Final temperature
Final temperature – Change in temperature = Starting temperature
Final temperature – Starting temperature = Change in temperature

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.NS: Apply and extend previous understandings of operations with fractions.
7.EE: Use properties of operations to generate equivalent expressions.
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 7, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson students work individually on an assessment task designed to reveal their current understanding. You review their responses and write questions to help them improve their work.
- After a whole-class introduction, students work together in small groups on a card-matching task.
- In the same small groups, they then create directed number calculations to correspond to temperature changes or city temperatures. To end the lesson there is a whole-class discussion.
- In a follow-up lesson, students again work alone on a task similar to the assessment task.

MATERIALS REQUIRED

- Each student will need a copy of the assessment task *Temperature Changes* and *Temperature Changes (revisited),* a copy of the *Temperature Scale*, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need a set of cut-up cards from the sheets *City Temperatures* and *Changes in Temperature*, a glue stick, and a large sheet of paper for making a poster.
- You will also need one copy of the cut-up cards *Introduction to Negative Numbers*.

TIME NEEDED

15 minutes before the lesson, a 100-minute lesson (or two 55-minute lessons), and 20 minutes in a follow-up lesson. Timings are approximate and will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Temperature Changes* (15 minutes)

Have students complete this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the *Temperature Changes* task.

> *Spend 15 minutes on your own, answering these questions.*
>
> *Explain all your answers carefully.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task that should help them to progress.

Temperature Changes

1.

The temperature was +3°C at midday.
By evening, the temperature was −5°C.

(a) Some of the calculations below show how to figure out the temperature change.

Circle any that apply.

3 − 5	3 + (−5)	(−5) − 3	(−5) + 3

(b) What was the change in temperature? Explain your answer.

....

....

2.

The temperature was −7°C at midnight.
By the next day, the temperature had risen by 11°C.

(a) Some of these calculations show how to figure out the temperature the next day.

Circle any that apply.

7 − 11	11 + (−7)	(−11) − (−7)	(−7) + 11

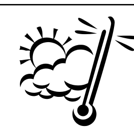
(b) What was the temperature the next day? Explain your answer.

....

....

3. Here is a calculation:

5 − (−11)



(a) Figure out the answer to the calculation:

....

....

(b) Which of these story questions fits the calculation? Circle any that apply.

A	It was really cold at midnight. During the morning, the temperature rose 5°C. By midday, it had reached -11°C. What was the temperature at midnight?
B	The temperature at midnight was −11°C. By midday, the temperature was 5°C. What was the temperature change?
C	At midday, the temperature was 5°C. The temperature then fell by 11°C. What was the final temperature?

Explain your answer:

....

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We recommend that you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students in the follow-up lesson.

Common issues:	Suggested questions and prompts:
Chooses an expression that does not represent the story (Q1, 2) For example: The student chooses the 1st, 2nd or 4th expressions to represent the temperature change (Q1). Or: The student chooses the 1st or 3rd expressions to represent the next day temperature (Q2).	• Write what you are asked to figure out in words. How can you write this calculation using math? • Mark the start and finish temperatures on your *Temperature Scale*. Does the temperature rise or fall? What is the difference between the two temperatures? Is the temperature change positive or negative? • What does the negative sign represent in −7? What does the middle negative sign represent in (−11) – (−7)?
Makes a calculation error For example: The student writes '(−5) – 3 = – 2' (Q1). Or: The student writes '(−5) + (−11) = (−6)' (Q3).	• Mark the beginning and end temperatures on your *Temperature Scale*. From the first figure, which direction do you move in? Does your solution make sense?
Chooses a story that is not represented by the expression (Q3) For example: The student connects the expression 5 – (−11) with Story A or Story C.	• How does the calculation represent what happens to the temperature in the story? How can you write that in words? • (Story A) What is the temperature at midday? Mark this on your *Temperature Scale*. By how much does the temperature change to reach that temperature? Write this story as a calculation. • (Story C) What was the temperature at midday? Mark this on your *Temperature Scale*. By how much does the temperature change? How would you write this story as a calculation?
Provides little or no explanation For example: The student writes correct numerical solutions but does not provide any explanation of how he/she generated them. Or: The student writes correct procedures but does not explain the links between context and operations.	• Imagine students in a lower Grade are going to read your solution. How can you explain very clearly, so that they understand? • How do you know that this calculation matches that story? Explain your answer clearly.
Answers all questions correctly	• Make up a calculation using positive and negative numbers. Now write a story to fit that calculation. Explain how the calculation and story fit together.

SUGGESTED LESSON OUTLINE

Whole-class interactive introduction (10 minutes)

Give each student a mini-whiteboard, a pen, and an eraser.

Explain to the class that in the lesson they will be working with positive and negative numbers in the context of temperatures, in degrees Celsius.

In degrees Celsius, at what temperature does water boil? [100°C.]

In degrees Celsius, at what temperature does water freeze? [0°C.]

In degrees Celsius, what is an average temperature for a summer's/winter's day?

Give each student one copy of the sheet *Temperature Scales*.

Ask four students to come to the front of the room and give each of them a card from the sheet *Card Set: Introduction to Negative Numbers*.

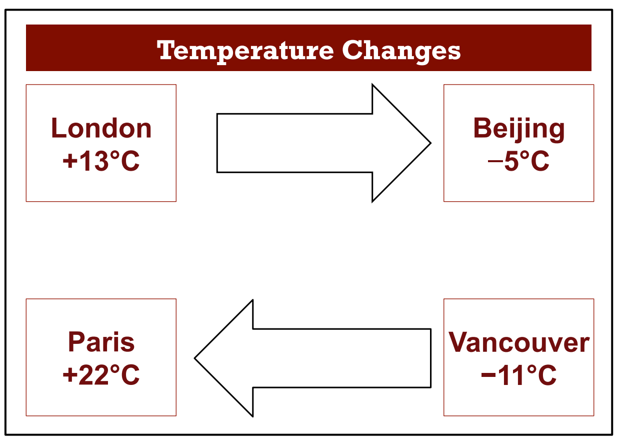
Ask the four students to stand in order of temperature from coldest to hottest.

Who thinks they are in the correct order? Who thinks they are not in the correct order?

If students disagree with the order, ask one or two to explain their thinking. Many students find it difficult to come to terms with the notion that –11°C is colder than –8° C.

Ask the students to return to their seats.

Now project Slide P-1 of the projector resource onto the board (or draw the diagrams on the board):



Ask students to write on their mini-whiteboards the answers to questions such as the following. Each time, ask students to explain their method:

If I fly from London to Beijing, does the temperature rise or fall?
What is the change in temperature? [–18°C.] How do you know?

If I fly from Vancouver to Paris, does the temperature rise or fall?
What is the change in temperature? [+33°C.]

The temperature change is +24°C. What was my journey? [Vancouver to London.]

The temperature change is –6°C. What was my journey? [Beijing to Vancouver.]

Cairo is 30°C warmer than Beijing. What is the temperature in Cairo? [+25°C.]

Write the temperature changes between the relevant cities on the board. Draw in new arrows as necessary. The figures should go in the arrows.

Collaborative work 1 (20 minutes)

Organize the class into groups of two or three students.

Give each group a large sheet of paper for making a poster, the *Card Set: City Temperatures*, and *Card Set: Changes in Temperature*.

> *What do you think the two sets of cards represent?*

Explain to students how they should work together:

> *Take turns to place cards.*
>
> *When it is your turn, connect two City Temperatures cards using an arrow from the Changes in Temperature cards. Figure out any missing temperatures.*
>
> *Write the calculations and give reasons explaining your calculations.*
>
> *If others in your group disagree, they should explain why. Then figure out the answer together.*
>
> *When you have reached an agreement, write your temperature in the box on the card.*

To remind students about these instructions, you could use Slide P-2, *Instructions for Working Together*. Tell students to ignore the dashed lines on the cards for now; these will be used later.

While students are working, you have two tasks: to find out about students' work and to support their reasoning.

Find out about students' work

As you move around the room, listen to students' explanations. Are they able to interpret the operations in terms of this context? Notice the vocabulary that they use to describe operations on numbers in the context. Do they find it more difficult to find a missing temperature, or the change in temperature? Do they make any errors, either in the choice of calculation, or in their arithmetic?

Note any difficulties that emerge for more than one group; these can be discussed later in the lesson.

Support student reasoning

If you hear students providing incorrect explanations, try not to make suggestions that move them towards a particular approach to the task or correct their errors directly. Instead, ask questions that help students to identify their own errors and re-direct their thinking.

> *What does the number on the arrow card stand for? [The change in temperature.]*
>
> *Jordan wrote this temperature on this card. Della, do you think Jordan is correct? Why?*
>
> *Is it hotter in [Washington or Las Vegas]? Does this mean that the change in temperature when traveling from Washington to Las Vegas will be positive or negative?*
>
> *What does a minus sign on the Changes in Temperature card mean? [The temperature dropped.]*
>
> *How can you check you've calculated the correct temperature for this city? [Use a different route to get to the city.]*

Prompt students to focus on the complex and multiple relationships between how the equations and expressions are written and the English language sentences or 'stories'. Each equation represents a relationship between three numbers. Students may mistakenly think that there is just one way of saying each equation in English, or that there is just one equation to match any particular sentence.

> *Are there other ways you could write the math for that story? Which way do you like best? Why?*
> *What is the story for that equation? Are there other ways you could express that?*

You may want to also use the questions in the *Common issues* table to support your questioning.

If students are struggling, you might refer one group of students to another group that has dealt with an issue well. If several groups of students are finding the same issue difficult, you might write a suitable question on the board and organize a brief whole-class discussion focusing on that aspect of mathematics.

When students are satisfied with their cards, give each small group a glue stick and ask them to glue their cards onto the poster paper.

Sharing posters (10 minutes)

As students finish connecting the cards, ask them to share their solutions with a neighboring group.

Check to see if the figures on any of the cards are different from your own.

Someone in each group needs to explain the reasoning behind these figures. If anything is unclear, ask for clarification.

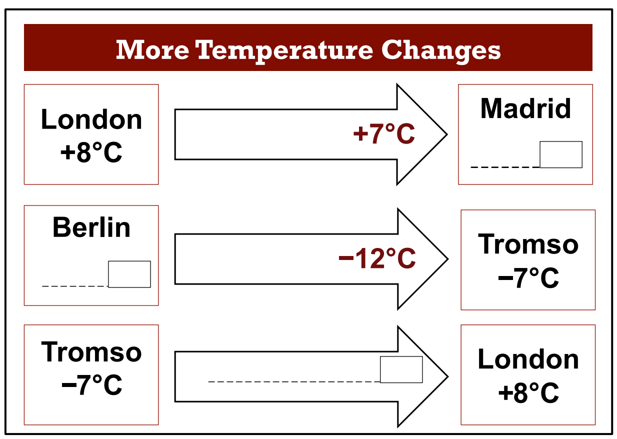
Then, all working together, consider if you should change any of your answers.

It is important that everyone in both groups understands the math. You are responsible for each other's learning.

Whole-class discussion (15 minutes)

This activity allows students to write the different temperatures as an arithmetic sum.

Project or draw Slide P-3 of the projector resource onto the board:



Ask each of the following questions in turn. If you think your students will struggle to write the arithmetic, you could first ask them to describe in words how they calculated each answer and emphasize the structure.

What is the temperature in Madrid? [15°C.] Write this in the box below 'Madrid'.
What calculation gives this answer? [(+8) + (+7).] Write this on the dotted line.

What is the temperature in Berlin? [5°C.] Write this in the box below 'Berlin'.
What calculation gives this answer?

Students may use different strategies to figure out the answer:

- Some may find the temperature in Berlin by flying from Tromso to Berlin: (–7) + 12 = + 5.
- Others may use *final temperature – change = initial temperature*: (–7) – (–12) = +5. Write both answers along the dashed line below 'Berlin'.
- Others may try to do the equivalent of solving $x - 12 = -7$ algebraically.

What is the temperature change when you fly from Tromso to London? [15°C.] Write this in the box in the arrow.
What calculation gives this answer? [(+8) – (–7) = +15.] Write this on the dotted line.

Write the calculation along the dashed line in the third arrow.

You may also want to ask:

Suppose you don't know the final temperature. For example, you didn't know the temperature in Madrid. What is a general method used to figure out a missing final temperature?
[Starting temperature + Change in temperature = Final temperature.]

Suppose you don't know the starting temperature. For example, you didn't know the temperature in Berlin. What is a general method used to figure it out?
[Final temperature – Change in temperature = Starting temperature.]

Suppose you don't know the temperature change, for example, when flying from Tromso to London. What is a general method used to figure it out?
[Final temperature – Starting temperature = Change in temperature.]

Extending the lesson over two days

If you are taking two days to complete the unit you might want to end the first lesson here. Make sure that students have glued the cards they have agreed upon onto their posters before collecting the posters in. Then, at the start of the second day, return the posters to students and give them a chance to familiarize themselves with their work, before they work together to write a calculation for each of their completed temperatures.

Collaborative work 2 (25 minutes)

Now ask students to write a calculation for each of the temperatures they have already written in the boxes on either the *City Temperatures* cards or the *Changes in Temperature* cards. These calculations should be written on the dashed lines.

Support the students as you did in the previous collaborative activity. As students finish the task, ask them to share their solutions with a neighboring group, as they did before.

Whole-class discussion (20 minutes)

Organize a discussion about what has been learned. Try to focus the discussion on any common errors and misconceptions you noticed in the collaborative work. You may want to draw on the questions in the *Common issues* table to support your own questioning. The intention is that students develop their understanding by talking about and correcting each other's errors. You should try to resist simply explaining the mistakes students have made yourself.

Ask each pair of students to report back on one of their calculations. Concentrate on the arithmetic students found difficult.

Matt, how did you calculate this temperature/temperature change?

Mary, do you have a different method for obtaining the same temperature/temperature change?

You may also want to investigate combining changes:

If I fly from Detroit to Anchorage, does the temperature rise or fall? By how much? [–20°C.]
Write a calculation for this answer. Can you now write a different calculation?

How can you calculate the fall in temperature? [15 – 35 = –20°C.]

Make up a journey you've not already looked at. Tell me about the temperatures. Tell me the temperature change.

Ask students to try to generalize rules for adding or subtracting positive and negative quantities from these or similar examples.

What is the effect of adding/subtracting a positive/negative number? Give me an example.

Follow-up lesson: reviewing the assessment task (20 minutes)

Give each student a copy of the review task *Temperature Changes (revisited)* and their original scripts from the assessment task, *Temperature Changes*. If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Read through your script from Temperature Changes and the questions [on the board/written on your script]. Answer these questions and revise your response.

Now look at the new task sheet, Temperature Changes (revisited). Can you use what you've learned to answer these questions?

SOLUTIONS

Assessment task: *Temperature Changes*

The assessment is designed to check whether students can make sense of simple arithmetic operations on negative numbers in context. In addition to using operations on negative integers, students need to be able to deal with ambiguities in the English language of mathematical operations.

1. (a) The calculations $3 - 5$, $3 + (-5)$, and $(-5) + 3$ all give the answer -2. Only the third calculation, $(-5) - 3 = -8$, fits the story, indicating by how much the temperature has changed. This uses the structure: final temperature – initial temperature = change in temperature.

   (b) When reading a student's chosen method, notice whether they just write a number, use a jotted strategy such as writing how they counted down, or use a written method.

2. (a) $7 - 11 = -4$, $(-11) - (-7) = -4$ are incorrect. The calculations $11 + (-7) = 4$ and $(-7) + 11 = 4$ show how to find the final temperature by adding the starting temperature to the temperature change.

   (b) Again, notice whether the student just writes a number, uses a jotted strategy such as writing how to count up from -7, or uses a written method.

3. (a) $5 - (-11) = +16$. Noticing the method of calculation is as important as checking the correct numerical solution.

   (b) *Story B* fits the calculation: final temperature – initial temperature = change in temperature.

   Story A gives the change in temperature and the final temperature. To calculate the initial temperature you could use $(-11) - 5 = -16$ or an equivalent calculation.

   Story C gives the initial temperature and the change in temperature. To find the final temperature, you would need to calculate $5 + (-11)$.

Assessment task: *Temperature Changes (revisited)*

1 (a) The second and third calculations could both be used to calculate how much the temperature has changed. The second calculation suggests that the student is using the idea of subtracting the initial temperature from the final temperature. The third calculation might suggest that the student is using a number line, moving to zero then finding the size of the negative component.

(b) When reading a student's chosen method, notice whether they just write a number, use a jotted strategy such as writing how they count down, or use a written method.

2. (a) The second and fourth calculations, $9 + (-2) = 7$ and $(-2) + 9 = 7$, are both ways of figuring out the final temperature.

   (b) Again, notice whether the student just writes a number, uses a jotted strategy, or uses a written method.

3. (a) $9 - (-4) = 13$. Noticing the method of calculation is as important as checking the correct numerical solution.

   (b) *Story B* fits the calculation: by subtracting the initial temperature from the final temperature, you figure out the change in temperature: $9 - (-4) = 13$.

   Story A gives the change in temperature and the final temperature. To calculate the start temperature you would add the negative change in temperature to the initial temperature:
   $9 + (-4)$ or $9 - 4$.

   Story C gives the final temperature and change in temperature. To find the initial temperature, you subtract the change from the final temperature. That calculation would be $(-4) - 9 = -13$.

Lesson Task

Students may use different routes to figure out the temperatures.

Detroit **(−5) + (+15) = +10°C**	New York to Detroit +15°C	New York −5°C
Detroit to Honolulu **(+25) − (+10) =** **+15°C**	**Honolulu to New York** (−5) − (+25) = −30°C	Anchorage to New York **(−5) − (−10) = +5°C**
Honolulu +25°C	Honolulu to Anchorage −35°C	Anchorage **(+25) +(−35) = −10°C**
Fairbanks to Honolulu +50°C	**Fairbanks to Anchorage** (−10) − (−25) = +15°C	Anchorage to Washington +30°C
Fairbanks **(+20) + (−45) =−25°C**		Washington **(−10) + (+30) = +20°C**
Las Vegas to Fairbanks −45°C	Washingto to Las Vegas **(+20) − (+20) = 0**	Washington to New Orleans −10°C
Las Vegas +20°C	New Orleans to Las Vegas **(+20) − (+10) = +10°C**	New Orleans +10°C

Temperature Changes

1.

The temperature was +3°C at midday.

By evening, the temperature was −5°C.

(a) Some of the calculations below show how to figure out the temperature change.

Circle any that apply.

3 − 5	3 + (−5)	(−5) − 3	(−5) + 3

(b) What was the change in temperature? Explain your answer.

2.

The temperature was −7°C at midnight.

By the next day, the temperature had risen by 11°C.

(a) Some of these calculations show how to figure out the temperature the next day.

Circle any that apply.

7 − 11	11 + (−7)	(−11) − (−7)	(−7) + 11

(b) What was the temperature the next day? Explain your answer.

3. Here is a calculation:

$$5 - (-11)$$

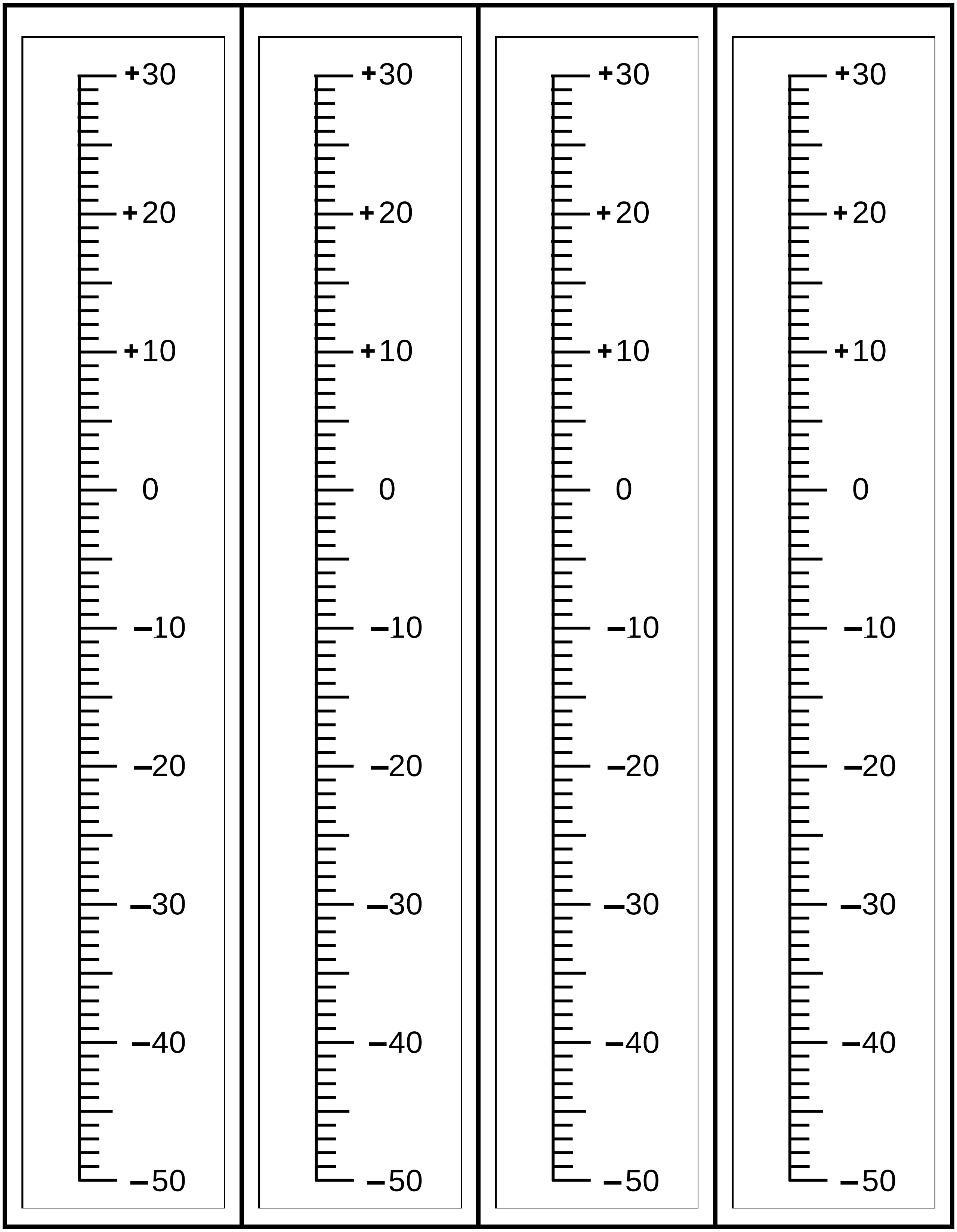
(a) Figure out the answer to the calculation:

(b) Which of these story questions fits the calculation? Circle any that apply.

A	It was extremely cold at midnight. During the morning, the temperature rose by 5°C. By midday, it had reached -11°C. What was the temperature at midnight?
B	The temperature at midnight was −11°C. By midday, the temperature was 5°C. What was the temperature change?
C	At midday, the temperature was 5°C. The temperature then fell by 11°C. What was the final temperature?

Explain your answer:

Temperature Scales



Card Set: Introduction to Negative Numbers

London **+13°C**	**Paris** **+20°C**
Vancouver **–11°C**	**Beijing** **–5°C**

Card Set: City Temperatures

Anchorage [ ] _ _ _ _ _ _ _ _	**New York** **–5°C**
Washington [ ] _ _ _ _ _ _ _ _	**Fairbanks** [ ] _ _ _ _ _ _ _ _
New Orleans **+10°C**	**Honolulu** **+25°C**
Las Vegas **+20°C**	**Detroit** [ ] _ _ _ _ _ _ _ _

Card Set: Changes in Temperature

Washington to Las Vegas _ _ _ _ _ _ _ _ _ _ ☐	Washington to New Orleans **–10°C**
New Orleans to Las Vegas _ _ _ _ _ _ _ _ _ _ ☐	Anchorage to New York _ _ _ _ _ _ _ _ _ _ ☐
Honolulu to Anchorage **–35°C**	New York to Detroit **+15°C**
Detroit to Honolulu _ _ _ _ _ _ _ _ _ _ ☐	Las Vegas to Fairbanks **–45°C**
Anchorage to Washington **+30°C**	Fairbanks to Honolulu **+50°C**

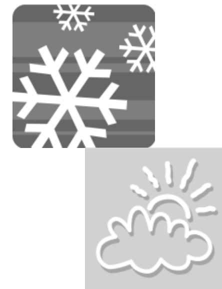
Card Set: Changes in Temperature (continued)

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ (−5) − (+25) = **−30°C**	_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ (−10) − (−25) = +**15°C**

Temperature Changes (revisited)

1.

> **The temperature was +5°C at midday.**
>
> **By evening, the temperature was −3°C.**



(a) Some of these calculations show how to figure out the temperature change.

Circle any that apply.

$5 - 3$	$(-3) - (+5)$	$(-5) - 3$	$(-5) + 3$

(b) What was the change in temperature? Explain your answer.

....

....

2.

> **The temperature was −2°C at midnight.**
>
> **By the next day, the temperature had risen by 9°C.**

(a) Some of these calculations show how to figure out the temperature the next day.

Circle any that apply.

$(-2) - 9$	$9 + (-2)$	$(-9) - (-2)$	$(-2) + 9$

(b) What was the temperature the next day? Explain your answer.

....

....

3. Here is a calculation:

$$9 - (-4)$$

(a) Figure out the answer to the calculation:

(b) Which of these story questions fits the calculation?

Circle any that apply.

A	At midday, the temperature was 9°C. The temperature then fell by 4°C. What was the final temperature?
B	At midnight, the temperature was −4°C. By next morning, the temperature was 9°C. What was the change in temperature?
C	It was extremely cold at midnight. During the morning, the temperature rose by 9°C. By midday, the temperature was −4°C. What was the temperature at midnight?

Explain your answer:

Temperature Changes

London
+13°C

Beijing
−5°C

Paris
+22°C

Vancouver
−11°C

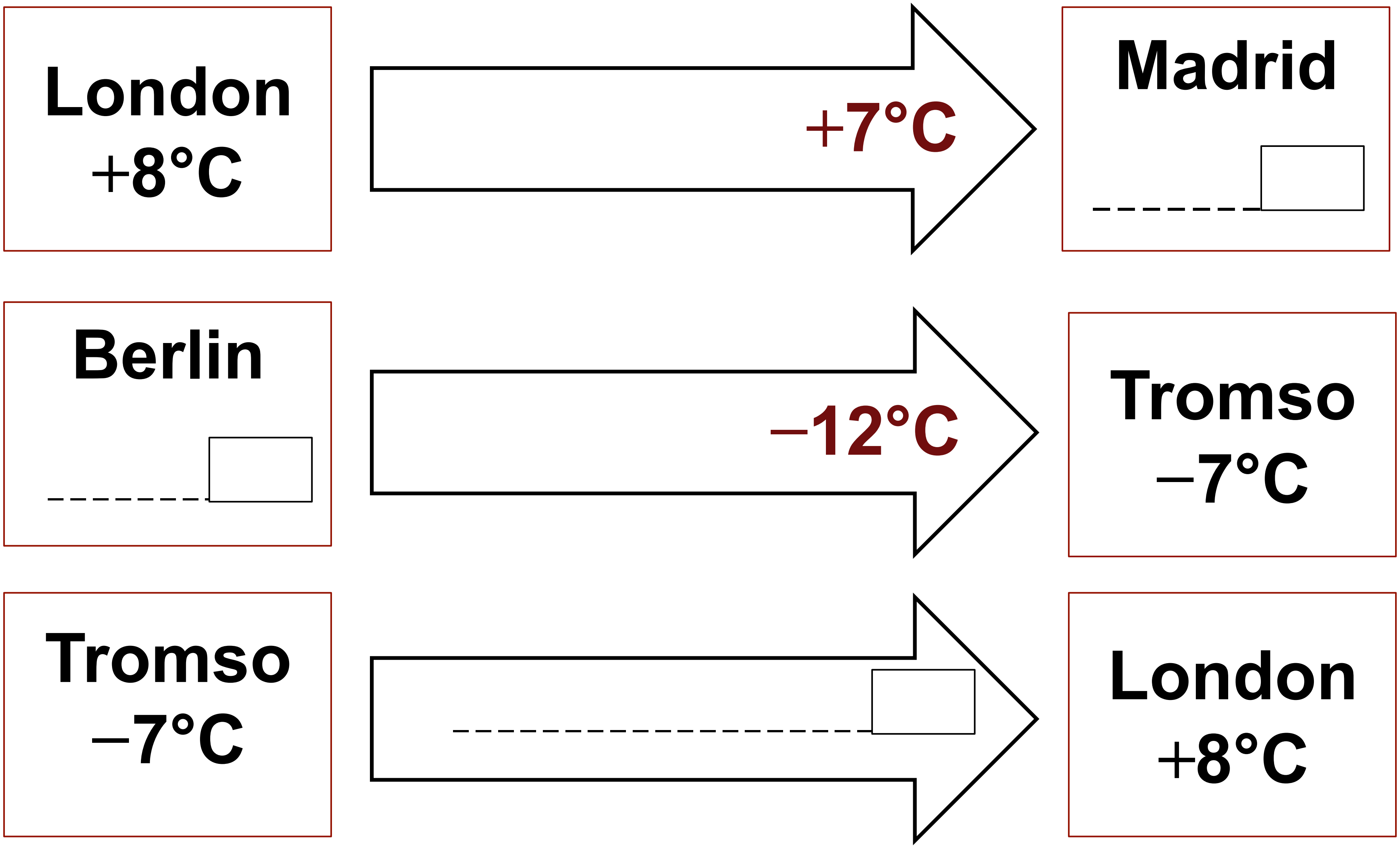
Instructions for Working Together

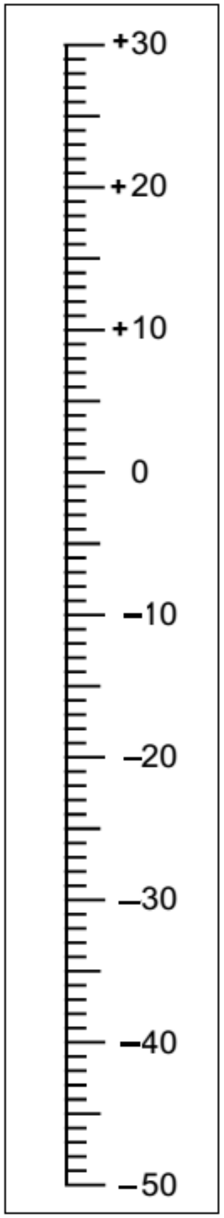
1. Take turns to place cards.

2. When it is your turn, connect two *City Temperature* cards using a *Changes in Temperature* arrow *card*.
   - Figure out any missing temperatures.
   - Explain your calculations.

3. If others in your group disagree, they should explain why. Then figure out the answer together.
4. When you have reached an agreement, write your solution in the box on the card.

More Temperature Changes




+30
+20
+10
0
–10
–20
–30
–40
–50

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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University of Nottingham, England:

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Nichola Clarke, Clare Dawson, Sheila Evans, Colin Foster, and **Marie Joubert**
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and **Daniel Pead**

We are grateful to the many teachers and students, in the UK and the US,
who took part in the classroom trials that played a critical role in developing these materials

The classroom observation teams in the US were led by
David Foster, Mary Bouck, and **Diane Schaefer**

This project was conceived and directed for
The Mathematics Assessment Resource Service (MARS) by
Alan Schoenfeld at the University of California, Berkeley, and
Hugh Burkhardt, Daniel Pead, and **Malcolm Swan** at the University of Nottingham

Thanks also to Mat Crosier, Anne Floyde, Michael Galan, Judith Mills, Nick Orchard, and Alvaro Villanueva who contributed to the design and production of these materials

This development would not have been possible without the support of
Bill & Melinda Gates Foundation

We are particularly grateful to
Carina Wong, Melissa Chabran, and **Jamie McKee**

The full collection of Mathematics Assessment Project materials is available from
http://map.mathshell.org