PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Modeling Relationships: *Car Skid Marks*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Modeling Relationships: *Car Skid Marks*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to use an approximate mathematical model of a real-world situation and use data to test, critique, and compare models. In particular this unit aims to identify and help students who have difficulty using variables to represent quantities and analyze the relationship between these variables using tables, graphs, and equations.

COMMON CORE STATE STANDARDS

This lesson relates to **all** the *Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 1, 2, 3, and 4:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6.EE: Represent and analyze quantitative relationships between dependent and independent variables.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the *Car Skid Marks* task individually. You then assess their responses and formulate questions that will prompt them to review their work.
- At the start of the lesson, students think individually about their responses to the questions set. They then work in small groups to combine their thinking to produce a collaborative solution to the *Car Skid Marks* task, in the form of a poster.
- Working in the same small groups, students evaluate and comment on sample responses, identifying strengths and weaknesses in each approach and comparing them with their own work. In a whole-class discussion, students compare and evaluate the methods they have seen and used.
- In a follow-up lesson, students reflect on their work and what they have learned.

MATERIALS REQUIRED

- Each student will need a copy of the *Car Skid Marks* task, the *How Did You Work?* questionnaire, a mini-whiteboard and eraser, and some plain or squared paper to work on. Calculators and graph paper should be available on request.
- Each small group of students will need a large sheet of paper and enlarged copies of the *Sample Responses to Discuss*.

TIME NEEDED

20 minutes before the lesson, a 100-minute lesson (or two 55-minute lessons), and 15 minutes in a follow-up lesson (or for homework). Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Car Skid Marks* (20 minutes)

Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the assessment task *Car Skid Marks* and introduce it. Check that students understand the context:

Has anyone experienced braking suddenly in a car? Did the car skid?

What happens to a car when it skids? [When a driver brakes hard, the wheels lock, but the car continues to slide.]

Explain that there are often skid marks at the scene of a traffic accident, even if the road conditions are good.

Do you think the speed of a car when the driver brakes affects the length of the skid mark? How?

Accident investigators use the lengths of skid marks to estimate the speed of the car before it started to brake.

Explain that on a dry test track, a car is driven at different speeds, in miles per hour (mph). Each time it brakes as hard as possible. The numbers in the table represents the relationship between the speed of the car and the length of the skid marks.

Car Skid Marks

Dek and Mani are traffic accident investigators.
Their job is to find out how and why accidents happen.
To do this they gather evidence from the scenes of accidents.

When a car suddenly brakes to a stop, it can leave skid marks.
These marks can be used to figure out the speed of the car.
This might give evidence that the driver was going over the speed limit.

On a dry test track, a car is driven at different speeds, in miles per hour (mph).
Each time it brakes as hard as possible.
The skid length is then measured in feet.
Here are the results:

Speed (mph)	0	19	27	29	37	39	49	54	56	60	66	69	74	76	80	85	89	93	98
Length (feet)	0	20	37	42	61	68	100	120	131	150	180	200	230	240	270	300	330	360	400

The relationship shown in the table looks complicated, so Dek and Mani both try to work out a 'rule of thumb' for estimating the speed of a car from the length of the skid marks:

Dek

I've got an easy rule.
Halve the length of the skid mark in feet.
This gives an estimate for the speed in miles per hour.

Mani

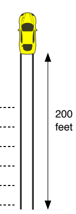
My rule is more complicated.
I use the formula:

$$y = \frac{x}{4} + 30$$

y is the speed of the car in miles per hour
x is the length of the skid mark in feet

Car Skid Marks (continued)

A car was travelling on a dry flat road with brakes in good condition.
The skid marks for the car measured 200 feet.



1. Which rule gives the best estimate for the speed of the car: Dek's or Mani's?
   Show all your work.

   ....

2. Dek and Mani argue about which rule is the best one to use for ***any*** traffic accident.
   What is your advice?
   Show your work and explain your reasoning.

Using the table, what would be a good estimate for the speed of a car when the skid length is 150 feet?

As the relationship looks a little complicated, the accident investigators Dek and Mani have come up with some simple rules of thumb for estimating this.

You may want to ask students if they know the meaning for 'rule of thumb'. The term is thought to originate from wood workers who used the width of their thumbs rather than rulers for measuring things. It is a quick and simple rule for calculating something.

It is important that, as far as possible, students work on the task without your assistance. If students are struggling to get started then ask questions that help them understand what is required, but make sure you do not do the task for them.

Remind students:

If you need resources like a calculator or some graph paper, then please ask.

The point is to make these resources available without giving the impression that they have to be used.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. Research suggests that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the beginning of the lesson.

Common issues	Suggested questions and prompts
Has difficulty getting started For example: They do not understand the relationship between speed and length of skid mark.	• What do you need to know? What do you know? • For the data in the table, how does the length of a skid mark change as the speed changes? • Use Dek's rule of thumb to figure out a car's speed for a skid length of 200 feet. How does this compare with the data in the table?
Has difficulty evaluating Mani's 'rule of thumb' For example: The student makes an error when substituting 200 into the formula (Q1).	• Describe the equation by replacing the variables y and x with words. • Check your answers. Do your answers make sense?
Evaluates rules based on an inappropriate or insufficient sample of data For example: The student only checks the slowest and fastest speeds.	• What assumptions have you made? Can you justify those assumptions? • Is the data in the table linear? How do you know? Does this make a difference to the number of comparisons you need to make? • How do you know you have compared enough values?
Assumes the relationship in the table is linear For example: The student only plots two points on a graph of the data in the table.	• Have you made any assumptions when plotting the data in the table? • What do you know about the relationship between the length of the skid marks and the speed of the car?
Checks all values for each rule against the data in the table	• This looks like a lot of work. Can you think of a quicker method?
Does not work systematically and/or their work is disorganised For example: The student randomly compares values.	• Would someone unfamiliar with the problem understand your solution? • Can you use a more organized method?
Obtains values for the rules but does not draw conclusions from these values	• What conclusions can you now make about the two rules?
Concludes that either Dek's or Mani's rule is always the best	• Is this rule **always** the best? When is the other rule better?
Does not consider a graphical approach	• Can you think of an alternative method that could help you see when one rule is closer than another?
Uses an inefficient method when plotting the two rules on a graph For example: The student plots several points for each 'rule of thumb'.	• What do you know about the two rules? How can you use what you know to think of a more efficient method to plot the two rules?

SUGGESTED LESSON OUTLINE

This lesson assumes students are familiar with using variables in an equation to represent quantities in a real-world problem.

Reviewing individual solutions to the problem (10 minutes)

Give each student a mini-whiteboard, a pen and an eraser and return their work on the *Car Skid Marks* task.

If you have not added questions to individual pieces of student work, either give each student a printed version of your list of questions with the questions that relate to their work highlighted, or write your list of questions on the board so that students can select questions that are appropriate to their own work.

Recall what we were working on previously. What was the task about?

I have had a look at your work and have some questions I would like you to think about.

On your own, carefully read through the questions I have written.
I would like you to use the questions to help you think about ways of improving your work.

Use your mini-whiteboards to make a note of anything you think will help to improve your work.

You will be sharing these notes with a partner later on.

If mini-whiteboards are not available, students may want to use the back of their assessment task to jot down their ideas about ways to improve their work.

This is an opportunity for students to review their own work before working collaboratively on producing a group solution.

While students are reviewing their work, it may be appropriate to ask individual students questions that help them to clarify their thinking. However, the purpose of the activity is not to address misconceptions; there will be opportunities for students to deal with these collaboratively later in the lesson.

Collaborative small-group work: making posters (35 minutes)

Organize the class into groups of two or three students.

Today you are going to work together on the Car Skid Marks task to produce a joint solution that is better than your individual work.

Before students have another go at the task, they need to discuss what they have learned from reviewing their individual solutions.

Show and explain to students Slide P-1 of the projector resource:

Sharing Individual Solutions

1. Take turns to share your work with your partner.
2. Share the notes you made on how you might improve your work.
3. Listen carefully to each other, asking questions if you don’t understand.
4. Notice similarities or differences between the methods described.

Explain to students that this activity will enable them to decide an approach to collaboratively pursue. Students may decide to use a different approach or improve one of their existing approaches.

Once students have had a chance to discuss their work, hand out to each group a sheet of poster paper. Display Slide P-2 of the projector resource and explain to students how to work collaboratively:

Joint Solution: Making Posters

1. In your group agree on the best method for completing the problem.
2. Produce a poster that shows a joint solution to the Car Skid Marks task that is better than your individual work.
3. State on your poster any assumptions you have made.
4. Give clear reasons for your choice of method.

While students are working in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches

In particular, note whether students' original methods are the same or different. If they are different, how do they decide which method to use for their joint solution? What are their reasons for the choice of method? Are students aware of any assumptions they have made? Do they justify these assumptions? How do they organize their work? Are they able to substitute figures correctly into the equation? Do they figure out the 'rule of thumb' speeds for all skid mark lengths or just some? If the latter, how do they select the lengths? Do students realize the two rules of thumb represent linear relationships between the skid length and speed, but that is not the case with the data in the table? Do they draw a graph of the situation? Are they concerned with the practical, real-life aspects of the problem?

Support student problem solving

If students are struggling to produce a joint solution to the task, try not to make suggestions that move them towards a particular approach. Instead, ask questions to help students clarify their thinking, encouraging them to identify the strengths and weaknesses of the methods employed in their individual solutions.

What have you done that you both [all] agree on? Why have you chosen this method?

What decisions have you already made? Why did you make those decisions?

What else do you need to find out?

Have you used all the information given in the task? Why/Why not?

What do you now know that you didn't know before?

How can you organize your work?

If several students in the class are struggling you might want to write a relevant question on the board or hold a brief whole-class discussion.

You may also want to use some of the questions in the *Common issues* table to support your own questioning.

Sharing posters (10 minutes)

Once students have finished their posters, ask them to share their work by visiting another group. This gives all students the opportunity to actively engage with another group's reasoning, as well as voicing their own thinking.

Show Slide P-3 and explain how students are to share their work and the purpose of the activity:

Sharing Posters

1. One person from each group get up and visit a different group.
2. If you are staying with your poster, explain your work to the visitor, giving reasons for your choice of method.
3. If you are the visitor, look carefully at the work, asking clarifying questions to help you to understand the method used.
4. Discuss whether or not the method described on the poster is similar to the visitor's method.
5. The visitor is to write on a post-it note suggestions on how the work could be improved.

Extending the lesson over two days

If you are taking two days to complete the unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of the problem before moving on to the collaborative analysis of sample responses.

Collaborative analysis of *Sample Responses to Discuss* (25 minutes)

Once students have had sufficient time to discuss their joint solutions, distribute to each group, copies of the *Sample Responses to Discuss*.

Display and explain to students Slide P-4 of the projector resource:

Sample Responses to Discuss

1. Read each piece of sample student work carefully.
2. Try to understand what they have done. You may want to add annotations to the work to make it easier to follow.
3. Think about how the work could be improved. Take turns explaining your thinking to your partner.
4. Listen carefully and ask clarifying questions.
5. When your group has reached its conclusions, write your answers to the questions underneath the work.

Explain to students that this activity gives them an opportunity to evaluate a variety of possible approaches to the task and to notice any differences and/or similarities with their own work.

The task encourages students to be flexible in their approach and recognize relationships among different approaches.

Ezra uses a table to figure out the difference between the 'real' speeds and the two 'rule of thumb' speeds for a number of skid lengths.

Ezra could improve the clarity of the table by including units of measure.

I will carefully select a range of skid lengths to test Dek and Mani's rules of thumb.

Length of skid	Correct speed	Dek's speed	Mani's speed	Best rule of thumb
0	0	0 (Error 0)	30 (30)	Dek
42	29	21 (Error 29-21 = 8)	40.5 (40.5-29=11.5)	
100	49	50 (Error 50-49 = 1)	55 (55-49 = 6)	
150	60	75 (75 - 60 = 15)	67.5 (67.5 - 60 = 7.5)	
200	69	100 (100 - 69 = 31)	80 (80 - 69 = 11)	
240	76	120 (120 - 76 = 44)	90 (90 - 76 = 14)	
300	85	150 (150 - 85 = 65)	105 (105 - 85 = 20)	
360	93	180 (180 - 93 = 87)	120 (120 - 93 = 27)	
400	98	200 (200 - 98 = 102)	130 (130 - 98 = 32)	

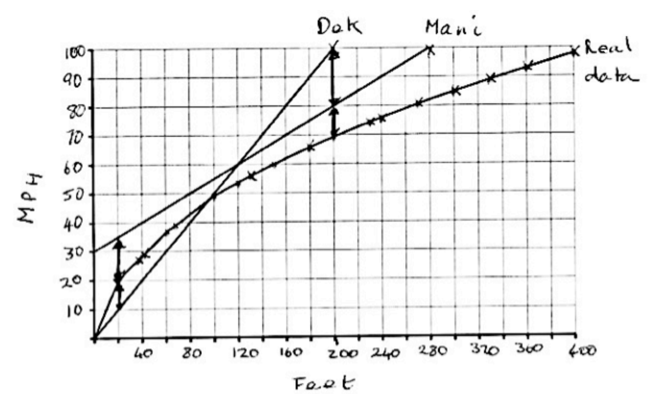
Ezra has carefully selected a wide range of skid lengths. It is clear that Dek's 'rule of thumb' is more accurate for the shorter skid lengths and Mani's is more accurate for the longer ones. However, because of the limited sample, it is difficult to appreciate in detail how the errors change as the speed changes.

Why do you think Ezra selected those particular lengths?

Do you agree with his selection? Why/Why not?

For different skid lengths, how does the size of the error change?

Leanne has drawn a graph of the situation.



The speed is the independent variable, as its magnitude causes the length of the skid. Leanne has, however, decided to plot speeds along the *y*-axis. This is reasonable; it is the skid lengths that are measured at the scene of an accident not the speeds.

Why has Leanne decided to plot the speeds along the y-axis?

What is the independent variable?

She has noted that the two rules of thumb are linear and so has only plotted two points for each straight line.

Why has Leanne plotted just two points for each rule of thumb, but several points for the data in the table?

Leanne has not explained why she has drawn the vertical lines on the graph.

Why do you think Leanne has drawn vertical lines on the graph?
What do they show?[For a given skid length, the lines measure the difference between the speed derived from the rule of thumb and the speed derived from the data in the table.]

Extension task

If students finish the task quickly, you may want to suggest they try to figure out a better ‘rule of thumb’ than Dek’s or Mani’s.

How is your rule of thumb better than Dek’s and Mani’s?

Have you tested your rule of thumb?

Is your rule of thumb better than Dek’s and Mani’s for a wide range of skid lengths?

Some students may decide to combine Dek and Mani’s ‘rule of thumb’, using Dek’s for the shorter skid lengths and Mani’s for the longer ones. This is fine, although it is making the ‘rule of thumb’ quite complicated!

Whole-class discussion: comparing different approaches (20 minutes)

Hold a whole-class discussion to consider the different approaches used within the sample responses. Look at each response in turn and ask students to comment on their strengths and weaknesses.

It may be helpful to display Slides P-5 and P-6 during this discussion.

What did Ezra/Leanne do?

Once you have discussed each piece of work, ask students to compare and evaluate the different methods.

Which piece of work did you find easiest/most difficult to understand? Why was that?
What are the advantages or disadvantages of each approach?

Which method do you think has the most potential? Please explain.

Which method is the most efficient? Please explain.

Do both responses use all the data? Please explain.

How is Ezra’s/Leanne’s work similar/different to what you did?

Did analyzing the responses enable anyone to see ways in which they could improve their own work? Please explain.

Is there a need to check speeds over the state speed limit? Please explain.

[Some students may argue that skid marks over a certain length show that the driver was definitely breaking the law, so there is no need to use a rule of thumb.]

Did anyone come up with their own rule of thumb?
Please describe it.
How did you test it?

Some students may decide that Dek’s ‘rule of thumb’ is fine as it is easy to use and ‘reasonably’ accurate to a speed of 60mph.

If students have created a complicated but accurate ‘rule of thumb’, you may want to discuss the tension between simplicity of method and accuracy of result.

Follow-up lesson (or possible homework): individual reflection (15 minutes)

Once students have had a chance to discuss the sample responses as a whole-class, distribute the questionnaire *How Did You Work?*

Ask students to spend time, individually, answering the questions.

Think carefully about your work in this lesson and the different methods you have seen and used.

On your own, answer the review questions as carefully as you can.

If you notice students writing generic answers such as ‘We explained it better’, encourage them to add more detail specific to their work or the sample responses. Some teachers give this as homework.

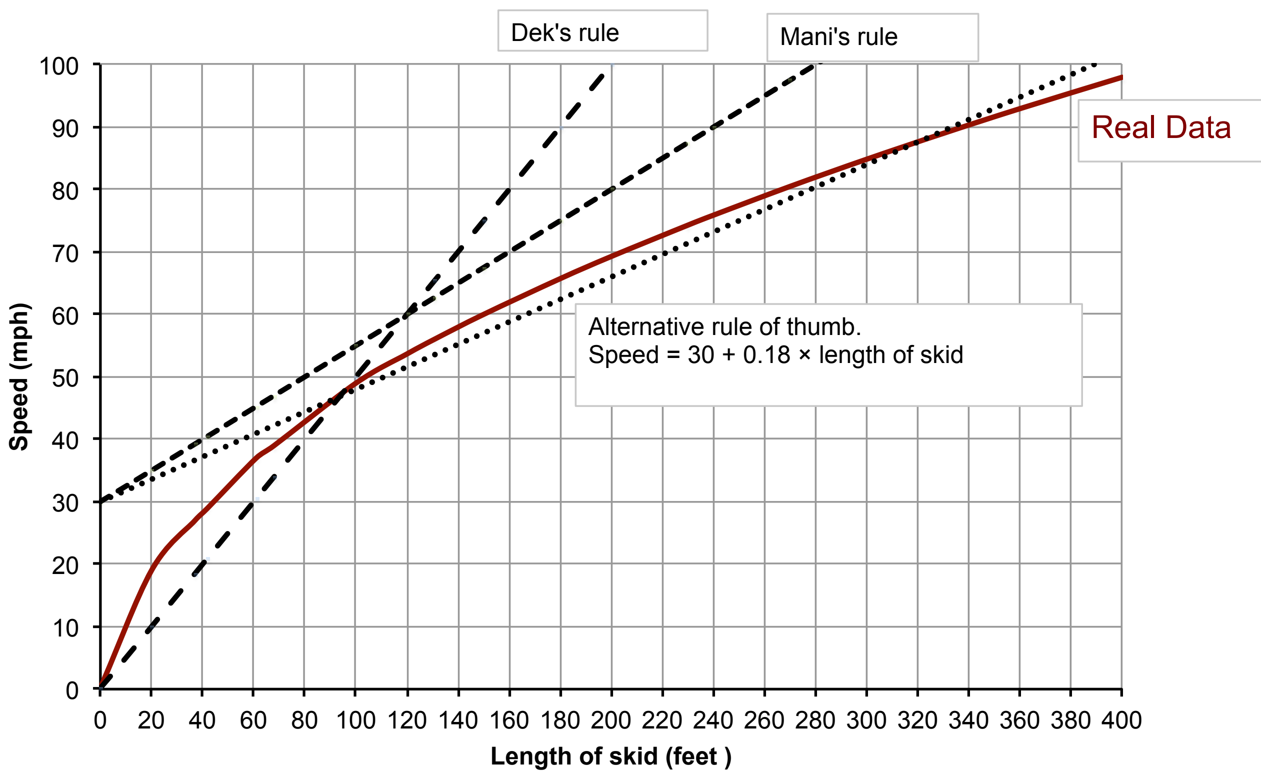
SOLUTIONS

1. Using the table, a skid mark of length 200 feet, implies a speed of 69 mph.
   Dek's 'rule of thumb' gives a speed of 100 mph
   Mani's 'rule of thumb' gives a speed of 80 mph.
   So Mani's rule gives the best estimate.

2. There are several ways students could test the 'rules of thumb'.

   Graphing the 'rules of thumb' and the data in the table is an efficient method.
   Students can see clearly the variation in accuracy of each 'rule of thumb'.

   For skid marks under 120 feet Dek's 'rule of thumb' is preferable, whereas Mani's is the more accurate for skid marks over 120 feet. As Mani's estimates are all too high, a simple subtraction of 10 or 15 would provide a better estimate.



The graph above also shows a possible alternative 'rule of thumb' that is quite accurate for speed between 35 mph and 100 mph, but it is quite complicated!

If students decide to use a table then they will need to test a large proportion of the data. This is a lot of work! Here is a table showing all the data:

Length of skid (feet)	**Actual Speed of car (mph)**	**Dek's Estimated Speed (mph)**	**Error**	**Mani's Estimated Speed (mph)**	**Error**
0	0	0	0	30	+30
20	19	10.0	-9	35.0	+16
37	27	18.5	-8.5	39.3	+12.3
42	29	21.0	-8	40.5	+11.5
61	37	30.5	-6.5	45.3	+8.3
68	39	34.0	-5	47.0	+8
100	49	50.0	+1	55.0	+6
120	54	60.0	+6	60.0	+6
131	56	65.5	+9.5	62.8	+6.8
150	60	75.0	+15	67.5	+7.5
180	66	90.0	+24	75.0	+9
200	69	100.0	+31	80.0	+11
230	74	115.0	+41	87.5	+13.5
240	76	120.0	+44	90.0	+14
270	80	135.0	+55	97.5	+17.5
300	85	150.0	+65	105.0	+20
330	89	165.0	+76	112.5	+23.5
360	93	180.0	+87	120.0	+27
400	98	200.0	+102	130.0	+32

The table shows clearly the size of the error for each skid length. It is clear from the table that Dek's 'rule of thumb' is better than Mani's for skid marks under 120 feet. Dek's 'rule of thumb' is preferable, whereas Mani's is the more accurate for skid marks over 120 feet. The sizes of the errors for both 'rules of thumb' increase as the speed increases.

Note to teacher: A more advanced model for this situation is:

$$\text{Speed} = \sqrt{k \times \text{length skid mark}}$$

k is a constant. It's value depends on factors such as the condition of the road and the conditions of the car brakes.

In this case $k = 24$. The formula represents an approximate line of best fit for a car driving on a dry road with good brakes. The formula depends on many factors, such as the road surface, the condition of tires, the weather conditions and so on. This will be beyond the capacity of most Grade 6 students, but we include this here for completeness.

Car Skid Marks

Dek and Mani are traffic accident investigators.
Their job is to find out how and why accidents happen.
To do this they gather evidence from the scenes of accidents.

When a car suddenly brakes to a stop, it can leave skid marks.
These marks can be used to figure out the speed of the car.
This might give evidence that the driver was going over the speed limit.

On a dry test track, a car is driven at different speeds, in miles per hour (mph).
Each time it brakes as hard as possible.
The skid length is then measured in feet.
Here are the results:

Speed (mph)	0	19	27	29	37	39	49	54	56	60	66	69	74	76	80	85	89	93	98
Length (feet)	0	20	37	42	61	68	100	120	131	150	180	200	230	240	270	300	330	360	400

The relationship shown in the table looks complicated, so Dek and Mani both try to work out a 'rule of thumb' for estimating the speed of a car from the length of the skid marks:

Dek

I've got an easy rule.

Halve the length of the skid mark in feet.

This gives an estimate for the speed in miles per hour.

Mani

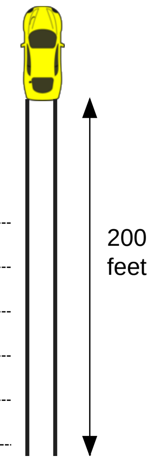
My rule is more complicated.
I use the formula:

$$y = \frac{x}{4} + 30$$

y is the speed of the car in miles per hour
x is the length of the skid mark in feet

Car Skid Marks (continued)

A car was travelling on a dry flat road with brakes in good condition.
The skid marks for the car measured 200 feet.



1. Which rule gives the best estimate for the speed of the car: Dek's or Mani's?
   Show all your work.

2. Dek and Mani argue about which rule is the best one to use for ***any*** traffic accident.
   What is your advice?
   Show your work and explain your reasoning.

Sample Responses to Discuss: Ezra

I will carefully select a range of skid lengths to test Dek and Mani's rules of thumb.

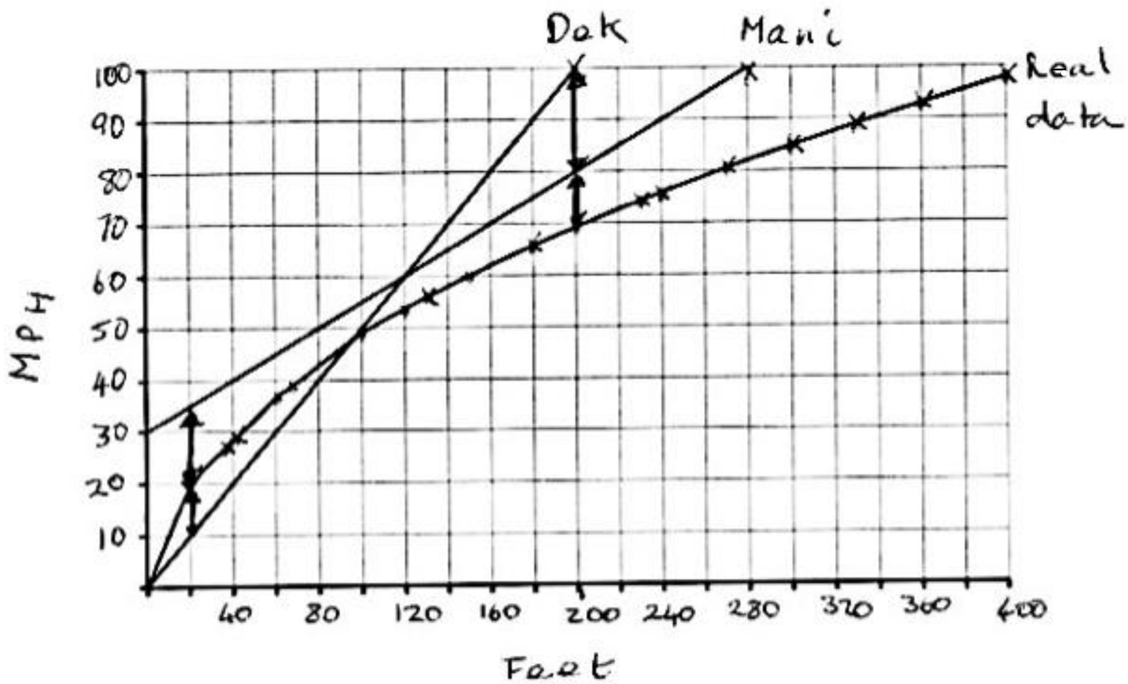
Length of skid	Correct speed	Dek's speed	Mani's speed	Best rule of thumb
0	0	0 (Error 0)	30 (30)	Dek
42	29	21 (Error 29-21=8)	40.5 (40.5-29=11.5)	
100	49	50 (Error 50-49=1)	55 (55-49=6)	
150	60	75 (75-60=15)	67.5 (67.5-60=7.5)	
200	69	100 (100-69=31)	80 (80-69=11)	
240	76	120 (120-76=44)	90 (90-76=14)	
300	85	150 (150-85=65)	105 (105-85=20)	
360	93	180 (180-93=87)	120 (120-93=27)	
400	98	200 (200-98=102)	130 (130-98=32)	

Clearly explain Ezra's method. (You do not need to check Ezra's arithmetic; it is correct).

Use Ezra's method to complete the final column.
What conclusion could Ezra make?

How could Ezra improve his work? Fully explain your answer.

Sample Responses to Discuss: Leanne



Clearly explain Leanne's method.

What conclusion could Leanne make?

How could Leanne improve her work? Fully explain your answer.

How Did You Work?

Complete the sentences and mark the boxes that apply to your work.

1. The method I used to complete the task on my own was ....

....

I advised Dek and Mani that ....

2. The method we used in our group was ....

....

We advised Dek and Mani that ....

3. My method is similar to one of the sample responses ☐ OR My method is different from **both** the sample responses ☐

My method is similar to ______ *(Add name of sample response)*

Because .... | Because ....

.... | ....

4. Our method is similar to one of the sample responses ☐ OR Our method is different from **both** the sample responses ☐

Our method is similar to ______ *(Add name of sample response)*

Because .... | Because ....

.... | ....

5. A 'rule of thumb' that is better than Dek's or Mani's is ....

....

Sharing Individual Solutions

1. Take turns to share with your partner.

2. Share the notes you made on how you might improve your work.

3. Listen carefully to each other, asking questions if you don’t understand.

4. Notice any similarities or differences between the methods described.

Joint Solution: Making Posters

1. In your group agree on the best method for completing the problem.

2. Produce a poster that shows a joint solution to the Car Skid Marks task that is better than your individual work.

3. State on your poster any assumptions you have made.

4. Give clear reasons for your choice of method.

Sharing Posters

1. One person from each group get up and visit a different group.

2. If you are staying with your poster, explain your work to the visitor, giving reasons for your choice of method.

3. If you are the visitor, look carefully at the work, asking clarifying questions to help you to understand the method used.

4. Discuss whether or not the method described on the poster is similar to the visitor's method.

5. The visitor is to write on a post-it note suggestions on how the work could be improved.

Sample Responses to Discuss

1. Read each piece of sample student work carefully.

2. Try to understand what they have done. You may want to add annotations to the work to make it easier to follow.

3. Think about how the work could be improved. Take turns explaining your thinking to your partner.

4. Listen carefully and ask clarifying questions.

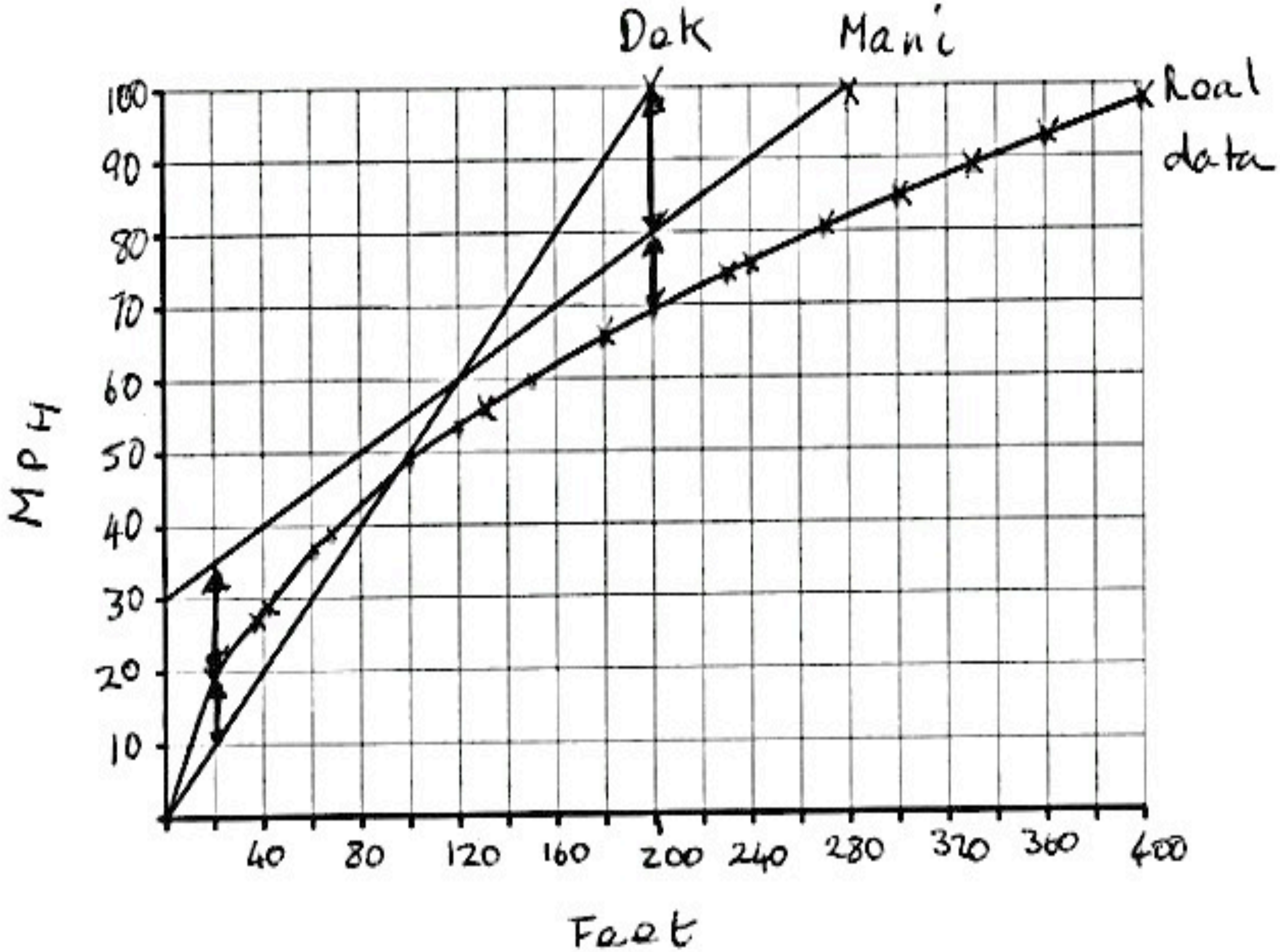
5. When your group has reached its conclusions, write your answers to the questions underneath the work.

Sample Responses to Discuss: Ezra

I will carefully select a range of skid lengths to test Dek and Mani's rules of thumb.

Length of skid	Correct speed	Dek's speed	Mani's speed	Best rule of thumb
0	0	0 (Error 0)	30 (30)	Dek
42	29	21 (Error 29-21 = 8)	40.5 (40.5-29=11.5)	
100	49	50 (Error 50-49 = 1)	55 (55-49 = 6)	
150	60	75 (75 - 60 = 15)	67.5 (67.5 - 60 = 7.5)	
200	69	100 (100 - 69 = 31)	80 (80 - 69 = 11)	
240	76	120 (120 - 76 = 44)	90 (90 - 76 = 14)	
300	85	150 (150 - 85 = 65)	105 (105 - 85 = 20)	
360	93	180 (180 - 93 = 87)	120 (120 - 93 = 27)	
400	98	200 (200 - 98 = 102)	130 (130 - 98 = 32)	

Sample Responses to Discuss: Leanne



Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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