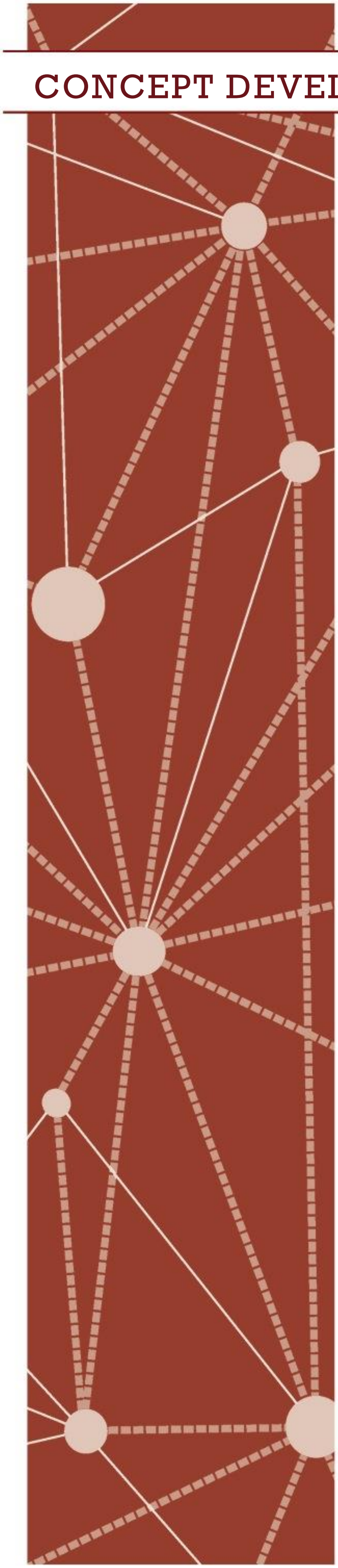


CONCEPT DEVELOPMENT

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Using Standard Algorithms for Number Operations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

For more details, visit: http://map.mathshell.org

Using Standard Algorithms for Number Operations

MATHEMATICAL GOALS

This lesson unit is intended to help students to make sense of standard algorithms for addition, subtraction, multiplication, and division of positive integers. In particular it should assist them in the following areas:

- Improving conceptual understanding of why and how the algorithms work.
- Developing procedural fluency in carrying out the algorithms.
- Becoming more able to spot unreasonably sized answers and to debug errors in procedures.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6.NS: Compute fluently with multi-digit numbers and find common factors and multiples.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 3, 6, 7, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task, *Getting it Wrong*, designed to reveal their current understanding and difficulties. You review their solutions and create questions for students to answer in order to improve their work.
- The main lesson begins with a whole-class introduction, in which students critique systematic errors in standard algorithms. Students try to explain **why** what is being done does not work.
- Then students work in pairs or threes on a collaborative task to find errors in some sample student work. They are encouraged to go beyond ‘correcting’ the answers to conjecturing why the student might have made the error and explaining why the method used does not (always) work.
- In a whole-class discussion, students describe what they have learned from the task.
- Finally, students receive your comments on the assessment task and use these to attempt a similar task, approaching it with insights that they have gained from the lesson.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks *Getting it Wrong* and *Getting it Wrong (revisited)*, a mini-whiteboard, pen, and eraser.
- Each small group of students will need two cards from either *Card Set (1)* or *Card Set (2)* (already cut up) and some blank paper to work on.

TIME NEEDED

20 minutes before the lesson, an 85-minute lesson (or two shorter lessons), and 20 minutes in a follow-up lesson. Timings given are approximate and will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Getting it Wrong* (20 minutes)

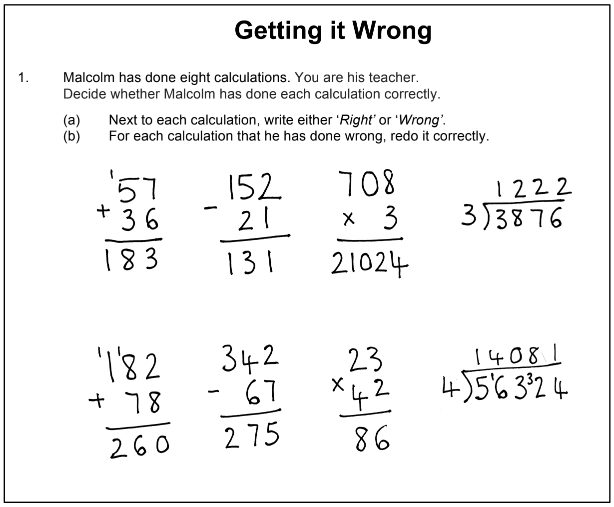
Have the students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of *Getting it Wrong*. Introduce the task briefly, helping the class to understand the problem. You could say:

> *You are going to look at some calculations that have been done by two students.*
> *Some of their answers are right and some are wrong.*
>
> *If a calculation is correct, write 'Right'; if it is incorrect, write 'Wrong'.*
>
> *On question 2, try to explain what Graham has done wrong and why it doesn't work.*
>
> *Try to predict what Graham would answer to 26 multiplied by 8.*
>
> *Then correct the calculations he has got wrong, writing your answers on the sheet.*

It is important that, as far as possible, students answer the questions without assistance. If students are struggling to get started, ask questions that help them understand what they are being asked to do, but do not do the task for them. The questions in the *Common issues* table on page T-3 may be helpful.

Students should not worry too much if they cannot understand or do everything, because there will be a lesson related to this, which should help them. The intention is that students will learn about common errors in standard algorithms, so that they can more easily spot and avoid them in their own work.



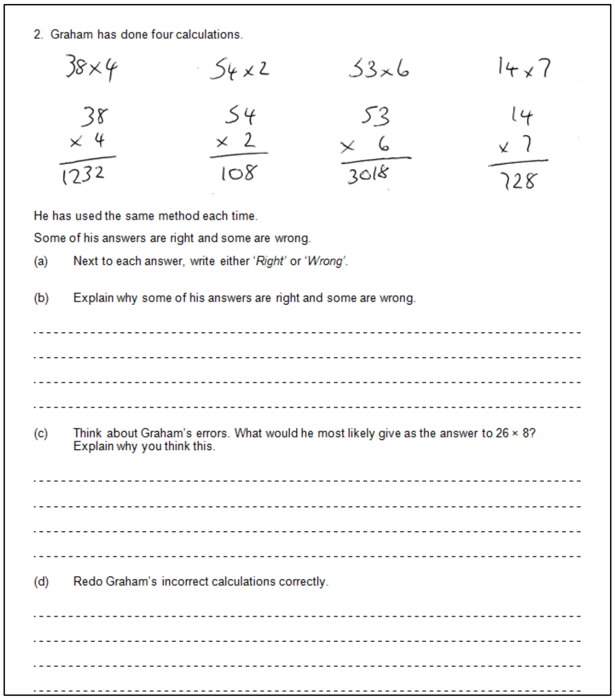
Getting it Wrong

1. Malcolm has done eight calculations. You are his teacher.
Decide whether Malcolm has done each calculation correctly.

(a) Next to each calculation, write either '*Right*' or '*Wrong*'.
(b) For each calculation that he has done wrong, redo it correctly.

$\begin{array}{r} {}^{1}57 \\ +\ 36 \\ \hline 183 \end{array}$ $\begin{array}{r} 152 \\ -\ 21 \\ \hline 131 \end{array}$ $\begin{array}{r} 708 \\ \times\ 3 \\ \hline 21024 \end{array}$ $\begin{array}{r} 1222 \\ 3\overline{)3876} \end{array}$

$\begin{array}{r} {}^{1}1{}^{1}82 \\ +\ 78 \\ \hline 260 \end{array}$ $\begin{array}{r} 342 \\ -\ 67 \\ \hline 275 \end{array}$ $\begin{array}{r} 23 \\ \times 42 \\ \hline 86 \end{array}$ $\begin{array}{r} 14081 \\ 4\overline{)5{}^{1}63{}^{3}24} \end{array}$



2. Graham has done four calculations.

38×4 54×2 53×6 14×7

$\begin{array}{r} 38 \\ \times\ 4 \\ \hline 1232 \end{array}$ $\begin{array}{r} 54 \\ \times\ 2 \\ \hline 108 \end{array}$ $\begin{array}{r} 53 \\ \times\ 6 \\ \hline 3018 \end{array}$ $\begin{array}{r} 14 \\ \times\ 7 \\ \hline 728 \end{array}$

He has used the same method each time.

Some of his answers are right and some are wrong.

(a) Next to each answer, write either '*Right*' or '*Wrong*'.

(b) Explain why some of his answers are right and some are wrong.

(c) Think about Graham's errors. What would he most likely give as the answer to 26 × 8?
Explain why you think this.

(d) Redo Graham's incorrect calculations correctly.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding.

We suggest that you do not score students' work. Research suggests that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table.

We recommend that you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these questions on the board when you return the work to the students in the follow-up lesson.

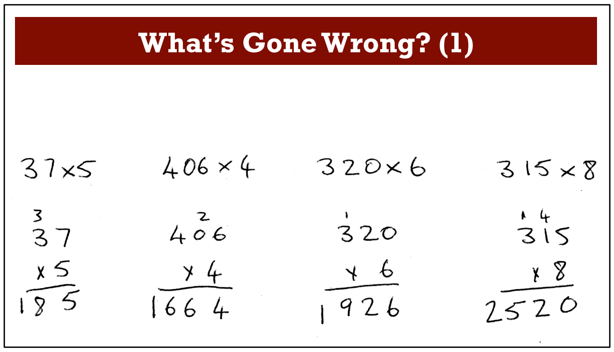
Common issues	**Suggested questions and prompts**
Does not identify the correct and incorrect answers	• Can you check you have found all the correct answers? • Have you looked at the size of the answers? Are there any answers that look way too big or way too small?
Has difficulties identifying 'carrying' errors For example: The student considers 57 + 36 = 183 to be correct and/or 3876 ÷ 3 = 1222 to be correct (Q1).	• Can you do these calculations a different way? • Can you do 57 + 36 in your head? How could you write this method? • How could you check your answer to this division using multiplication?
Decides all of the answers are incorrect/correct For example: The student writes 'wrong' next to each calculation (Q2a).	• Are you sure that all of these are wrong/right? Check carefully.
Does not give explanations or explanations are not sufficient For example: The student just calculates the correct answers for the calculations they think are incorrect (Q2b).	• You have corrected Graham's work, but can you *explain* what he was doing wrong and why it didn't work?
Gives the correct answers For example: The student writes 208 (Q2c).	• You have got the right answer, but what mistake do you think Graham might have made with this question?
Gives the wrong answers For example: The student makes Graham's error, or another systematic error, or a variable random error (Q2d).	• Can you check these answers? Are you sure that they are right?
Completes all the questions correctly	• Can you work out *how* Malcolm obtained the incorrect answers in question 1? • Can you think of a different kind of error that a student might make with calculations like these?

SUGGESTED LESSON OUTLINE

Whole-class introduction: finding and explaining errors (20 minutes)

Throughout this introduction, encourage students to work on the problem on their own for a few minutes, before sharing their ideas with a partner. This may deter one student from dominating the discussion.

Give out mini-whiteboards, pens and erasers. Display Slide P-1:



What's Gone Wrong? (1)

37×5	406×4	320×6	315×8
3 37 ×5 185	2 406 ×4 1664	1 320 ×6 1926	1 4 315 ×8 2520

A student has done these calculations using the same method each time.

Which ones are incorrect? [The middle two.]

Give students plenty of thinking time.

How did you determine which ones were incorrect?

Did anyone do anything other than work through the calculation?

What should the answers be for the ones the student has got wrong? [1624 and 1920.]

Can you work out what they are doing wrong?
If you think you know, don't tell anyone, but show me on your whiteboard what this student would give as an answer to 420 × 5 [2105.]

What is the correct answer? [2100.]

If students find this hard, you could repeat this with a few more examples (e.g. 304 x 7; 2016 x 8).

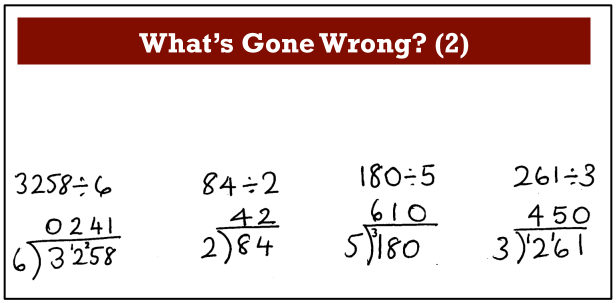
They could discuss with a neighbor if no one has any ideas. It is important that students realize that the errors are not random 'careless' slips and that there is a consistent method being used each time, which sometimes works and sometimes doesn't.

Why did the method fail only ***sometimes****, not always?*

What do you think this student would give as the answer to 43 × 0?

This student has a problem whenever there are zeroes in the two numbers being multiplied. They have the '0 × *n* = *n*' misconception. This has no effect in the first and last calculations, so those ones are correct.

Display Slide P-2:



What's Gone Wrong? (2)

3258÷6	84÷2	180÷5	261÷3
0241 6)3258	42 2)84	610 5)180	450 3)261

Here is some work from another student.
Again, they have used the same method each time.

Which ones are incorrect? [All, except the second one.]
What are the correct answers? [543, 36, and 87.]

Can you work out what they did wrong? Why is it wrong?

Students may find it difficult to answer the second question about **why** it is wrong. The student has worked through all of the divisions from right to left, rather than from left to right. So in the first one the student has thought: ‘6 into 8 goes once remainder 2’ and written ‘1’ over the 8 and the 2 in front of the 5. Where all the divisions go without remainder, as in the second one, the answer is correct.

Again, give students plenty of thinking time. They should notice that the sizes of the other answers are all inappropriate for the calculation being done. If students are struggling to identify what the student is doing wrong, you might like to ask:

What do you do first when dividing using the standard algorithm? Why?
What do you do next? Why?

How could you do this method incorrectly?
How could you start differently?
What could you forget to do?

Collaborative task: finding and explaining errors (45 minutes)

Ask students to work in pairs or threes. The cards from *Card Set (1)* and *Card Set (2)* should be cut out and distributed. Give each group two cards from either *Card Set (1)* or *Card Set (2)* and some blank paper to work on.

Try to ensure that each of the six cards is being tackled by at least two pairs of students, as this will help with the discussion later. Students will need to have some familiarity with negative numbers to be able to tackle Mia’s card (from *Card Set (2)*). Have some spare cards for students who finish their first two cards.

You may or may not wish to provide calculators to assist students in checking answers. This might depend on how difficult you expect students to find the task and what the class is used to.

Display and explain Slide P-3, which summarizes instructions on how students should work on the task:

Collaborative Work

1. Look at each card and indicate which calculations are correct and which are incorrect.
2. Work out what the student is doing that is wrong. They are using the same method for each calculation.
3. Explain **why** the student’s method does not give the correct answer.
4. Do the wrong calculations correctly.

Encourage students to spend a few minutes working on each card on their own before sharing their thinking with a partner.

While students are working, you have two tasks: to notice their approaches to the task and to support student problem solving.

Make a note of student approaches to the task

Notice how students work on the task. You can use this information to focus a whole-class discussion towards the end of the lesson. Do they use estimation or calculators to see which answers are correct, or do they begin by trying to work out the method that the student is using? Are they analytical about the method or do they just dismiss it as 'wrong'?

Support student problem solving

Try not to make suggestions that push students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, you could write one or two relevant questions on the board and/or hold a brief whole-class discussion.

The following questions and prompts may be helpful:

> *Do any of the answers look like they are not reasonable?*
>
> *Look at each digit that the student has written down. Where did that digit come from? Why did they place it in that position? What do you think that they were thinking?*
>
> *Why is their method not giving the right answer?*

Whole-class discussion (20 minutes)

Encourage students to share the different errors, as not all students will have seen all errors. (It might be fun for students to present their reports in the style of a doctor reporting on a sick patient! – "My student was really in a bad way; it's a very sad case…" etc. If you have a spare white laboratory coat handy, this could add to the role.)

> *What was the reasoning behind your student's method?*
> *Why did their method give incorrect answers?*
> *Did their method always fail or only sometimes? Why?*
>
> *Make up two more calculations that use their method.*
> *Can you find an example where the answer is correct, although the method is wrong?*
>
> *What is the correct method for doing this calculation?*
> *How does it work?*
> *Why does it give the right answer?*

Slides P-4 to P-9 are available, showing the six cards, to assist in the discussion. Encourage students to talk about why the student's consistent approach did not work.

If there is time, ask students to make up a sum for the rest of the class to work out using the 'student's' method.

Optional discussion (10 minutes)

If you have the Internet available, you may be able to locate the video clip of Abbott and Costello arguing about whether the product 7 × 13 makes 28. The clip lasts about 2 minutes 30 seconds[1].

> *I want you to watch this short clip.*
>
> *Try to make sense of what's going on.*
>
> *I am going to ask you some questions about it afterwards.*

---

[1] At the time of writing the clip was on Youtube at: http://www.youtube.com/watch?v=XnICFjDn97o

You may like to make some notes while you are watching.

Hopefully students will enjoy the clip and find it amusing. Ask students to reconstruct what happened and account for it:

What was the first calculation that they did? [28 ÷ 7.]

Why did it give the wrong answer?

What was the second calculation that they did? [13 × 7.]

Why did it give the wrong answer?

What was the third calculation that they did? [13 + 13 + 13 + 13 + 13 + 13 + 13.]

Why did it give the wrong answer?

The reason for the mistake each time is that place value is confused. Carry digits are not placed in the correct columns; tens are treated as units, etc. See how much students can explain.

Follow-up lesson (20 minutes)

Return the initial assessment task, *Getting it Wrong*. If you have not chosen to write feedback questions on individual student papers, display your list of questions on the board.

Here are my comments on the work you did [a few days ago].
Work individually, please, answering my questions to improve your work. Write your responses on the back if there isn't space.

Give students a copy of the similar task, *Getting it Wrong (revisited)*.

Now see if you can use what you have learned to produce a better answer to this task. Try to analyze Shan's mistakes as well as you can.

This task could be given as homework.

SOLUTIONS

Assessment task: *Getting it Wrong*

1. 57 + 36 = 183 is incorrect. The correct answer is 93. The 10 that has been carried has become 100.

   152 – 21 = 131 is correct.

   708 × 3 = 21024 is incorrect. The correct answer is 2124. An unnecessary zero has appeared from working out 3 × 0 but not attending to place value.

   3876 ÷ 3 = 1222 is incorrect. The correct answer is 1292. The calculation has been carried out left to right but with remainders being discarded at each stage.

   182 + 78 = 260 is correct.

   342 – 67 = 275 is correct.

   23 × 42 = 86 is incorrect. The correct answer is 966. Units digits have been multiplied together, as have 10s digits, but the other two products have not been carried out.

   56324 ÷ 4 = 14081 is correct.

2. (a) The second calculation is the only one that is correct.

   (b) Graham's consistent error is that he multiplies each digit of the two-digit number by the one-digit number and concatenates the two answers side-by-side rather than carrying. For example, in the first multiplication when multiplying the unit digits 8 and 4, to obtain 32, he writes down 32 rather than writing the 2 in the units column and carrying the 3 in the tens column. Since multiplying the units in the second multiplication gives a total less than 10, his answer is correct as no carrying is required.

   (c) Since 6 × 8 = 48 and 2 × 8 = 16, concatenating the two answers side-by-side rather than carrying would give an answer of 1648.

   (d) The correct answers to the other three calculations are 152, 318 and 98.

Collaborative work

Some explanations of the errors are given below, together with the correct answers to the ones that are wrong.

<table>
<tr><th colspan="4">Card</th><th>Description of error</th></tr>
<tr><td colspan="4">Jacob</td><td rowspan="2">Jacob always subtracts the smaller digit from the larger. This gives the correct answer only when the larger digit is in the minuend (the first number in the subtraction).</td></tr>
<tr><td>58-35
58
-35
23
Right</td><td>46-27
46
-27
21
Wrong
19</td><td>874-321
874
-321
553
Right</td><td>687-508
687
-508
181
Wrong
179</td></tr>
<tr><td colspan="4">Ava</td><td rowspan="2">Ava discards the ‘carry’ digits in her calculations. For example, in the first sum when adding the digits 8 and 6, to obtain 14, she writes down the 4 but discards the 1. Since both columns total less than 10 in the second calculation, her answer is correct, as no carrying is necessary.</td></tr>
<tr><td>48+96
48
+96
34
Wrong
144</td><td>35+42
35
+42
77
Right</td><td>68+36
68
+36
94
Wrong
104</td><td>521+398
521
+398
819
Wrong
919</td></tr>
<tr><td colspan="4">Mason</td><td rowspan="2">Mason discards remainders, so the only question he gets right is Q3, where there are no remainders.</td></tr>
<tr><td>56÷2
23
2)56
Wrong
28</td><td>876÷6
111
6)876
Wrong
146</td><td>633÷3
211
3)633
Right</td><td>605÷5
101
5)605
Wrong
121</td></tr>
<tr><td colspan="4">Aiden</td><td rowspan="2">Aiden multiplies the units digit by the units digit to obtain the units digit of the answer, and adds any carried tens digit to the tens digit in the top number. He gets the correct answer for Q2, perhaps because there is a zero in the tens digit there.</td></tr>
<tr><td>35×4
$\overset{2}{3}5$
×4
50
Wrong
140</td><td>803×6
$8\overset{1}{0}3$
×6
4818
Right</td><td>74×4
$\overset{1}{7}4$
×4
86
Wrong
296</td><td>92×7
$\overset{1}{9}2$
×7
104
Wrong
644</td></tr>
</table>

Card	Description of error
Abigail 804÷4 $\begin{array}{r} 21 \\ 4\overline{)804} \end{array}$ Wrong 201 750÷5 $\begin{array}{r} 150 \\ 5\overline{)750} \end{array}$ Right 3620÷4 $\begin{array}{r} 95 \\ 4\overline{)3620} \end{array}$ Wrong 905 6005÷5 $\begin{array}{r} 121 \\ 5\overline{)6005} \end{array}$ Wrong 1201	Abigail ignores zeroes in the number that is being divided, unless it is on the end, as in Q2. It is quite common for students to have problems with zeroes in the middle of numbers.
Mia 754−28 $\begin{array}{r} 754 \\ -\ \ 28 \\ \hline 73\,^{-}4 \end{array} = 730-4 = \underline{\underline{726}}$ Right 625−37 $\begin{array}{r} 625 \\ -\ \ 37 \\ \hline 6\,^{-}1\,^{-}2 \end{array} = 600-3 = \underline{\underline{597}}$ Wrong 588 5392−685 $\begin{array}{r} 5392 \\ -\ \ 685 \\ \hline 5\,^{-}3\,1\,^{-}3 \end{array} = 5000+1-6 = \underline{\underline{4995}}$ Wrong 4707	Mia carries out the subtraction in each column correctly, writing negative answers when the bottom number (subtrahend) is larger than the top number (minuend). Her consistent error is that she ignores the place value of the digits that she obtains, treating them always as 'units'. In the second case, for example, she should subtract 10 and 2 from 600, not 1 and 2. In the first calculation, the only negative digit is in the units column, so her method works.

Assessment task: *Getting it Wrong (revisited)*

1. 38 + 93 = 131 is correct.

   85 – 57 = 32 is incorrect. The correct answer is 28. In the units column, the smaller digit has been subtracted from the larger.

   35 × 20 = 700 is correct.

   8064 ÷ 6 = 144 is incorrect. The correct answer is 1344. The zero has been ignored.

   167 + 29 = 196 is correct.

   321 – 155 = 276 is incorrect. The correct answer is 166. The 1 above the 2 in the tens column should replace the 2, but instead the two digits have been treated as 12.

   602 × 4 = 248 is incorrect. The correct answer is 2408. The zero has been ignored.

   6135 ÷ 5 = 1227 is correct.

2. (a) The first calculation is the only one that is correct.

   (b) Shan always divides the larger number by the smaller (or the longer by the shorter). Only in the first case has the division been performed correctly. The remaining answers are the reciprocals of the correct ones.

   (c) Shan would write: $\begin{array}{r} 50 \\ 13\overline{)650} \end{array}$

   (d) The correct answers to the other three calculations are 0.0125, 0.05, 0.04.

Getting it Wrong

1. Malcolm has done eight calculations. You are his teacher.
   Decide whether Malcolm has done each calculation correctly.

   (a) Next to each calculation, write either '*Right*' or '*Wrong*'.

   (b) For each calculation that he has done wrong, redo it correctly.

$$\begin{array}{r} {}^{1}57 \\ +\ 36 \\ \hline 183 \end{array} \qquad \begin{array}{r} 152 \\ -\ 21 \\ \hline 131 \end{array} \qquad \begin{array}{r} 708 \\ \times\ \ 3 \\ \hline 21024 \end{array} \qquad \begin{array}{r} 1222 \\ 3\overline{)3876} \end{array}$$

$$\begin{array}{r} {}^{1}1{}^{1}82 \\ +\ 78 \\ \hline 260 \end{array} \qquad \begin{array}{r} 342 \\ -\ 67 \\ \hline 275 \end{array} \qquad \begin{array}{r} 23 \\ \times 42 \\ \hline 86 \end{array} \qquad \begin{array}{r} 14081 \\ 4\overline{)5{}^{1}63{}^{3}24} \end{array}$$

2. Graham has done four calculations.

38×4	54×2	53×6	14×7
38	54	53	14
× 4	× 2	× 6	× 7
1232	108	3018	728

He has used the same method each time.

Some of his answers are right and some are wrong.

(a) Next to each answer, write either ‘*Right’* or ‘*Wrong’*.

(b) Explain why some of his answers are right and some are wrong.

(c) Think about Graham’s errors. What would he most likely give as the answer to 26 × 8? Explain why you think this.

(d) Redo Graham’s incorrect calculations correctly.

Card Set (1)

Jacob

58−35

$$\begin{array}{r} 58 \\ -35 \\ \hline 23 \end{array}$$

46−27

$$\begin{array}{r} 46 \\ -27 \\ \hline 21 \end{array}$$

874−321

$$\begin{array}{r} 874 \\ -321 \\ \hline 553 \end{array}$$

687−508

$$\begin{array}{r} 687 \\ -508 \\ \hline 181 \end{array}$$

Ava

48+96

$$\begin{array}{r} 48 \\ +96 \\ \hline 34 \end{array}$$

35+42

$$\begin{array}{r} 35 \\ +42 \\ \hline 77 \end{array}$$

68+36

$$\begin{array}{r} 68 \\ +36 \\ \hline 94 \end{array}$$

521+398

$$\begin{array}{r} 521 \\ +398 \\ \hline 819 \end{array}$$

Mason

56÷2

$$\begin{array}{r} 23 \\ 2\overline{)56} \end{array}$$

876÷6

$$\begin{array}{r} 111 \\ 6\overline{)876} \end{array}$$

633÷3

$$\begin{array}{r} 211 \\ 3\overline{)633} \end{array}$$

605÷5

$$\begin{array}{r} 101 \\ 5\overline{)605} \end{array}$$

Card Set (2)

Aiden

35×4

$$\begin{array}{r} \overset{2}{3}5 \\ \times\ 4 \\ \hline 50 \end{array}$$

803×6

$$\begin{array}{r} 8\overset{1}{0}3 \\ \times\ 6 \\ \hline 4818 \end{array}$$

74×4

$$\begin{array}{r} \overset{1}{7}4 \\ \times\ 4 \\ \hline 86 \end{array}$$

92×7

$$\begin{array}{r} \overset{1}{9}2 \\ \times\ 7 \\ \hline 104 \end{array}$$

Abigail

804÷4

$$\begin{array}{r} 21 \\ 4\overline{)804} \end{array}$$

750÷5

$$\begin{array}{r} 150 \\ 5\overline{)750} \end{array}$$

3620÷4

$$\begin{array}{r} 95 \\ 4\overline{)3620} \end{array}$$

6005÷5

$$\begin{array}{r} 121 \\ 5\overline{)6005} \end{array}$$

Mia

754−28

$$\begin{array}{r} 754 \\ -\ 28 \\ \hline 73^{-}4 \end{array}$$

$= 730-4$

$= 726$

625−37

$$\begin{array}{r} 625 \\ -\ 37 \\ \hline 6^{-}1^{-}2 \end{array}$$

$= 600-3$

$= 597$

5392−685

$$\begin{array}{r} 5392 \\ -\ 685 \\ \hline 5^{-}31^{-}3 \end{array}$$

$= 5000+1-6$

$= 4995$

Getting it Wrong (revisited)

1. Hugh has done eight calculations. You are his teacher.
   Decide whether Hugh has done each calculation correctly.

$$\begin{array}{r} \overset{1}{3}8 \\ +\ 93 \\ \hline 131 \end{array} \qquad \begin{array}{r} 85 \\ -\ 57 \\ \hline 32 \end{array} \qquad \begin{array}{r} {}^{1}35 \\ \times\ 20 \\ \hline 700 \end{array} \qquad \begin{array}{l} \quad 144 \\ 6\overline{)80{}^{2}6{}^{2}4} \end{array}$$

$$\begin{array}{r} 1\overset{1}{6}7 \\ +\ 29 \\ \hline 196 \end{array} \qquad \begin{array}{r} 3\overset{1}{2}1 \\ -\ 155 \\ \hline 276 \end{array} \qquad \begin{array}{r} 602 \\ \times\ \ \ 4 \\ \hline 248 \end{array} \qquad \begin{array}{l} \quad 1227 \\ 5\overline{)6{}^{1}1{}^{1}3{}^{3}5} \end{array}$$

2. Shan was given four division calculations to do:

105 ÷ 7 =	15 ÷ 1200 =	85 ÷ 1700 =	14 ÷ 350 =
$\begin{array}{r}15\\7\overline{)105}\end{array}$	$\begin{array}{r}80\\15\overline{)1200}\end{array}$	$\begin{array}{r}20\\85\overline{)1700}\end{array}$	$\begin{array}{r}25\\14\overline{)350}\end{array}$

She has used the same method each time.

Some of her answers are right and some are wrong.

(a) Next to each answer, write either '*Right'* or '*Wrong'*.

(b) Explain why some of her answers are right and some are wrong.

(c) Think about Shan's errors. What would she most likely give as the answer to 13 ÷ 650? Explain why you think this.

(d) Redo Shan's incorrect calculations correctly.

What’s Gone Wrong? (1)

37×5

$$\begin{array}{ccc} & 3 & \\ & 3 & 7 \\ & \times & 5 \\ \hline 1 & 8 & 5 \end{array}$$

406×4

$$\begin{array}{cccc} & & 2 & \\ & 4 & 0 & 6 \\ & & \times & 4 \\ \hline 1 & 6 & 6 & 4 \end{array}$$

320×6

$$\begin{array}{cccc} & 1 & & \\ & 3 & 2 & 0 \\ & \times & & 6 \\ \hline 1 & 9 & 2 & 6 \end{array}$$

315×8

$$\begin{array}{cccc} & 1 & 4 & \\ & 3 & 1 & 5 \\ & & \times & 8 \\ \hline 2 & 5 & 2 & 0 \end{array}$$

What’s Gone Wrong? (2)

$$3258 \div 6$$

$$\begin{array}{r} 0\,2\,4\,1 \\ 6\overline{)3\,{}^{1}2\,{}^{2}5\,8} \end{array}$$

$$84 \div 2$$

$$\begin{array}{r} 4\,2 \\ 2\overline{)8\,4} \end{array}$$

$$180 \div 5$$

$$\begin{array}{r} 6\,1\,0 \\ 5\overline{){}^{3}1\,8\,0} \end{array}$$

$$261 \div 3$$

$$\begin{array}{r} 4\,5\,0 \\ 3\overline{)^{1}2\,{}^{1}6\,1} \end{array}$$

Collaborative Work

1. Look at each card and indicate which calculations are correct and which are incorrect.

2. Work out what the student is doing that is wrong. They are using the same method for each calculation.

3. Explain **why** the student's method does not give the correct answer.

4. Do the wrong calculations correctly.

Jacob

58-35

$$\begin{array}{r} 58 \\ -35 \\ \hline 23 \end{array}$$

46-27

$$\begin{array}{r} 46 \\ -27 \\ \hline 21 \end{array}$$

874-321

$$\begin{array}{r} 874 \\ -321 \\ \hline 553 \end{array}$$

687-508

$$\begin{array}{r} 687 \\ -508 \\ \hline 181 \end{array}$$

Ava

$48+96$ $35+42$ $68+36$ $521+398$

$$\begin{array}{r} 48 \\ +96 \\ \hline 34 \end{array} \qquad \begin{array}{r} 35 \\ +42 \\ \hline 77 \end{array} \qquad \begin{array}{r} 68 \\ +36 \\ \hline 94 \end{array} \qquad \begin{array}{r} 521 \\ +398 \\ \hline 819 \end{array}$$

Mason

56÷2

$$\begin{array}{r} 23 \\ 2\overline{)\,56} \end{array}$$

876÷6

$$\begin{array}{r} 111 \\ 6\overline{)\,876} \end{array}$$

633÷3

$$\begin{array}{r} 211 \\ 3\overline{)\,633} \end{array}$$

605÷5

$$\begin{array}{r} 101 \\ 5\overline{)\,605} \end{array}$$

Aiden

35×4

$$\begin{array}{r} \overset{2}{3}5 \\ \times\ 4 \\ \hline 5\ 0 \end{array}$$

803×6

$$\begin{array}{r} 8\overset{1}{0}3 \\ \times\ 6 \\ \hline 4818 \end{array}$$

74×4

$$\begin{array}{r} \overset{1}{7}4 \\ \times\ 4 \\ \hline 8\ 6 \end{array}$$

92×7

$$\begin{array}{r} \overset{1}{9}2 \\ \times\ 7 \\ \hline 104 \end{array}$$

Abigail

$804 \div 4$

$$\begin{array}{r} 21 \\ 4\overline{)804} \end{array}$$

$750 \div 5$

$$\begin{array}{r} 150 \\ 5\overline{)750} \end{array}$$

$3620 \div 4$

$$\begin{array}{r} 95 \\ 4\overline{)3620} \end{array}$$

$6005 \div 5$

$$\begin{array}{r} 121 \\ 5\overline{)6005} \end{array}$$

Mia

$754-28$

$$\begin{array}{r} 754 \\ -\ \ 28 \\ \hline 73\,{}^{-}4 \end{array} = 730-4$$

$$= \underline{\underline{726}}$$

$625-37$

$$\begin{array}{r} 625 \\ -\ \ 37 \\ \hline 6\,{}^{-}1\,{}^{-}2 \end{array} = 600-3$$

$$= \underline{\underline{597}}$$

$5392-685$

$$\begin{array}{r} 5392 \\ -\ \ 685 \\ \hline 5\,{}^{-}3\,1\,{}^{-}3 \end{array}$$

$$= 5000+1-6$$

$$= \underline{\underline{4995}}$$

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
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