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Uses of Video in Understanding and Improving Mathematical Thinking and Teaching

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Abstract

This article characterizes my use of video as a tool for research, design and development. I argue that videos, while a potentially overwhelming source of data, provide the kind of large bandwidth that enables one to capture phenomena that one might otherwise miss; and that although the act of taping is in itself an act of selection, there is typically enough shown in a video that it rewards multiple watching and supports the kinds of arguments over data that are essential for theory testing and replication. In pragmatic terms, video presents phenomena in ways that have an immediacy that is tremendously valuable. I discuss ways in which videos help students and teachers focus on phenomena that might otherwise be very hard to grapple with.

This article begins with a brief review of my uses of video, almost 40 years ago, for research and development in problem solving. It then moves to the discussion of very fine-grained research on learning and decision making. The bulk of the article is devoted to a discussion of the Teaching for Robust Understanding (TRU) Framework, which was derived in large measure from the extensive review of classroom videotapes, and which serves as the basis for an extensive program of pre-service and in-service professional development. The professional development relies heavily on the use of videos to convey the key ideas in TRU, and to help teachers plan and review instruction.

Key Words

Video, professional development; mathematics teaching; mathematics learning; teaching for robust understanding

1. Introduction: Video in research and theory, and then in practice

From my earliest days as a researcher studying thinking and learning, I have believed in the value of getting as close as possible to the phenomena under investigation. That led me, in 1978, to include videotape equipment in my request to the US National Science Foundation for a research grant to study mathematical problem solving. Before the grant was awarded I had to respond to a series of questions from the Foundation. The primary issue that reviewers raised concerned videotapes – why were they necessary? My response was that there were aspects of problem solving behavior that could not be captured any other way. Those included references to specific drawings as students were working on them ("What if we draw the line from here to here?") and the evolution of those figures, indications of shared attention or independent processing, and more. Perhaps most important, many aspects of exchanges are fleeting and ephemeral; if I didn't "capture" them on tape, they would be lost forever, subject to faulty memory, and more. It wasn't that videos would be my only source of data, but rather that they offered data that were irreplaceable.

I should note that there were, back then, legitimate concerns about the use of videos and more generally about reports of thinking and problem solving. At that time the psychological community had not yet come to the understanding that the things people *say* could be treated as data. Whether one could do so was a major controversy, one that Nobel Prize Winner Herb Simon (and co-author of the foundational volume *Human Problem Solving;* Newell & Simon, 1972) felt compelled to address in a paper (Ericson and Simon, 1980) and a volume entitled *Protocol Analysis: Verbal Reports as Data* (Ericson and Simon, 1984). They wrote to counter arguments about the limited value of such data made, for example, by Nisbett and Wilson (1977). From another perspective, my friend and colleague Jim Greeno questioned my need for a video lab when, in 1985, we were building the Education in Mathematics, Science, and Technology program at Berkeley. The challenge, Jim said, was that video data were *too* rich. How could one narrow things down to a manageable focus when confronted with such broad bandwidth? Most psychological and educational researchers at the time preferred to control things in advance, constraining both circumstances and data collection to provide data that could be analyzed "objectively" using extant (typically statistical) tools. There wasn't a history of video analyses.

My 1985 book *Mathematical Problem Solving*, derived in large measure from the work done on the NSF grant, clearly demonstrated the value of the videotapes – some of which had been reviewed more than 50 times as I strove to capture and represent what took place in them. It was in watching students go awry, both in pursuing unprofitable approaches far longer than was reasonable, and by failing to pursue potential leads that they themselves had uncovered, that I came to understand what I called "control" (and is now called monitoring and self-regulation, an aspect of metacognition). It was in watching the struggles of students working on tasks that they *knew* how to solve that I came to understand the role and power of belief systems. Once I had such insights, it was possible to use a series of measurement techniques – some classical (i.e., pretests and posttests), some developed in response to the new phenomena (i.e., "timeline" graphs of student trajectories through problems) to document those insights in reliable and replicable ways.

Over the years the use of video has become fundamental to my practice, both in my role as a researcher and as someone who works with pre-service and in-service teachers. Beginning with the paper "Learning" (Schoenfeld, Smith, and Arcavi, 1993), video became a prime source of phenomenological data. In that paper we had the 7-hour video record of a series of tutoring sessions; our goal was explain, in fine-grained detail, how the tutee's understandings changed over those seven hours. That is: the student's learning, most completely represented by everything she said and did over the seven hours of videotape, became the direct object of analysis and a primary means of theory building and theory testing. Our object was to model the tutee's evolving understanding. Of necessity, a model is the instantiation of a theoretical stance: the objects in the model represent what the theorists believe "counts" in a student's developing understanding. The Learning paper (Schoenfeld, Smith, and Arcavi, 1993) was one of the field's first microgenetic analyses (micro in the sense of very fine-grained, genetic in the Piagetian sense of focusing on the growth and change – genesis – of knowledge).

The use of videos for fine-grained theory building and testing culminated in my book *How We Think* (Schoenfeld, 2010), in which I describe four microgenetic analyses (three of teaching, one of a doctor-patient conversation) that served as a way of developing and testing a theory of human decision making. As in the Learning paper, the primary data were recordings of the focal players in action – in teaching or in a medical consultation. This time, there were very explicit theoretical goals: to show that the constructs of an individual's *resources* (including but not limited to that person's knowledge), *orientations* (a generalization of beliefs that includes beliefs, dispositions, values, and preferences), and *goals* were sufficient to explain that person's actions in "well practiced" domains such as teaching or medical practice, where the individual had had enough time to build up well established routines and practices. Here too, video was essential as a source of data: the goal of the analyses was to explain every utterance of the people who were being studied. In that way, the tapes were a source of potential falsification for the theoretical ideas: the theorists' obligation was to explain every utterance that was not consistent with the proposed model of the individuals' decision making.

In all of my work, research and development have been deeply intertwined. Thus, my ideas about problem solving were developed both in my lab and while I taught my problem solving courses. Similarly over the years, my thoughts about decision making (primarily in teaching) were informed by my involvement in teacher preparation and professional development, and fed into those efforts. As indicated above, video has been central as a source of data. It has also been important as a mechanism for communicating ideas once they had become apparent. One of the major issues in my problem solving courses was to convey the importance of metacognition in decision making. For that reason, I showed my students some of the videotapes (without prior comment) that had made me aware of the importance of monitoring and self-regulation. As the students in the video perseverated at what seemed to be obvious dead ends and ignored potential leads, the viewers got increasingly perturbed – "how can they be so blind?" was a common sentiment. Then, one student said that he probably did the same thing. That kind of awareness made it much easier for us to pursue issues of "control" as a community. (The results of such efforts are given in Schoenfeld, 1985, 1987.)

In a similar vein, Abraham Arcavi and I wondered about ways in which we could make the idea of beliefs and orientations tangible to teachers and coaches of teachers. Having teachers watch videos of classrooms is a delicate matter: it is easy for a viewer to be judgmental, focusing on what he or she would have done in the given context. Arcavi had the clever idea of re-framing the observations. We make the basic assumption that teachers do what they do in the best interests of their students. That being the case, what understandings would have led the teachers in the videotapes to make the choices they did? This framing led to productive discussions of beliefs and orientations.

In what follows I discuss my work over the past half dozen years or so. In this work we – a large team including the Algebra Teaching Study (see http://ats.berkeley.edu/) and the Mathematics Assessment Project (see http://map.mathshell.org/) – began once again with video analyses of as a major strand of our work – but then moved to the much more extensive use of video in our current and planned work with pre-service and in-service teachers.

2. What makes for powerful teaching?

Some years ago, having come to the understanding of teacher decision making represented in Schoenfeld (2010), I felt that I was ready to tackle the issue of what makes for "powerful" classrooms – classrooms that produce students who are effective mathematical thinkers and problem solvers. This work, like my problem solving work, would involve close links between theory and practice. That video would be central both for deriving theory and for influencing practice was a given.

In many ways, the goals for this endeavor parallel the goals of the problem solving work. At the very top level, one wants a parsimonious characterization of "what counts." In the problem solving work (Schoenfeld, 1985, 1992), I argued that a small number of categories – the knowledge base, use of problem solving strategies, metacognition, and beliefs and practices – were necessary and sufficient to understand success or failure in problem solving. They were necessary in that if one did not attend to all of the categories, one might miss the cause of success or failure. They were sufficient in that no other categories would be necessary: in all problem solving episodes, the cause of success or failure would be found in one or more of those four categories. That there were a small number of somewhat independent categories meant that the problem solving framework was "actionable:" one could focus on a small number of things to improve people's problem solving. A list of twenty categories is unmanageable in pragmatic terms.

In this work we seek the same kind of characterization of mathematics classrooms in general. That is, our goal is to produce a framework for characterizing mathematics classrooms that has the following attributes:

- the framework contains a small number of categories;
- each category is necessary;
- taken together, the categories are comprehensive;
- classrooms that do increasingly well on these categories produce students who are increasingly powerful mathematical thinkers and problem solvers; and,
- the framework is actionable, in that its parsimony and the somewhat independent nature of the dimensions make it possible to map out a reasonable improvement process, and to measure progress.

It goes without saying that had such a framework existed when we began, we would not have needed to embark on this work. There did exist a substantial number of frameworks for the analysis of teaching (or more broadly, for the analysis of classroom environments). Among them were the Framework for Teaching (Danielson, 2011), the Classroom Assessment Scoring System (Pianta, La Paro, & Hamre, 2008), the Protocol for Language Arts Teaching Observations (Institute for Research on Policy Education and Practice, 2011), the Mathematical Quality of Instruction framework (University of Michigan, 2006), the UTeach Teacher Observation Protocol (Marder & Walkington, 2012), the Instructional Quality Assessment, (Junker et al., 2004), the Performance Assessment for California Teachers (PACT Consortium, 2012), and the Systematic Classroom Analysis Notation (Beebe, Burkhardt, & Caddy, 1980). Each of these frameworks had its virtues, but none of them had the collective set of properties that we sought – some were domain-general and did not support a focus on mathematics, some were focused on particular aspects of the classroom (e.g., the content-richness of classroom dialogue) but were not comprehensive; some were comprehensive but contained so many categories of classroom behavior and activity that they were unwieldy at best. I do note that the MET study (Measures of Effective Teaching, 2012), which had not been undertaken when we began our work, did show that such schemes, in general, correlate with student learning. That they do is unsurprising: we have known in general terms that a large number of classroom attributes contribute to learning. Our goal was to obtain a coherent and manageable perspective meeting the criteria listed above[1].

The evolution of the framework is described in some detail in Schoenfeld (2013). We began by watching numerous classroom videos, with my expectation being that we would be able to create a "quick and easy to use" version of the analytic tools used for the analyses of teacher decision making in Schoenfeld (2010). This attempt failed for various reasons, among them the fact that it was labor intensive and that it was too teacher-centered. (The classroom environment was taken into account, but through the teacher's eyes.) Any framework we developed would need to have the properties described above, but also, if it was to be used for large-scale work, would need to be efficient. Our previous focus on understanding a teacher's resources, goals, and orientations turned out not to be directly useful (although, of course, when one turns to issues of professional development, the question of how to have a positive impact on teachers' resources, goals, and orientations is central).

We spent a number of years trying to be both analytic (tying our observations to what we observed in a range of classroom videos and the literature) and comprehensive. The result was a framework that grew increasingly complex and unwieldy: in trying to capture all the important things we saw in classroom videotapes, we got lost in the details. None of the details were *wrong*, in that they did matter – but, it was hard to see what mattered at a level of abstraction that could provide a clear focus for classroom assessment and for professional development. Finally, we engaged in the mathematical activity of sorting the more than 100 categories of activity in our observational system into equivalence classes. Once we did, a small number of dimensions of classroom activities emerged. One representation of those dimensions is given in Figure 1.

[1] I should note that our goals were not this clear when we began the work. The presentation in this paper is a function of their evolution over some years.

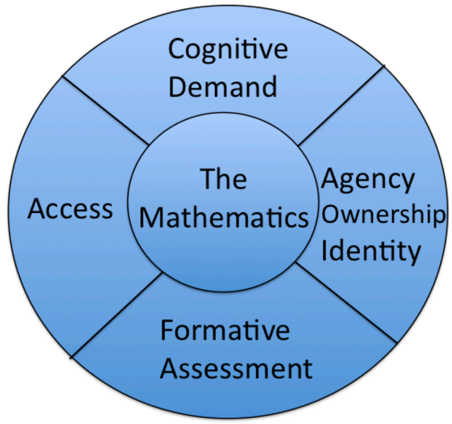

Figure 1. The five dimensions of the Teaching for Robust Understanding of Mathematics (TRUmath) Framework

These dimensions are arranged spatially in Figure 1 to illustrate both the individual dimensions and their connections – everything is connected, but each dimension has its own integrity.

The Mathematics is in the center for the obvious reason that the mathematics that is the focus of classroom must be of high quality. If it is not, then no matter how good the other aspects of a lesson may be, the students will not be engaging with, and therefore will not be learning, powerful mathematics. However, the quality of the mathematics is a necessary but not sufficient condition for powerful mathematics learning: we have all been in lectures or classrooms where the mathematics presented was beautiful and elegant, but precious few of those listening could understand it! What counts is what the students learn. The other four dimensions pertain to the students' engagement with the mathematics, and what they gain (or should gain) from that engagement. We begin somewhat arbitrarily at the top.

Cognitive Demand represents the opportunity students have for engaging meaningfully in mathematical sense making, in what has been called productive struggle. If the students are insufficiently challenged and engaged, or if the mathematics is too far beyond them, they will not learn very much. In this context we see the importance of *Formative Assessment*. Learning is enhanced when the teacher attends to student thinking and the lesson is adjusted so that students engage with the content at a productive level of cognitive demand. In this way, the top and bottom dimensions are linked (through the mathematics).

There is a similar linkage between the left and right hand dimensions. We take it as a given that a powerful classroom is one in which all students have meaningful opportunities to learn powerful mathematics; a classroom in which three students excel and get most of the teacher's attention is a classroom in which all but three students have been denied equitable opportunities to engage in mathematics. This is the issue of *Access*.

Finally, there is the issue of students' engagement with the mathematics. How do students see themselves as doers of mathematics? We have all met people who say "I gave up on mathematics in fourth (or seventh, or…) grade. I got OK grades, but it didn't make sense. I'm just not a math person." That sense of identity – I am, or am not, a math person – relates to the dimension *Agency, Ownership, and Identity*. Do students have the opportunity to engage productively in mathematics, and feel that they can do so (Agency)? Do they have the opportunity to make the content their own? Do they have they have opportunities to see themselves as people who can do mathematics, and to develop positive mathematical identities?

The framework can thus be outlined as in Figure 2.

The Five Dimensions of Powerful Classrooms

The Content	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which the content students engage with represents our best current disciplinary understandings (as in CCSS, NGSS, etc.). Students should have opportunities to learn important content and practices, and to develop productive disciplinary habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' disciplinary development. Students should be able to engage in sense making and "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core content being addressed by the class. No matter how rich the content being discussed, for example, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to "walk the walk and talk the talk," building on each other's ideas, in ways that contribute to their development of agency (the willingness to engage) and ownership over the content, resulting in positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent instruction responds to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Figure 2. The TRUmath framework: The five dimensions of powerful mathematics classrooms

Our hypothesis, for which there is an increasing amount of evidence (see below), is that classrooms that support the five dimensions in Figures 1 and 2 will produce students who are powerful mathematical thinkers.

3. Tools for supporting pre-and in-service teachers in creating more powerful learning environments.

Concurrent with the development of the TRU framework has been the development of a set of tools intended to help teachers teach in ways consistent with TRU. We have a protocol for the use of videos in pre-service and in-service work with teachers, and evidence of some of the impact of those materials. I begin by discussing the tools. I then turn to some of our uses of video, after which I describe some of the impact of the work.

Before proceeding, however, I should stress that TRU is not simply a set of tools. As documented below, some of the tools are quite powerful. However, an important aspect of TRU is that it provides a perspective on “what counts” in classrooms – a set of lenses through which to view instruction, and a language for discussing such issues. I elaborate on this point in the discussions of the Conversation Guide, Observation Guide and classroom rubric.

Tools

Over the past half dozen years our research teams have developed a number of tools to support teachers in the kinds of ambitious teaching that reflects the TRU Framework. These tools are available on the Mathematics Assessment Project and Algebra Teaching Study web sites, at <http://ats.berkeley.edu/> and <http://map.mathshell.org/> respectively.

The Mathematics Assessment Project (MAP) supported the development of the TRU Framework, along with the development of 100 closely related “Formative Assessment Lessons” (FALs). Every FAL is fundamentally aligned with TRU, in that the activity structures in the lessons embody ways of doing well on the five TRU dimensions. The mathematics in each lesson has been selected to represent central content and practices in the Common Core Standards for Mathematics (Common Core Standards Initiative, 2010), the de facto set of curriculum and evaluation standards across the United States. The name of the lessons, “Formative Assessment Lessons,” indicates the focus on student thinking and formative assessment that lies at the heart of each lesson. The lessons, which typically take 2-3 days to implement, are preceded by a diagnostic task that helps the teacher determine how students currently understand the material. On the basis of prior research and multiple field trials, the research team is aware of typical student difficulties and designs for them, aiming for an appropriate level of cognitive demand. The lesson plans contain descriptions of “common student issues” and “suggested questions and prompts,” which are intended to help teachers provide scaffolding but not simply provide answers – a way of maintaining cognitive demand. The lessons themselves provide multiple opportunities for students to work collaboratively and to present their work, thus providing affordances for access and the development of agency, authority, and identity[2]. We are now working with professional developers across the state of Arkansas and through the U.S. Southern Regional Education Board (SREB) to offer TRU-based professional development for the FALs. In addition to live coaching, this PD makes use of videotapes for TRU-based teacher discussions.

The TRU Math Conversation Guide (Baldinger and Louie, 2014) was designed to support teachers, coaches, and professional learning communities in planning and reflecting on their lessons – no matter what lessons they are teaching. As noted above, TRU provides a framework for thinking about what is important in mathematics instruction. Each dimension of TRU can thus be used as a lens for inquiry – in planning lessons, enacting them, and reflecting on them.

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[2] One must note that a lesson plan – even one that is 20-30 pages long, like most of the Formative Assessment Lessons – can at best provide affordances (that is, offer structured opportunities) for the dimensions of TRU. A teacher *could* lecture on the content, give students procedures when they get stuck, etc. The most we can say is that the lessons provide opportunities consistent with TRU.

Regarding *The Mathematics*, one can ask, “How are the mathematical ideas from this unit (this course) developed in this lesson or lesson sequence?” Regarding *Cognitive Demand,* one can ask, “What opportunities do student have in this lesson to make sense of their own mathematical ideas?” Regarding *Access to the Mathematical Content*, one can ask, “Who does and does not participate in the mathematical work of the class, and how?” Regarding *Agency, Authority, and Identity*, one can ask, “What opportunities do students have to explain their own and respond to each other's mathematical ideas?” And regarding *Formative Assessment,* one can ask, “What do we know about each student's current mathematical thinking, and how can we build on it?”

The Conversation guide is intended to facilitate conversations between teacher and coach, or among teachers in a Professional Learning Community, or by a teacher planning lessons by him- or herself. It offers expanded sets of questions aligned with those above. For example, the expanded set of questions related to the *Agency, Ownership, and Identity* dimension is given in Figure 3.

Agency, Ownership, and Identity

Core Question: What opportunities do students have to explain their own and respond to each other’s mathematical ideas?

Many students have negative beliefs about themselves and mathematics, for example, that they are “bad at math,” or that math is just a bunch of facts and formulas that they’re supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and authority. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

Agency, Ownership, and Identity

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist in the lesson for students to explain their own and respond to each other’s mathematical ideas?	What opportunities did students have to explain their own and respond to each other’s mathematical ideas?	What opportunities can we create in future lessons for more students to explain their own and respond to each other’s mathematical ideas?

Think about:

- Who generates the mathematical ideas that get discussed.
- Who evaluates and/or responds to others' ideas.
- How deeply students get to explain their ideas.
- How the teacher responds to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.).
- How norms around students' and teachers' roles in generating mathematical ideas are developing.
- How norms around what counts as mathematics (justifying, experimenting, practicing, etc.) are developing.
- Which students get to explain their own and respond to others' ideas in a meaningful way.

Figure 3. An expanded discussion of Agency, Authority, and Identity

The TRU Math Rubric (Schoenfeld, Floden, & the Algebra Teaching Study and Mathematics Assessment Project, 2014) characterizes classroom activity structures along each of the five dimensions on a 3-point scale (1 = “basic,” 2 = “proficient,” 3 = “distinguished”). The rubric has two functions. First, it can be used to score classroom performance. As such, the rubric can be used to examine the hypothesis that increasingly rich classroom activities along the five dimensions (as scored on the rubric) will result in students who are increasingly powerful mathematically (as reflected by scores on tests of mathematical thinking and problem solving). Equally important, however, is the fact that the rubric itself characterizes increasing levels of sophistication along the five dimensions. It thus represents a developmental progression. Teachers can reflect on their classroom practices, and contemplate "next steps" in their growth as teachers. The most important use of a yardstick, after all, is to measure growth. Tools such as the TRU Math Rubric and Conversation Guide are intended to support formative assessment regarding teachers' classroom practices.

The TRU Math Rubric offers separate scoring rubrics for a range of classroom activity structures – whole class, small group, individual work, and presentations. The summary rubric is given in Figure 4.

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Formative Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
Basic	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
Proficient	Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
Distinguished	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Figure 4. The summary rubric for the five dimensions of TRU Math

The TRU Observation Guide (Schoenfeld et. al., 2016) is a tool used by coaches of both pre-service and in-service teachers, as well as professional learning communities, to frame coaching conversations around teaching activities using the five TRU dimensions. In brief, the idea is that teacher and mentor/coach/PLC plan an observation in advance, and that the mentor/coach/PLC takes notes using the observation guide. As discussed below, the debriefing often includes the use of video. A sample page from the observation guide is given in Figure 5.

AGENCY, OWNERSHIP, AND IDENTITY	
The extent to which every student has opportunities to explore, conjecture, reason, explain, and build on emerging ideas, contributing to the development of agency (the willingness to engage academically) and ownership over the content, resulting in positive disciplinary identities.	
Each student…	Teachers…
• Takes ownership of the learning process in planning, monitoring, and reflecting on individual and/or collective work • Asks questions and makes suggestions that support analyzing, evaluating, applying and synthesizing ideas • Builds on the contributions of others and help others see or make connections • Holds classmates and themselves accountable for justifying their positions, through the use of evidence and/or elaborating on their reasoning	• Provide time for students to develop and express their ideas • Work to make sure all students have opportunities to have their voices heard • Encourage student-to-student discussions and promote productive exchanges • Assign tasks and pose questions that call for marshaling, analyzing and synthesizing evidence, and for students to explain their reasoning • Employ a range of techniques that attribute ideas to students, to build student ownership and identity
• Other focal points for observation:	
What opportunities do all students have to see themselves and others as proficient disciplinary thinkers, to grapple with challenges and construct new understandings, to build on others' ideas, and demonstrate their understandings? How can more of these opportunities be created?	
Goal: All students build productive disciplinary identities through taking advantage of opportunities to engage meaningfully with the discipline and share and refine their developing ideas.	

Figure 5. The observation page for "agency, ownership, and identity"

4. Uses of Video

Video has played a prominent role in our TRU work with pre-service and in-service teachers. I begin by describing a generic TRU workshop, which has been used with both pre-service and in-service teachers, as well as with mathematics coaches (who then implement the workshop themselves).

4.1 The "Introduction to TRU" video workshop.

We have chosen a sequence of three video clips, each approximately five minutes long. The first clip is from very traditional teacher-centered lesson, in which the content is straightforward if not rote and the teacher maintains complete control, asking students questions whose answers are usually a number or a phrase. It is rare to hear a student utter a full sentence in that class. The second clip is from a classroom in which very different norms have been established. The teacher poses a question, and waits; students indicate when they have arrived at an answer, and the teacher does not move on until the vast majority of students have done so. The teacher then has the students explain their work to their tablemates, before calling on volunteers – who go to the front of the classroom and give extended explanations of their thinking. In the third video clip, the teacher is not seen at all. We watch a group of sixth graders arguing seriously and respectfully about mathematical content. Their agency (as well as the robustness of their understanding) is palpable.

After each of the tapes has been viewed we have workshop participants write down notes, and then discuss the tapes with their neighbors. After the third tape, we open things for general discussion. Workshop participants describe what they saw, and how it might affect the students. We take notes, in front of the group. After they are done, we organize the notes.

At one typical implementation of this workshop (for representatives from 14 California school districts, which teach more than 1 ½ million students) we had five people taking notes, each at one flip chart. What the participants did not know was that note-taker 1 was recording comments that pertained to the mathematics; note-taker 2 was recording comments that pertained to cognitive demand, note-taker 3 access, and so on. Every comment fit one, and sometimes more than one category. When we were done recording, I labeled each of the flip charts and told the workshop participants that they had just invented TRU Math. Figure 5 shows one of the flip charts, which was labeled (after the fact) as "cognitive demand."

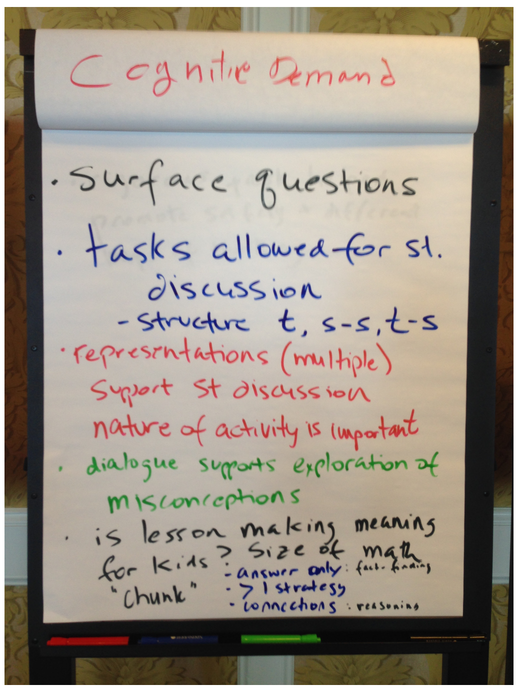

Figure 5. The flipchart corresponding to Cognitive Demand

The point was clear: all of the comments made by workshop participants fit comfortably into the TRU Framework. That substantiated our fundamental claim, that in a sense, TRU offered nothing new – no "silver bullets" or other radical suggestions for professional development. Rather, it *organized* much of what people knew into a coherent framework, and provided them with a perspective and a language for discussing instruction. Thus experience provides a base for subsequent TRU-focused professional development. Three versions of such PD follow.

4.2 Video work with pre-service teachers.

The University of California at Berkeley offers a pre-service teacher preparation program called MACSME, in which students can earn a Masters and Credential in Science and Mathematics Education at the secondary level – see https://gse.berkeley.edu/cognition-development/macsme. MACSME is now TRU-based.

At the very beginning of the program students experience the workshop described above. In their seminars on teaching methods, they discuss videos of practice using the TRU framework (and the tools discussed above) as a way of conceptualizing teaching practice. Students learn to observe lessons with the help of the observation guide, specifically with the perspective introduced in Figure 6, below – the framing being that one is to see lessons "through the eyes of the student."

Observe the lesson through a student's eyes	
The Content	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored?
Agency, Ownership, and Identity	• Do I get to explain, to present my ideas? Are they built on? • Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	• Do classroom discussions include my thinking? • Does instruction respond to my thinking and help me think more deeply?

Figure 6. The lens through which pre- and in-service teachers are invited to view instruction

MACSME students oberve and discuss videotapes using this perspective. Their practice teaching is planned using the conversation guide, observed with the help of the observation guide, and videotaped. Discussions of and reflections on the videotapes make use of the TRU framework and tools. This is a core, ongoing practice in the program.

4.3 Video work with in-service teachers.

We begin our work with high school a mathematics department (or more generally, a professional learning community) with the video workshop described above. At a second workshop, teams of teachers each pick a TRU dimension and (cf. Figure 6) observe classroom videotapes through the lens of that dimension. (Collectively, the groups cover all five dimensions.) For the third workshop, each group plans instruction with a focus on its chosen dimension. Group members observe each other teaching and bring back artifacts (usually student work) for discussion. Once the PLC has developed a sense of trust and the norms for communal discussions of each other's work have been established, teachers plan collaboratively for lessons using the conversation guide. The lessons are videotaped and the teachers select clips from those videotapes for discussion by the collective. This becomes the modus operandi for ongoing collective work by the PLC.

4.4 Video work with coaches and districts.

We have collaborated with school districts and state and regional organizations on TRU-related professional development. These collaborations rely heavily on the use of videotapes (and the other tools discussed above). The project team is in the process of putting together a professional development package that replicates the opening workshop described above. This video workshop will be made available to a participating network of California school districts that serves more than 1.5 million students; the idea is that in each district, district staff can orient district personnel to the TRU framework by leading the workshop.

The city of Chicago has been conducting extensive TRU-related professional development, using videotapes of children from Chicago schools and discussing student performance using TRU-related tools. There are video sessions with teachers, and video sessions with administrators, so that the administrators learn to view instruction through the "TRU lens" and support instruction in that way. Chicago (which houses 660 public schools) has just begun a summer institute for new mathematics coaches, where more than 60 mathematics coaches are introduced to TRU through video.

A large network of schools across the U.S. has been using materials produced by the project, their work having begun with the Formative Assessment Lessons described above. Subgroups of that network are now using TRU-based video as a way of planning for and reflecting on those lessons. In May 2016 the author gave an intensive video-based professional development course for mathematics coaches across the state of Arkansas. That workshop, which covered the full spectrum of TRU tools for teaching and professional development, was videotaped. The tapes are being converted into a video course for mathematics coaches. Similarly, the Southern Regional Education Board (see http://www.sreb.org/), which serves a large part of the southeastern United States, is planning video-based TRU seminars to enhance its professional development work.

4.5 Indications of impact

As noted above, project work as a whole includes a range of tools (the Formative Assessment Lessons, the Conversation Guide, the TRU Rubric, the Observation guide, etc.) and video-based workshops, all of which reflect and support the use of the TRU framework. To date approximately 6,000,000 Formative Assessment Lessons (FALs) have been downloaded from the Mathematics Assessment Project's web site, <http://map.mathshell.org/>. The Gates Foundation, which provided funding for the development of TRU and the Formative Assessment Lessons, commissioned independent evaluations of the FALs.

One study by the CRESST evaluation center evaluated the work of professional development efforts related to the FALs by the Mathematics Design Collaborative (MDC). Here is a summary of the CRESST findings:

> Participating teachers were expected to implement between four and six Challenges [Formative Assessment Lessons], meaning that students were engaged only 8-12 days of the school year.
>
> Nonetheless, the studies found statistically significant learning effects… the approximate equivalent of 4.6 months. (Herman, Epstein, Leon, La Torre Matrundola, Reber, & Choi, 2014, p. 10)

These data raise the question of how so few lessons (an 8-12 day intervention) could have so large an impact (4.6 months of mathematical growth). Our conjecture is that the FALs, combined with the professional development supporting them, scaffolded significant changes in the teachers' pedagogy, even when the teachers were using their standard curriculum. Some evidence in favor of this conjecture was obtained in a dissertation study by Kim (2015), which made heavy use of video analysis of teacher practices during instruction using the FALs and during regular instruction. This conjecture is also given indirect support in a policy brief published by Research For Action (2015), which obtained survey responses from more than 600 teachers. Among the findings in the policy brief:

- 94% of the teachers surveyed said that "MDC Classroom Challenges [FALs] are effective in providing a curricular resource for teachers in addressing the CCSS." (p. 3)
- 98% of the teachers surveyed "agreed that the teacher taking on the role of facilitator coach strengthens students' mathematical understanding." (p. 3)
- 85% of the teachers surveyed "agreed that using the MDC Classroom Challenges raised their expectations for students' mathematical work." (p. 4)
- 91% of the teachers surveyed agreed that "using the MDC Classroom Challenges helped me create an environment that promotes mathematical discourse." (p. 4)
- 85% of the teachers surveyed agreed that "I use MDC strategies during non-MDC instruction." (p. 5)
- 87% of the teachers surveyed agreed that "using the MDC Classroom Challenges has helped me learn new ways to include formative assessment in my classes" (p. 5)
- 65% of the teachers surveyed agreed that "MDC Classroom Challenges help me differentiate instruction for ELL students"; 80% for struggling students; and 96% for advanced students. (p. 6)
- 95% of the teachers surveyed agreed that "MDC Lessons are effective in making instruction more engaging to students." (p. 8)

All of these statements relate to the core values of the TRU framework. The results are heartening, but as a researcher I have some concerns about questionnaire-based research and about relying on teacher perceptions. My research group currently has some much more fine-grained studies under way. These, by looking closely at videotapes of classroom practices, suggest some clear advances in pedagogy (e.g., increased use of questions that demand more than a few words in answer, more student time giving explanations, and improved presentations) but also reveal some challenges (e.g., a loss of some disciplinary control when the class moves from teacher-centered to student-centered, or teachers choosing to focus on one aspect of their practice and not improving on others).

I have mentioned the use of video-based TRU work in Arkansas and by SREB. In addition, there are two major organizations of school districts in California. The California Office to Reform Education (CORE) districts describe themselves as follows (see http://coredistricts.org/):

> CORE is a nonprofit organization that seeks to improve student achievement by fostering highly-productive, meaningful collaboration and learning between its 10 member school districts: Clovis, Fresno, Garden Grove, Long Beach, Los Angeles, Oakland, Sacramento, San Francisco, Sanger and Santa Ana Unified. Together these districts serve more than one million students and their families.

The Math-in-Common districts overlap with the CORE districts. In addition to Garden Grove, Long Beach, Oakland, Sacramento, San Francisco, Sanger and Santa Ana Unified, they include Dinuba, Elk Grove, and Oceanside Unified. They describe themselves as follows (see http://collaborate.caedpartners.org/display/MICBooth/Welcome+to+the+Math+in+Common+Homepage):

> Ten school districts in California have been awarded significant support from the S.D. Bechtel, Jr. Foundation to help the transition to the Common Core State Standards in Mathematics (CCSS-M) in grades K-8. These districts will become leaders in the statewide transition to the CCSS-M, and participate in a learning community where they share plans, lessons learned, and tools with other grantee districts and statewide.

Both the CORE districts and the Math-in-Common districts have adopted the TRU Framework as their organizing framework for professional development in mathematics. An ongoing project is developing video-based sets of tools, including the workshop discussed above, to enable those districts to provide video-based professional development aligned with TRU to both district-level personnel and mathematics teachers.

Finally, as mentioned above, the city of Chicago has conducted intensive video-centered professional development work using TRU as its focus. According to Jessica Mahon, Director of mathematics for the Chicago Public Schools, that work has deeply permeated the district, and had powerful results. Mahon reported (personal communication, January 10, 2016) that for the first time in her experience, first grade teachers, eighth grade teachers, and school principals were able to communicate with each other using the same language about classrooms – the five dimensions of TRU. In a conference earlier this year, Brownell, Mahon, and Steward (2016) presented videos of TRU-based classrooms and videos of teachers and administrators using those videos as a mechanism for professional growth. They reported that Chicago test scores in mathematics had gone up, while those in the state of Illinois had gone down.

4.6 Plans for video development use, and study

TRU video work is still in its early phases. We have plans for additional tools and documentation over the coming years.

In Berkeley's teacher preparation program, we will be having students create video portfolio documentation of their teaching, annotating their videos using the TRU framework. These video records will also provide data that will better enable us to understand beginning teachers' evolving conceptions of what it means to teach and their capacity to do so.

In collaborative work with the SERP Institute (see http://serpinstitute.org/), Catherine Lewis from Mills College, the Oakland, California, Unified School District (OUSD), we are working on a synthesis of the TRU framework with key aspects of lesson study. Beginning in one high school, and ultimately at all high schools in the district, we plan to develop, study, and "package" a set of TRU-related resources for general use, including video-based resources for collaboration and for lesson study. Here too, video data will provide opportunities to begin to chart teachers' developmental trajectories.

5. Conclusion

Two things have remained constant throughout my career: my wish to understand mathematical thinking, teaching, and learning in ways that would support as many students as possible in becoming powerful mathematical thinkers and problem solvers, and my desire to be as "close" to those phenomena as possible, in order to best understand and support them.

In the 1970s we had yet to come to grips with the very processes entailed in thinking and problem solving. For example, terms such as metacognition and belief systems had not yet become part of our working vocabulary. Over the ensuing decades, the field gained much greater clarity about individual problem solving, about teachers' decision making, and about the attributes of productive learning environments and how to support them.

Throughout all of this, video has played a fundamental role. It goes without saying that making video recordings, like every other type of documentation, is an act of selection, and not an "objective" record of all that happened. At the same time, videos offer a particularly rich window into the phenomena we explore. In my work on problem solving and decision making – and prospectively, my research on teachers' developmental trajectories – videos enabled me to watch what was happening myriad times, until I thought I had an idea of what was taking place; then I could try to model what was visible in those videotapes, to see if my nascent understandings were "close." In my work with teachers, videos convey the immediacy of classroom actions in ways that support rich conversations about the nature of productive learning environments. I can no more, today, predict what the precise focus of my research and development will be in five years than I could make such predictions 40 years ago – but I can predict that video will play a fundamental role in those efforts, as I seek to understand and support the kinds of teaching that enable students to become powerful thinkers and problem solvers.

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