Problem Solving Lessons
Do I stand back and watch, or intervene and tell them what to do?
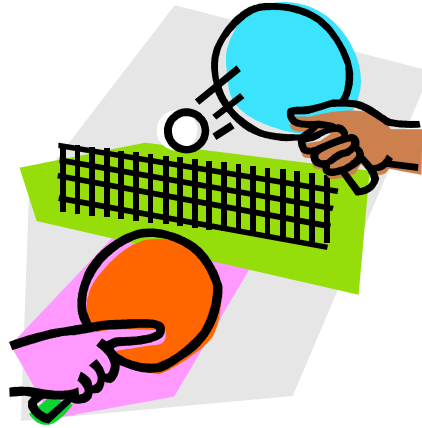### HANDOUTS FOR TEACHERS

Contents

Handout 1: Structured problems

Organizing a table tennis tournament

You have the job of organizing a table tennis tournament.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.

1. Call the players A, B, C, D, E, F, G
   Complete the list below to show all the matches that need to be played.

   A v B   B v C   ....
   A v C   B v D   ....
   ....

2. There are four tables at the club and each game takes half an hour.
   The first match will start at 1.00pm.

   Copy and complete the poster below to show the order of play,
   so that the tournament takes the shortest possible time.
   Remember that a player cannot be in two places at once!
   You may not need to use every row and column in the table.

| Start Time	Table on which the game will be played			
	1	2	3	4
1.00	A v B			
1.30				
2.00				
2.30				
3.00				
3.30				
4.00				
4.30				

Handout 1: Structured problems (continued)

Designing a box for 18 sweets

You work for a design company and have been asked to design a box that will hold 18 mints.
Each mint is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of card with as little cutting as possible.

On the grid paper below, show clearly how the card can be folded up and glued together to make the box.
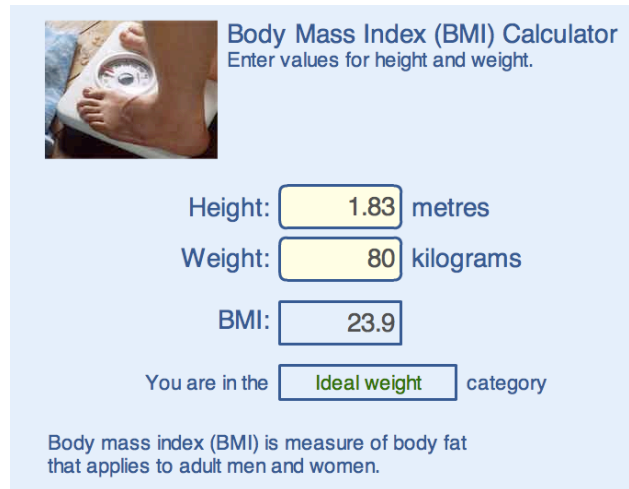
Make your box to check.
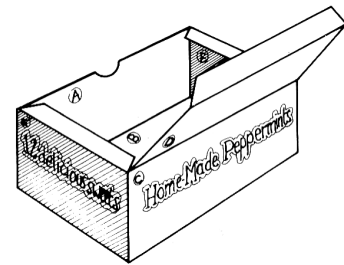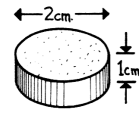
Handout 1: Structured problems (continued)

Calculating Body Mass Index

This calculator is used to help adults find out if they are overweight.

Body Mass Index (BMI) Calculator
Enter values for height and weight.

Height: 1.83 metres
Weight: 80 kilograms
BMI: 23.9
You are in the **Ideal weight** category

Body mass index (BMI) is measure of body fat that applies to adult men and women.

1. Fix the height at 2 meters - a very tall person!
   Complete the table below and draw a graph to show your results.

Weight (kg)	60	70	80	90	100	110	120	130	140
BMI									

   (a) What is the largest BMI for which someone is underweight?
   (b) What is the smallest BMI for which someone is overweight?
   (c) When you double the weight, what happens to the BMI?
   (d) Can you find a rule for calculating BMI from the weight?

2. Fix the weight at 80 kilograms and try varying the height.

   (a) When you double the height, what happens to the BMI?
   (b) Can you find a rule for calculating BMI from the height?
   (c) Draw a graph to show the relationship between the height and the BMI.

Note for students: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

Handout 2: Unstructured versions of the problems

Organizing a table tennis tournament

You have the job of organizing a table tennis league.

- 7 players will take part.
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.
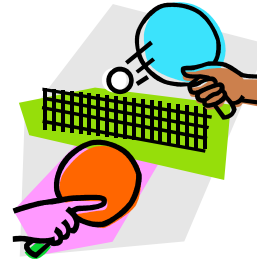
Plan how to organize the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do.

Designing a box for 18 sweets

You work for a design company and have been asked to design a box that will hold 18 sweets.
Each sweet is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of card with as little cutting as possible.
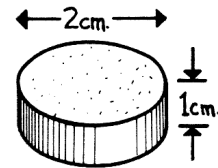
Compare two possible designs for the box and say which is best and why.

Make your box.

Calculating Body Mass Index

This calculator shown is used on websites to help an adult decide if he or she is overweight.
What values of the BMI indicate whether an adult is underweight, overweight, obese, or very obese?

Investigate how the calculator works out the BMI from the height and weight.

Body Mass Index (BMI) Calculator
Enter values for height and weight.

Height: 1.83 metres
Weight: 80 kilograms
BMI: 23.9
You are in the Ideal weight category

Body mass index (BMI) is measure of body fat that applies to adult men and women.

Note for students: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

Handout 3: Notes on the unstructured problems

Organizing a table tennis tournament

Students should quickly notice that it is impossible to use all four tables simultaneously as there are only seven players. On each occasion, therefore someone has to rest. One possible way of organizing the matches is shown below.

Start time	Table 1	Table 2	Table 3	
1.00	AvB	CvD	EvF	G rests
1.30	CvA	EvB	GvD	F rests
2.00	EvC	GvA	FvB	D rests
2.30	GvE	FvC	DvA	B rests
3.00	FvG	DvE	BvC	A rests
3.30	DvF	BvG	AvE	C rests
4.00	BvD	AvF	CvG	E rests

This solution was obtained by writing all the players' names on scraps of paper and placing them next to the three tables as shown. Every half an hour the players move one place clockwise. In this way each player plays against all the others once. It is also 'fair' in other ways; each player plays on each side of each table exactly once. Notice that if there were 8 players, the matches would not take any longer. The additional player could play the resting player.

A [ ] B
C [ ] D
E [ ] F
G rests
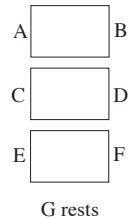
Designing a box for 18 sweets

18 sweets may be arranged in different ways. For example:
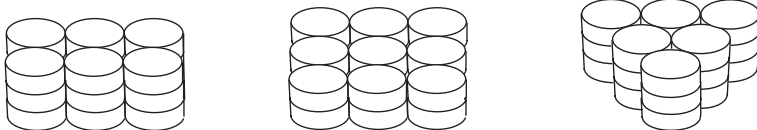
Each arrangement will lead to a different box design. Their dimensions may be calculated theoretically, or a more concrete approach may be adopted by drawing round sweets with appropriate dimensions. Furthermore any given design may be constructed from card in several different ways. Some possible box designs are illustrated below:
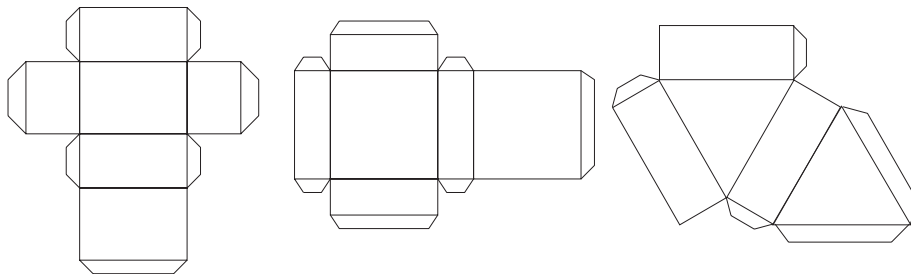
Handout 3: Notes on the unstructured problems (continued)

Calculating Body Mass Index

It is easy to find the boundaries at which someone becomes underweight/overweight/obese if one variable is held constant while the other is varied systematically. The boundaries occur at:

	BMI
Underweight	Below 18.5
Ideal weight	18.5 - 24.9
Overweight	25.0 - 29.9
Obesity	30.0 and Above

In order to find out how the calculator works, it is better to forget realistic values for height and weight and simply hold one variable constant while changing the other systematically. For example, if students hold the height constant at 2 meters (not worrying if this is realistic!), then they will obtain the following table and/or graph:

Weight (kg)	60	70	80	90	100	110	120	130
BMI	15	17.5	20	22.5	25	27.5	30	32.5
	Underweight		Ideal weight		Overweight		Obese	

From this it can be seen that there is a proportional relationship between weight and BMI. (If you double weight, you double BMI; Here BMI = Weight/4)

If they now hold the weight constant and double the height, they will see that the BMI decreases by a factor of 4. This is an inverse square law, which may be outside the experience of many students. They may be able to explore the relationship by graphing, however.

So, if the BMI is proportional to weight and inversely proportional to the square of the height, it makes sense to try the relationship BMI = k x (weight)/(height)^2. The result is that k = 1.

Handout 4: Practical advice for teaching problem solving

Allow students time to understand and engage with the problem Discourage students from rushing in too quickly or from asking you to help too soon.	• *Take you time, don't rush.* • *What do you know?* • *What are you trying to do?* • *What is fixed? What can be changed?* • *Don't ask for help too quickly - try to think it out between you.*
Offer strategic rather than technical hints Avoid simplifying problems for students by breaking it down into steps.	• *How could you get started on this problem?* • *What have you tried so far?* • *Can you try a specific example?* • *How can you be systematic here?* • *Can you think of a helpful representation?*
Encourage students to consider alternative methods and approaches Encourage students to compare their own methods.	• *Is there another way of doing this?* • *Describe your method to the rest of the group* • *Which of these two methods do you prefer and why?*
Encourage explanation Make students do the reasoning, and encourage them to explain to one another.	• *Can you explain your method?* • *Can you explain that again differently?* • *Can you put what Sarah just said into your own words?* • *Can you write that down?*
Model thinking and powerful methods When students have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	• *Now I'm going to try this problem myself, thinking aloud.* • *I might make some mistakes here - try to spot them for me.* • *This is one way of improving the solution.*